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PARAMETRIC IDENTIFICATION OF
NONLINEAR STOCHASTIC SYSTEMS APPLIED
TO OCEAN VEHICLE DYNAMICS

by

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Prof. M. A. Abkowitz (Supervisor)

August 20, 1971

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by

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PARAMETRIC IDENTIFICATION OF NONLINEAR STOCHASTIC SYSTEMS APPLIED TO OCEAN VEHICLE DYNAMICS

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Michael Ney Hayes

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ABSTRACT

This thesis shows that the techniques of model reference contouring and extended Kalman filtering are valid and useful for identifying parameters in ocean vehicle nonlinear dynamic mathematical models using noisy input-output data. The studies begin with the development of general, multiple degree of freedom, state space, nonlinear vector differential equation mathematical models for the overall motion of ocean vehicles in response to their effectors. These models are extended to include fluidic memory effects and higher order derivatives. Next, the specific equations for the two identification techniques are presented, and their use on ocean vehicle models is explained in great detail. Then the two identification techniques are applied to the six single-degree-of-freedom equations of motion for the Deep Submergence Rescue Vehicle (DSRV) for a large number of different types of input functions and noise characteristics. After presenting the detailed equations and Fortran IV subroutines for the 6 degree-of-freedom DSRV mathematical model, several selected coefficients are identified using 1, 2, 3, 4, 5, and 6 degree-of-freedom simulated sea trials and the results are discussed and compared. Finally a complete listing is provided of all of the identification computer programs used, and a bibliography of 276 references from the areas of system identification and ocean vehicle dynamics is included.

Thesis Supervisor: Martin A. Abkowitz

Title: Professor of Naval Architecture

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THESIS FORMAT AND MNEMONICS

SECTION	MNEMONIC LETTER	TITLE
0	T	TABLE OF CONTENTS AND ARRANGEMENT OF THESIS
1	I	INTRODUCTION
2	M	MATHEMATICAL MODELING OF OCEAN VEHICLE DYNAMICS
3	N	PARAMETRIC IDENTIFICATION OF NONLINEAR STOCHASTIC SYSTEMS
4	P	PARAMETRIC IDENTIFICATION OF OCEAN VEHICLE DYNAMICS
5	D	THE DSRV MATHEMATICAL MODEL
6	C	IDENTIFICATION STUDIES OF THE DSRV DYNAMIC COEFFICIENTS
7	S	SUMMARY AND CONCLUSION
8	A	APPENDICES
9	B	BIBLIOGRAPHY AND BIOGRAPHY

ALL CHAPTERS, SUBSECTIONS, TABLES, ILLUSTRATIONS, AND EQUATIONS IN THIS THESIS ARE IDENTIFIED IN THE ORDER:

MNEMONIC LETTER   CHAPTER NUMBER   .   ORDINAL NUMBER   .

FOR EXAMPLE: EQUATION M1.1 IS LOCATED IN SECTION ~~1~~ 2, CHAPTER 1 AND IS THE FIRST EQUATION IN THAT CHAPTER. REMEMBERING THE MEANING OF M, THE READER KNOWS, WITHOUT LOOKING IT UP, THAT M1.1, WHEREVER IT APPEARS, REFERS TO THE MATHEMATICAL MODELING SECTION OF THE THESIS.

ALL BIBLIOGRAPHY REFERENCES ARE IN PARENTHESIS AND ARE IN THE ORDER:

(FIRST LETTER OF AUTHOR'S LAST NAME - SEQUENCE NUMBER)

SECTION 0

TABLE OF CONTENTS AND ARRANGEMENT (T)

"I HAVE FOUND YOU AN ARGUMENT; I AM NOT OBLIGED TO FIND YOU AN UNDERSTANDING." - SAMUEL JOHNSON (1709-1784)

THE CENTRAL PURPOSE OF THIS THESIS IS THE DEVELOPMENT, PRESENTATION, DSRV UTILIZATION, AND ANALYSIS OF TECHNIQUES FOR THE IDENTIFICATION OF PARAMETERS IN OCEAN VEHICLE DYNAMIC MATHEMATICAL MODELS. THIS SECTION PRESENTS THE DETAILED PAGE STRUCTURE AND NOMENCLATURE USED. THE READER'S ATTENTION IS ESPECIALLY DIRECTED TO CHAPTER T2 WHICH DETAILS SOME OF THE USUAL AND UNUSUAL CONVENTIONS ADOPTED IN THIS THESIS AND PROVIDES AN EXPLANATION OF THE SYMBOLS, LANGUAGE AND TERMINOLOGY USED. THE NEXT SECTION PROVIDES A GENERAL DESCRIPTION OF THE RESEARCH IN THIS THESIS AND THEN SHOWS HOW THESE STUDIES RELATE TO THE GENERAL AREA OF MOTION CONTROL OF OCEAN VEHICLES.

TABLE OF CONTENTS

CHAPTER T2

CONVENTIONS USED IN THIS THESIS

This chapter lists in a somewhat random order a number of explanations of the symbols and language to be used throughout this thesis for the convenience of the reader. The ideas and research presented in this thesis run from the very general to the very specific.

T2.1 SYMBOLS USED

$\dot{x}$	dot over a letter means its time derivative d/dt
$\underline{x}$	underlined letter means it is a column vector
$\bar{x}$	overlined letter refers to its mean value, $E[x] = \bar{x}$
x	letter x can stand for x as a variable, a function x(t), or a number; x also refers to forward distance traveled by a vehicle (ft.)
A	capital letters in equations represent either specific vehicle coefficients or general matrices
A^T	indicates the transpose of the matrix A
A^{-1}	indicates the inverse of the matrix A
A^{-T}	indicates the inverse of A^T
$E[x]$	refers to the mean value of the random variable x
E	refers to the error covariance matrix used in Kalman filtering
u	vehicle surge (forward) velocity (feet/second) relative to a coordinate system moving with the vehicle; $u = x_1$ in the general state vector $\underline{x}$
u	vehicle input function such as propellor revolutions per second; u is used as an input variable in all state space

equations where the vehicle surge velocity is called x_1.

v vehicle sway (sidewise) velocity (ft/sec); $v = x_2$ in $\underline{x}$

v measurement noise input function for a state space ocean vehicle model, $E[v] = 0$; $v_n = v(t_n)$ for discrete samples

w vehicle heave (vertical) velocity (ft/sec); $w = x_3$ in $\underline{x}$

w process noise or input noise to the vehicle for a state space ocean vehicle model, $E[w] = 0$; $w_n = w(t_n)$ for discrete samples

p vehicle roll velocity (radians/sec); $p = x_4$ in $\underline{x}$

p general equation parameter in the state space mathematical model of an ocean vehicle

q vehicle pitch velocity (radians/second); $q = x_5$ in $\underline{x}$

q error covariance of the discrete process noise w_n

r vehicle yaw velocity (radians/second); $r = x_6$ in $\underline{x}$

r error covariance of the discrete measurement noise v_n

$\underline{x}$ general ocean vehicle state vector which may or may not have the vehicle parameters included within it

$\underline{u}$ general ocean vehicle input vector for state space models

$\underline{v}$ measurement noise input vector for state space models;

$E[\underline{v}(t)\ \underline{v}^T(t+\tau)] = R_c\ \delta(\tau)$; $R \approx R_c/\delta t$ for discrete noise

$\underline{w}$ vehicle input noise vector for state space models;

$E[\underline{w}(t)\ \underline{w}^T(t+\tau)] = Q_c\ \delta(\tau)$; $Q \approx Q_c\ \delta t$ for discrete noise

$\delta(\tau)$ Dirac delta function ; (B-7, pages 330-333)

R discrete measurement noise error covariance; used in simulations

Q discrete process or input noise error covariance; used in simulations

$\underline{p}$ vehicle parameter or coefficient vector; may be included or augmented into $\underline{x}$; all coefficients are parameters, but some vehicle parameters are not considered to be coefficients

$\underline{z}$ vehicle output vector for state space models

t time, seconds

$\underline{f}$ system structure vector in state space models

$\underline{h}$ measurement structure vector in state space models

$\underline{g}$ parameter structure vector in state space models

(x,y,z,ϕ,θ,ψ) ocean vehicle linear and angular coordinates or distances in feet or radians

(X,Y,Z,K,M,N) ocean vehicle forces and moments (sometimes called simply forces)

(u,v,w,p,q,r) ocean vehicle six degrees of freedom in linear and angular velocities; see Figure T2.1

$\underline{I}_x$ moment of inertia vector or matrix, $\underline{I}_x = (I_{xx}\ I_{yy}\ I_{zz})^T = (I_{x_1}\ I_{x_2}\ I_{x_3})^T$

i=1,6 comma designates intermediate values, i = 1,2,3,4,5,6

i,j=1,6 means i = 1,6 and j = 1,6

I the identifiability of a parameter or coefficient

M the modelability of an ocean vehicle

$\underline{X}$ vehicle forces and moments, $\underline{X} = (X\ Y\ Z\ K\ M\ N)^T$

C.G. center of gravity, $\underline{x}_G = (x_G\ \ y_G\ \ z_G)^T = (x_{1G}\ \ x_{2G}\ \ x_{3G})^T$

C.B. Center of buoyancy, $\underline{x}_B = (x_B\ \ y_B\ \ z_B)^T = (x_{1B}\ \ x_{2B}\ \ x_{3B})^T$

m mass of the ocean vehicle (slugs)

$\underline{X}_{ij}$ vector of the ij'th force and moment second degree coefficients; these coefficients correspond to those used in the literature (A-1) (S-9)

C Scalar cost functional, $C \geq 0$

e	measurement error
$\hat{x}$	estimated value of the variable x
$\hat{x}'$	next time increment estimate of the variable x
δt	time increment
D	dependent functions in the identifiability functional I(D)
S	system structure; structure in this thesis always refers to mathematical equation structure and never to the physical structure of a vehicle
K	Kalman filter gain matrix
$\underline{p}^*$	the optimum or best value of the parameter vector $\underline{p}$
σ	standard deviation of a gaussian random variable
ϵ	symbol meaning "is an element of"
$\{ \mid \}$	braces defining a set of numbers
N(a,b)	normally distributed or gaussian random variable of mean a and variance b; if a = 0, σ^2 = b in this notation; also written a $\pm$ σ
τ	equation time constant (seconds)
R	in a table indicates a relative minimum as opposed to an absolute minimum
θ	angular frequency of a sine wave, $\theta = 2\pi$/period
$\underline{X}_{grav}$	gravity forces and moments for an ocean vehicle
$\underline{X}_{sec}$	secondary drag forces and moments for the DSRV, Appendix A3
$\underline{X}_{cons}$	constant forces and moments for an ocean vehicle
$\underline{X}_{dist}$	disturbance (noise) forces and moments for an ocean vehicle
$\underline{\dot{x}}'$	untransformed DSRV accelerations
X_{ij}^k	the k'th second degree coefficient in $\underline{X}_{ij}$

(k,i,j) the location of X_{ij}^{k} in a storage vector for computation

F gradient matrix of an ocean vehicle, $F = \partial \dot{\underline{x}} / \partial \underline{x}$

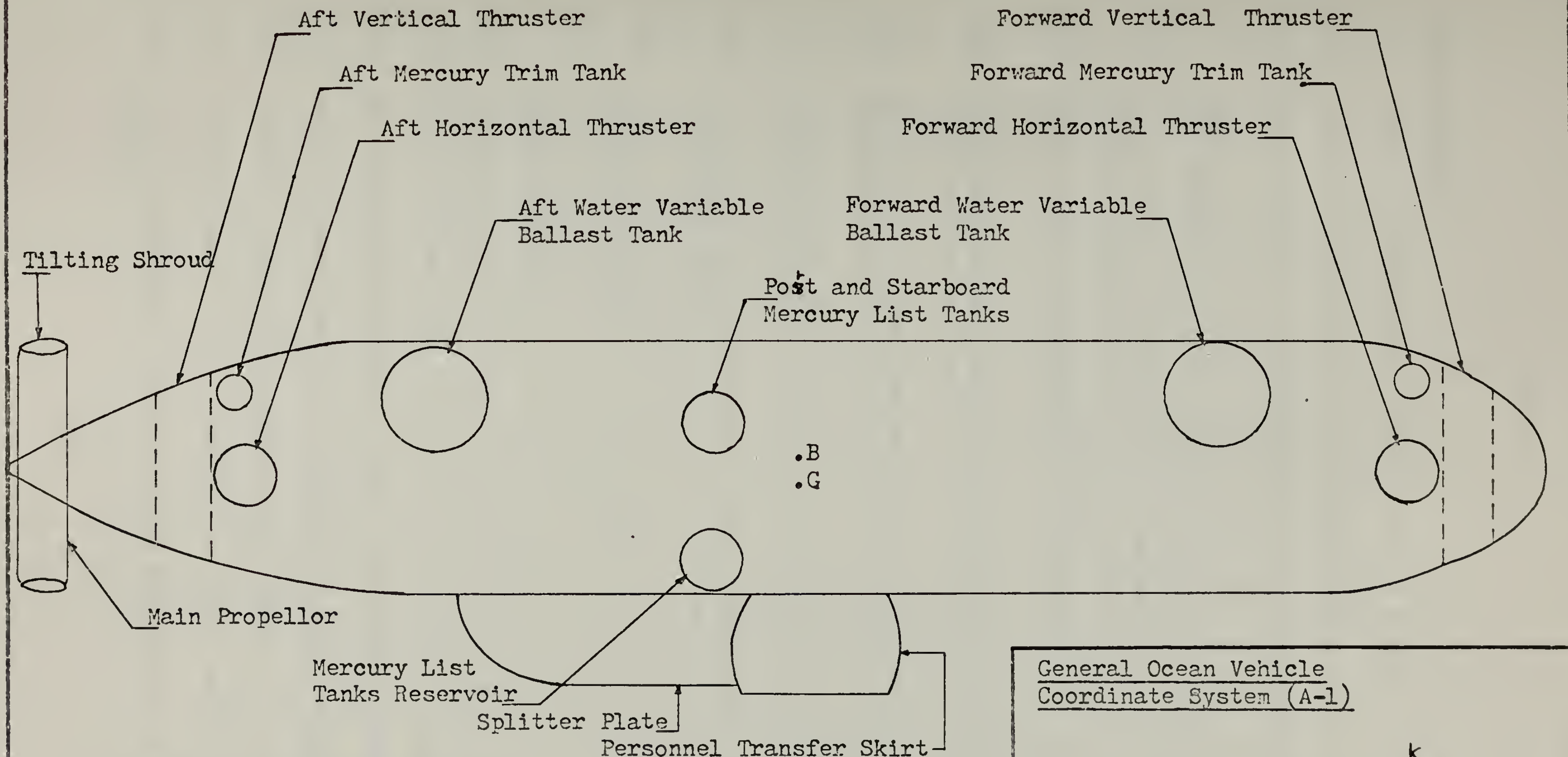
$a \pm \sigma$ mean $\pm$ standard deviation of a gaussian random variable, $N(a, \sigma^2)$

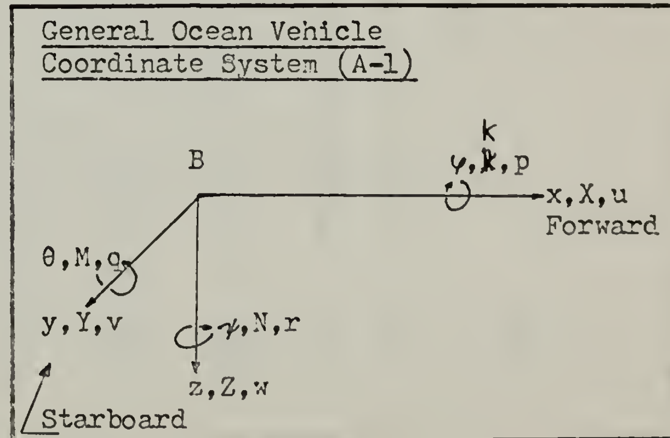
T2.2 LANGUAGE USED

parameters	general variables in the mathematical model equations; may be states, constants, or time variables
coefficients	ocean vehicle parameters which result from the Taylor series expansion of the hydrodynamic forces and moments
noise	random variable, stochastic process, unknown or uncertain input
and	A and B means both A and B
or	A or B means either A or B or both A and B; inclusive or
either...or	A or B but not both; exclusive or
neither...nor	means not A and not B
dynamics	the six primary velocities of an ocean vehicle; $(u\ v\ w\ p\ q\ r)^T$, unless otherwise stated
motion	" " "
behavior	" " "
overall motion	" " "
order	the highest derivative in a differential equation
degree	the power of the highest derivative in a differential equation or the power to which a term is raised in a polynomial

degree of freedom	one of the six primary velocities of an ocean vehicle
Chapter P3.2	refers to subsection P3.2 of Chapter P3
EKF	extended Kalman filter
DSRV	Deep Submergence Rescue Vehicle, Figure T2.1

DSRV STARBOARD (RIGHT) SIDE VIEW (B-9)





Characteristics of the DSRV

Length overall = 48.8 ft.
Weight (Equil.) = 140369 lbf.
z_{BG} = .1335 ft.
Cylindrical Diameter ≈ 8 ft.

Figure T2.1 THE DSRV AND A GENERAL COORDINATE SYSTEM

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LIST OF ILLUSTRATIONS PAGE

SECTION 1

INTRODUCTION (I)

"I DO NOT UNDERSTAND; I PAUSE; I EXAMINE." - MICHEL DE MONTAIGNE (1533-1592)

THE PURPOSE OF THIS SECTION IS TO PROVIDE THE READER WITH A GENERAL PERSPECTIVE FROM WHICH TO VIEW THE STUDIES AND RESULTS OF THIS THESIS. A VERY GENERAL DISCUSSION OF THE CENTRAL ISSUES INVESTIGATED IS PROVIDED, AND A BRIEF SUMMARY OF SOME OF THE RESULTS AND CONTRIBUTIONS IS GIVEN. THIS SECTION ANSWERS THE FOLLOWING QUESTIONS:

WHAT IS THE PROBLEM?

HOW IS IT SOLVED?

HOW IS ITS SOLUTION USEFUL?

WHY IS ITS SOLUTION NECESSARY?

WHAT MAKES IT DIFFICULT TO SOLVE?

HOW DOES THE SOLUTION RELATE TO THE REAL WORLD?

THE NEXT SECTION BEGINS THE ACTUAL THESIS STUDIES BY DEVELOPING STATE SPACE MATHEMATICAL MODELS FOR THE DYNAMIC MOTIONS OF OCEAN VEHICLES.

CHAPTER II

GENERAL ORIENTATION TO THE PROBLEM AREA

The studies and results presented in this thesis apply to the general areas of motion control and design of ocean vehicles. Both the motion control and the design of such vehicles are greatly facilitated by the capability of quantitatively predicting the dynamic behavior of the vehicle under the influence of its effectors. One way of developing this predictive capability is through the use of mathematical models and computer simulations which, in essence, run a mathematical version of the vehicle in the computer rather than running the actual vehicle in water. The computerized version of the vehicle allows cheaper, faster, and more flexible studies to be made of its motion characteristics than can be made with the full-scale vehicle or a physical model of it. However, it is important that the mathematical model behavior accurately represent the real vehicle behavior in order for the computer simulations to be useful in control or design of the real vehicle.

The studies in this thesis apply to the specific problem of determining accurate mathematical models for vehicle motion behavior. Such models are determined here by applying the techniques of parametric identification, from Modern Control Theory, to noisy input-output data from simulated vehicle maneuvers and to mathematical model differential equation structure, from Hydrodynamics, in order to evaluate the undetermined portions of the model. These techniques may be used both to evaluate the model parameters and to provide quantitative and qualitative measures of the mathematical model

accuracy using those parameters.

The mathematical model, once determined, may be used by the vehicle designer or by the vehicle control system. Such a model would be useful in the design process because design changes could be reflected directly in terms of the resulting vehicle motion characteristics changes. The model would be useful in the vehicle control system, within the vehicle during operation or outside of the vehicle during the design process, because its predictive capabilities would permit the control system to select the best vehicle effector inputs to carry out a specified maneuver.

There are many uncertainties involved in the process of developing mathematical models of vehicle behavior. Whenever vehicle inputs or outputs are measured by a data acquisition system, there is always some amount of noise present; and whenever the mathematical model structure is developed, there is uncertainty as to whether or not it will accurately represent the real vehicle. The parametric identification techniques used in this thesis are designed to operate on noisy input-output data and to give a "best" or "optimum" set of parameters in spite of the noise. Therefore, the identification methods applied in this thesis will facilitate both the design and the control of a vehicle in a noisy environment.

In the studies of this thesis a mathematically simulated ocean vehicle (specifically the DSRV, Deep Submergence Rescue Vehicle) with a fixed and known set of parameters is used to generate noisy vehicle input-output data. The same model structure, but with different or unknown parameters, is then used in the identification procedures in

attempts to redetermine the original or "true" set of parameters used in the data generation. The techniques are developed and utilized in such a way that not only are the parameters determined (identified) but they are also studied for the characteristics of ease and accuracy with which they may be determined (identifiability).

This combination of identifications and identifiability studies of vehicle parameters provides a "bridge" from the simulated vehicle to the real vehicle. If the "true" parameters are found again from noisy simulation data, if the real vehicle mathematical model differential equation structure is the same as that of the simulated vehicle, and if the parameters in the simulated vehicle are identifiable from the generated data, then the parameters found from noisy real vehicle data should be the "true" parameters for the real vehicle. The word "should" is the best that can be used in this logic chain because of the model and measurement uncertainties, because of the highly nonlinear nature of most ocean vehicle mathematical model structure, and because the proper inputs (sometimes unknown) must be used in many cases in order that the parameters be identifiable.

The specific layout of this thesis proceeds as follows (in brief):

Section 2(M) present the mathematical model equations,

Section 3(N) present the equations for the identification techniques,

Section 4(P) combine these equations for in-depth studies of simple models,

Section 5(D) present the DSRV 6 degree of freedom model,

Section 6(C) present selected parameter studies of the large DSRV model.

A diagramatic dependence chain of these thesis sections is shown in Figure II.1 in both number and mnemonic letter form. The general layout of the thesis is intended to run from very general ideas in

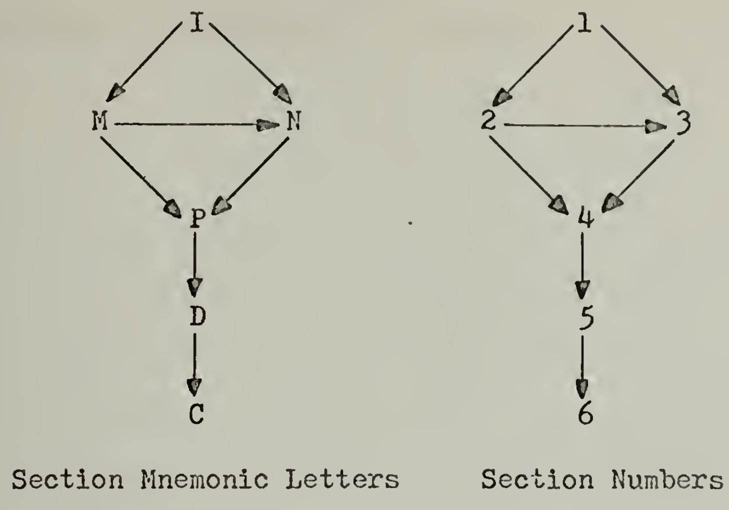


Figure II.1 SECTION RELATIONSHIP DIAGRAMS

the beginning sections to very specific ideas and evaluations in the later sections. The sections have been designed as completely independent blocks of ideas with interconnections being made through equation, figure, and chapter (subsection) references.

This chapter has presented a very brief introduction to the topics to be covered in this thesis and the order and structure in which these topics are to be discussed. The studies and results in this thesis apply to the general area of mathematically simulating the overall motion of ocean vehicles. The ultimate aims of mathematical simulation are to be able to quantitatively predict the vehicle dynamic behavior and to understand the specific features of the vehicle which cause it to behave as it does. This information shows the vehicle designer the results of his proposed vehicle directly referenced to its motion and allows the control system designer to use predictive control methods. The next chapter presents

a somewhat more detailed discussion of the parametric identification problem as it applies to this thesis.

CHAPTER 12

THESIS PROBLEM DESCRIPTION

The problem is to identify certain basic parameters in a mathematical model which simulates the dynamics of an ocean vehicle. To identify parameters means to determine their numerical values with sufficient accuracy to satisfy a stated criterion. The vehicle model to be used is a system of nonlinear first-order differential equations with some undetermined parameters or coefficients.

The ocean vehicle mathematical model, hereafter called the model, along with its set of identified parameters is designed to directly relate to a mathematically simulated vehicle. The simulated ocean vehicle, hereafter called the system, is designed to closely resemble a real ocean vehicle. A measure of closeness is chosen to specify the relationship between the model and the system, and the term "optimal" or "best" specifies the desired accuracy of the parameters.

The computer simulation of the vehicle then consists of solving the model differential equations for a given set of effector (propellor, rudder, etc.) inputs and taking input-output data measurements from the vehicle motions or maneuvers which result from those inputs. The uncertainties in the model structure, the inputs, and the measurements are simulated by adding noise, generated by the computer, to the model equations and to the measured data. The parameters used in the simulation are a fixed and known set of numbers.

This noisy data produced by the simulation is then used to identify the parameters in the mathematical model assuming that they are not known originally. The "best" set of parameters is determined

to be those which give the minimum closeness between the simulated vehicle and the mathematical model. This process of using the vehicle input-output data, mathematical model structure, and measure of closeness to determine the parameters in the model is called parametric identification.

Studies in parametric identification for a particular ocean vehicle must first begin with either the assumption or the determination that the parameters are, indeed, identifiable. For given input-output data, structure, and measure of closeness, the desired parameters may not be identifiable, and the assumption that they are identifiable will not be shown fallacious by the failure of a given procedure to find them. Identifiability studies are best made by using a fixed structure and measure of closeness and then varying the input-output data in the identification procedure.

It is in many cases both easier and less costly, for the purpose of studying identification techniques, to make these input-output data variations on a mathematically simulated ocean vehicle than on a physical model or full-scale vehicle because of the current capabilities of digital computers. For this reason it is desirable to use a simulated vehicle to study identifiability and to determine the type of data required for identification. The data from the physical model or full-scale vehicle can then later be used to identify parameters in the mathematical model.

In this thesis the Deep Submergence Rescue Vehicle (DSRV) is the particular vehicle studied for the purpose of identifying the parameters in its mathematical model. These parameters are first examined to determine their identifiability characteristics, and then they are

actually identified using several different kinds of input functions. Two identification techniques, model reference contouring and extended Kalman Filtering, are utilized to identify the parameters and to determine their capabilities for identification. A given set of DSRV parameters is used to simulate the real DSRV and to generate purposely noisy input-output data. Using this input-output data, studies are then made to identify the given set of DSRV parameters assuming that they are unknown or inaccurately known.

Identification of the DSRV parameters by the model reference contouring and extended Kalman filtering techniques requires extensive digital computation. The digital computer programs (FORTRAN IV) used for these identifications have been written for a very general class of ocean vehicles of which the DSRV is here used as an example. Because the computer programs are expected to have more general utility than for the DSRV, they are included and explained in this thesis.

This thesis is, in essence, the application of one body of knowledge to another. The basic structure of the mathematical models for ocean vehicles is derived from the Theory of Hydrodynamics. The techniques for parametric identification of dynamic systems are derived from Modern Control Theory. Lengthy derivations in each area are referenced, but only the basic equations to be used in this thesis are presented and discussed.

This chapter has presented a very general verbal statement of the problem to be considered in this thesis, parametric identification for ocean vehicle mathematical models, and the techniques to be

used to solve it, model reference contouring and extended Kalman filtering. The next chapter presents, by way of introduction, a verbal view of a general ocean vehicle as a black box (system) with inputs and outputs.

It is possible to approach system identification from either a microscopic or a macroscopic viewpoint (G-6). The microscopic viewpoint is one of identifying the system by determining the physical properties and basic laws of science which describe the behavior of the individual parts of the system and combining them. The next chapter presents the macroscopic or black box approach which uses the system's input-output information to determine its mathematical model. Most of the results of this thesis are developed using the macroscopic approach, and from that standpoint, they are most useful in the motion control of ocean vehicles. For the purpose of acquiring a complete understanding of the vehicle behavior, however, the microscopic viewpoint must eventually be used.

CHAPTER 13

THE OCEAN VEHICLE AS A BLACK BOX

For the purposes of simulation, control, and identification it is convenient and advantageous to view a general ocean vehicle as a black box with inputs and outputs. Simulation then answers the question "How does it behave?", control answers the question "How do we make it do what we want it to?", and identification answers the question "What's inside it?". Control often requires or is facilitated by simulation, and both control and simulation require identification in some form.

The subject of this thesis is identification, but it is often important to know how the identification is to be used in the overall system operation. Identification for control may have different objectives and accuracy requirements than identification for simulation. Identification for control might be an on-line parametric identification to allow the vehicle control system to adapt itself to changes in its environment. Identification for simulation might require extensive theoretical investigations to determine the model structure and then off-line data generation and processing to determine parameter identifiability or actual numerical values.

Ocean vehicles are usually only parts of larger systems which have been designed to complete specified missions in order to satisfy the goals of societies or individuals. In order to complete these missions, the vehicles must be controlled. Control requires knowledge of the behavior of the vehicle, and simulation and identification provide that knowledge. As the missions which are specified for vehicles in the ocean environment become more complex and sophisticated,

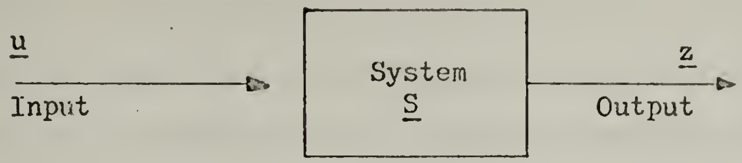
the precision with which the vehicle must be designed and controlled increases greatly. Very precise control of ocean vehicles often requires very accurate mathematical models, multiple degree of freedom control and modeling, and control systems which are capable of adapting to environmental and vehicular changes.

An ocean vehicle may be represented by the black box in Figure I3.1, where the input is designated by the vector $\underline{u}$ and the output by the vector $\underline{z}$. The sophistication of control systems increases in several stages. The simplest control system is the open loop system in which the input $\underline{u}$ is not dependent upon the output $\underline{z}$, but is merely a vector function of time. The next step upward in control system complexity is the closed loop system in which the input $\underline{u}$ is a function of both time and the output vector $\underline{z}$. In both open and closed loop configurations the system $\underline{S}$ may be partially known and partially unknown, but in neither case is its form alterable by the controller (Figure I3.1).

An adaptive controller is a major step above the open and closed loop controllers in complexity because limited and controllable system structural or parametric variations are introduced (H-2). The limited system variations are represented in Figure I3.1 by a parameter vector $\underline{p}$ which represents a set of system parameters which may be changed by the controller.

The adaptive controller can in a limited (by the number of elements in $\underline{p}$) way change both the input $\underline{u}$ to the system and some of the characteristics $\underline{p}$ of the system. In addition, some or all of the parameters $\underline{p}$ may be unknown and require identification by the controller. If the parameters are known, they may be treated as additional inputs

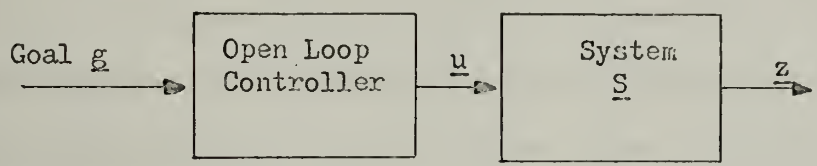
Black Box System



$z=\underline{S}(\underline{u})$ I3.1

S=System Functional

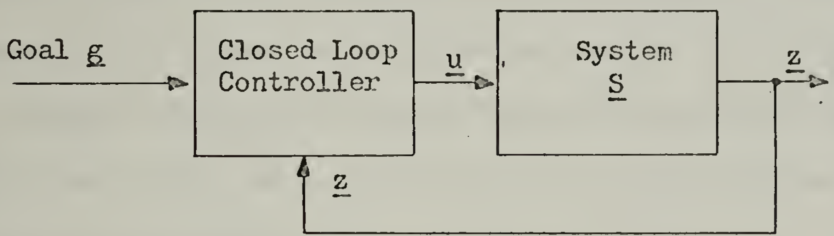
Open Loop Control System



$\underline{u}=\underline{C}(\underline{g})$ I3.2

$\underline{z}=\underline{S}(\underline{u})$ I3.3

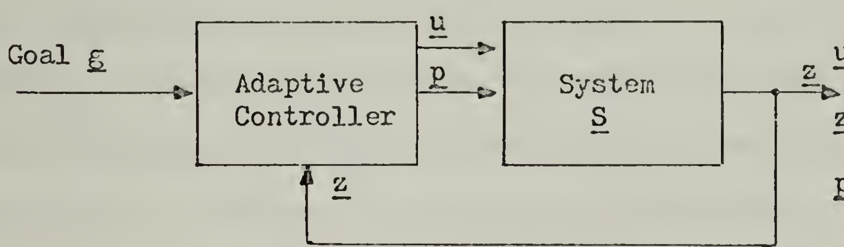
Closed Loop Control System



$\underline{u}=\underline{C}(\underline{g},\underline{z})$ I3.4

$\underline{z}=\underline{S}(\underline{u})$ I3.5

Adaptive Control System



$\underline{u}=\underline{C}(\underline{g},\underline{z},\underline{p})$ I3.6

$\underline{z}=\underline{S}(\underline{u},\underline{p})$ I3.7

$\underline{p}=\underline{A}(\underline{g},\underline{z},\underline{u})$ I3.8

Figure I3.1 GENERAL SYSTEM AND TYPES OF CONTROLLERS

to the system, and the adaptive controller is then in effect a closed loop controller. If the parameters are unknown, the adaptive controller must identify them, decide how to vary them to meet the goal, and modify the system by actually changing them and by varying the inputs. The change in the system is then reflected as a change or adaptation of the controller.

The past and present control systems used on ocean vehicles include all of these types: open loop, closed loop, and adaptive control systems. An example of open loop ship control would be a locked helm initially set to a desired course and then held fixed for a period of time. The loop could be closed in this control system by having a helmsman "feedback" information from the ship's compass and make helm changes to keep the ship on the desired course. In this case the helmsman becomes the closed loop controller.

Adaptive control has been primarily accomplished in the past by using human beings as the controllers on ocean vehicles. For example, the captain of a ship usually develops by experience a mental model of the ship's dynamic behavior in response to its inputs. He then uses this mental model to decide what bridge commands to issue in order to execute a maneuver, such as bringing the ship alongside a pier. If he has a two-screw ship and loses one propellor due to a casualty, he then "adapts" his mental model and control commands to execute the maneuver in spite of the casualty. The captain's acquisition of experience concerning the mental model of the ship's behavior may be likened to the process of identifying parameters in the vehicle mathematical model.

One of the most important aspects of adaptive control is that the model, whether it be a mental model in the captain's mind or a mathematical model in an on-board computer, permits the prediction of the response of the vehicle to a proposed input function. This predictive capability can be used to counteract the inherent time delays in many types of effectors, or it can be used to develop the best inputs to the vehicle to accomplish a desired maneuver.

This type of controller then suggests the idea of an ocean vehicle which contains on board a mathematical model of itself and the capability of identifying the parameters in that mathematical model using the input-output data of the vehicle itself. Any control system which employed the predictive capabilities of that mathematical model could then be made to automatically adapt to changes in the parameters of the vehicle and its environment. This form of control system is one of many possible uses to which the identification techniques of this thesis and the results of Modern Control Theory could be applied.

This chapter has presented a brief and general look at the ocean vehicle as a black box with input $\underline{u}$ and output $\underline{z}$. The types of control systems utilized for such systems were described and ocean vehicle examples of each type were discussed. Finally, the idea of a vehicle containing a model of itself which could be identified and used in the vehicle control system was described. The next chapter is an introduction to the general process of system identification.

CHAPTER I4

SYSTEM IDENTIFICATION

In its most general form system identification is the process of properly mathematically modeling the behavior of a given system. This means that it is the process of determining a set of variables and equations which will describe the performance of the system to a specified accuracy. The specified accuracy is usually determined by the ultimate purpose for which the system and the mathematical model are to be used.

For the studies in this thesis, system identification can quickly be specialized to input-output or macroscopic identification of stochastic state-determined dynamic systems (Section M). The most general form of this system is the continuous state-determined system in equations I4.1 and I4.2 which consists of a set of nonlinear first-order vector differential equations and a nonlinear vector measurement function.

Stochastic system identification for the dynamic system in equations I4.1 and I4.2 consists of finding $\underline{f}$ and $\underline{h}$ given $\underline{u}(t)$ and $\underline{z}(t)$ in the presence of the unknowns or noises $\underline{w}$ and $\underline{v}$. There are no general solutions to this problem nor are there any completely general techniques which may be used to attack it. Therefore, the next step is to assume or to develop from experience or theoretical studies the basic form of the two structure vectors $\underline{f}$ and $\underline{h}$ with the exception of a finite number of unknown or undetermined parameters $\underline{p}$. This step reduces the system identification problem to a parametric identification problem for which there exist techniques which may be

applied to specific systems.

$$\dot{\underline{x}} = \underline{f}\,(\underline{x}, \underline{u}, \underline{w}, t) \quad ; \quad \underline{x}\,(t_o) = \underline{x}_o \qquad \text{I4.1}$$

$$\underline{z} = \underline{h}\,(\underline{x}, \underline{u}, \underline{v}, t) \qquad \text{I4.2}$$

Where:

$\underline{x}$ = state vector, Chapter M2.1

$\underline{u}$ = input vector or control vector

$\underline{w}$ = system noise input vector

t = time scalar

$\bullet$ = time derivative d/dt

$\underline{f}$ = system differential equation structure vector

$\underline{h}$ = system measurement structure vector

$\underline{v}$ = measurement noise input vector

$\underline{z}$ = system output measurement vector

Section M in this thesis is designed to show that ocean vehicle mathematical models can be placed into the form of equations I4.1 and I4.2 with $\underline{f}$ and $\underline{h}$ known except for a parameter vector $\underline{p}$. Section N then presents the equations for the techniques which may be used to find the ocean vehicle parameters $\underline{p}$ from noisy input-output data. The model equations and the identification equations are then combined for the remainder of the thesis to show specifically how these techniques work for the DSRV.

CHAPTER I5

PRIMARY CONTRIBUTIONS OF THIS THESIS

The primary contributions of this thesis are to the area of motion control of ocean vehicles. The major contribution to that area is the verification that the techniques of model reference contouring and extended Kalman filtering are valid and useful for identifying parameters in mathematical models with several specific ocean vehicle characteristics and equation forms. This chapter presents, in a section by section format, those areas which the author believes are contributions to and extensions of the present theory of motion control of ocean vehicles.

Section 2(M) presents the rigid body and hydrodynamic equation structures for general ocean vehicles. These equations are then placed into a state-space form consisting of a single vector nonlinear differential equation in the vehicle primary states. The general state space equations in Chapter M2, their extension to include fluidic memory states in Chapter M3, and their extension to include higher derivative states of the primary variables in Chapter S2 are new and do not appear in the author's limited sampling of the literature in Chapter B1.

Section 3(N) presents the detailed equations and procedures for model reference contouring and extended Kalman filtering. These procedures come directly from the literature of Modern Control Theory and do not represent any contribution to the theoretical developments of that area. It has been mentioned (A-13) that the area of system identification needs specific applications and studies using its

available techniques so that better judgements may be made as to which techniques to select for a given problem. In this respect the application of model reference contouring and extended Kalman filtering to the specific DSRV models used in this thesis may provide this type of judgement information for someone considering these two techniques for his application. The equation simplifications in Chapter N3 for the extended Kalman filter when the parameters are constants and the modelability and identifiability considerations of Chapter N4 may also help in that application.

Section 4(P) is a detailed parameter study of the DSRV single degree of freedom equations using these two techniques. The results of the studies of this section show that these techniques work on these equations and give the details of how well they work. This knowledge represents a valuable contribution to the process of mathematically modeling the overall dynamic motions of ocean vehicles.

Section 5(D) presents the DSRV mathematical model, its gradient, and a description of the computer programs for calculating their numerical values. The DSRV mathematical model is taken from the literature, but the computer programs were developed specifically for the general ocean vehicle models of Section 2(M) with subroutines tailored specifically to the DSRV. The author hopes that these computer programs will be useful to someone whose ocean vehicle model requires them and fits their format. The manner of storing and addressing the DSRV second degree coefficients in Chapter D1 and the equations in Chapter D4 for use in identifying the A-matrix parameters are new to these applications.

Section 6(C) presents studies of several selected DSRV coefficients and parameters for multiple degree of freedom models to show that the parameters may be identified in more complicated ocean vehicle models than those of Section 4(P). The highly flexible computer programs for accomplishing this are discussed briefly concerning their structure-selectivity aspects and their utilization for general ocean vehicle models. As in Section 4(P) the studies of this section show that these techniques are valid for the DSRV mathematical model and as such represent some of the major contributions of this thesis.

Section 7(S) is a brief summary of the thesis and a description of a large number of areas for further research which may be conducted. Section 8(A) is a series of appendices containing numerical values, computer programs, and studies by the author of areas related to the research in this thesis. Section 9(B) is primarily a bibliography of 276 references from the areas of Ocean Engineering and Modern Control Theory. It is the author's intent that these three sections contain information which is ancillary to the main thesis research but useful for someone who desires to directly use the results of previous sections for his ocean vehicle investigations.

The purpose of this section has been to introduce the reader to the research and organization of this thesis by briefly and very generally discussing the areas to be covered. The next section begins these research studies by developing state space mathematical models for ocean vehicles.

SECTION 2

MATHEMATICAL MODELING OF OCEAN VEHICLE DYNAMICS (M)

"KNOWLEDGE IS MORE THAN EQUIVALENT TO FORCE." -

SAMUEL JOHNSON (1709-1784)

THE CENTRAL PURPOSE OF THIS THESIS IS THE DEVELOPMENT, PRESENTATION, DSRV UTILIZATION, AND ANALYSIS OF TECHNIQUES FOR THE IDENTIFICATION OF PARAMETERS IN AN OCEAN VEHICLE DYNAMIC MATHEMATICAL MODEL. THIS SECTION IS INTENDED TO PROVIDE A COMPLETE MATHEMATICAL MODELING FRAMEWORK, OF WHICH THE DSRV IS A SPECIAL CASE, IN ORDER THAT THE PARAMETRIC IDENTIFICATION TECHNIQUES OF THE NEXT SECTION MAY BE SHOWN TO APPLY GENERALLY TO SUCH OCEAN VEHICLE DYNAMIC MODELS. LATER SECTIONS THEN UTILIZE BOTH THE MODELS OF THIS SECTION AND THE IDENTIFICATION TECHNIQUES OF THE NEXT SECTION TO DETERMINE THE IDENTIFIABILITY OF SPECIFIC PARAMETERS FOR THE DSRV.

CHAPTER M1

GENERAL OCEAN VEHICLE DYNAMIC EQUATIONS

This section presents the development of the general state space equations for mathematically modeling overall ocean vehicle dynamic motions. It begins with a statement and referencing of the dynamic equations thus far developed in the literature, and then presents the details of their conversion to state space form. The peculiarities of ocean vehicle models such as nonlinearities, fluidic memory, measurement functions and partially unknown structure are then shown to fit the state space format and, therefore, to fit directly into the system identification problem discussed in Chapter I4. The section concludes with a discussion of the procedures to be followed in modeling uncertainties in mathematical model structure using stochastic processes.

M1.1 INTRODUCTION TO MODELING OCEAN VEHICLES

In order to understand the physical behavior of an ocean vehicle, man utilizes observations of the vehicle itself, a physical model of the vehicle, or a mathematical model of the vehicle. This understanding of vehicle behavior can then be utilized to help design, build, or operate the vehicle to meet its specified mission. Our concern here is with mathematical models to simulate overall motion, although the vehicle being mathematically modeled may be a full-scale ocean vehicle or a physical model of it for use in towing tank or self-propelled experiments.

A mathematical model for a dynamic system consists of two parts: equation structure and initial conditions. For ocean vehicles the

equation structure for overall motion usually consists of sets of differential equations, and the initial conditions represent the values of the variables in the differential equations at a beginning, specific time of interest to the observer. Once the equation structure and initial conditions for the dynamic system are known, the system can be simulated by solving the equations in some manner for a specified input.

This chapter presents one form of the basic equation structure for a general ocean vehicle and references its derivation in the literature. Much of the notation used is either described in the cited references or in Chapter T2. There appears to be two basic methods for arriving at these equations: energy methods and vector calculus. The energy method uses lagrangian state functions and Lagrange's equation and is described by Lamb (L-8), Dogan (B-2), and Tufts (T-1) to name only a few. A state space viewpoint of this method is given by Schultz and Melsa (S-7), Long (L-5) and many others. The vector calculus method consists in essence of a vector expansion of Newton's laws of motion and a Taylor series expansion of the hydrodynamic forces and moments. This method is derived in detail and presented by Abkowitz (A-1) (A-11). The vector calculus method will be briefly described in this chapter and its results used throughout this thesis because the method leads readily to the application of system identification techniques.

There are two basic types of dynamic equation structure to be developed for the mathematical simulation of an ocean vehicle: the rigid body structure and the hydrodynamic structure. The rigid body

structure is a collection of terms involving the vehicle mass, moments of inertia, velocities, and accelerations combined to satisfy Newton's Law. The hydrodynamic structure is a set of terms involving variables which express properties of the body of the vehicle, properties of its motion, and properties of the fluid through which it is moving (A-1). The ocean vehicle dynamic mathematical model is then expressed by equation M1.1 and some statement of the initial conditions of the vehicle at time t_o. Both sides of this equation

$$\boxed{\begin{array}{ll}\text{Rigid Body Structure} = \text{Hydrodynamic Structure} & \\ \text{Vehicle } (t_o) = \underline{x}_o & \text{M1.1}\end{array}}$$

represent the forces and moments on the vehicle.

M1.2 NEWTON'S LAW - THE LEFT SIDE OF EQUATION M1.1

The overall motion of an ocean vehicle when modeled as a rigid body motion must satisfy Newton's Law as in equations M1.2 and M1.3.

$$\frac{d}{dt}\,(\underline{\text{Momentum}}) = \underline{Fo} \qquad \text{M1.2}$$

$$\frac{d}{dt}\,(\underline{\text{Angular Momentum}}) = \underline{Mo} \qquad \text{M1.3}$$

The essence of these equations is that the forces and moments ($\underline{Fo}$, $\underline{Mo}$) acting on the vehicle must equal the time rate of change of the momentum and the angular momentum in a center-of-gravity coordinated system. The fact that the origin of the coordinate system is not at the center of gravity (C.G.) and is moving for a general ocean vehicle means that these equations must be expanded in terms of the vehicle mass, (m), vehicle velocities (u v w p q r)

relative to a coordinate system ($x\ y\ z\ \phi\ \theta\ \psi$) moving with the vehicle, and the vehicle accelerations ($\dot{u}\ \dot{v}\ \dot{w}\ \dot{p}\ \dot{q}\ \dot{r}$).

These equations and the complete details of the expansion for a vehicle of constant mass and time invariant center of gravity location are given by Abkowitz (A-1). The equations for a vehicle with an accelerating center of gravity are presented by Tufts (T-1, p. 17). The equations for a vehicle with changing mass can be developed by using equation M1.4 referenced to the derivation in reference (A-11, p. 36) and by taking into account the fact that the moments and cross

$$\frac{dm}{dt}\underline{U_G} + m\frac{d}{dt}\underline{U_G} = \underline{Fo} \qquad \text{M1.4}$$

products of inertia will also be time varying in equation M1.3. The variable mass, variable C.G. equations could also be fit into the identification procedures used in this thesis.

The force and moment equations to be examined in this thesis with reference to the DSRV are taken from Abkowitz (A-1) for a constant mass and constant center of gravity vehicle and listed as equations M1.5 - M1.10 for the six vehicle forces and moments (X Y Z K M N).

<u>Rigid Body Structure</u>

$$m[\dot{u} + qw - rv - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X \qquad \text{M1.5}$$

$$m[\dot{v} + ru - pw - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})] = Y \qquad \text{M1.6}$$

$$m[\dot{w} + pv - qu - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = Z \qquad \text{M1.7}$$

$$I_x\dot{p} + (I_z - I_y)qr + m[y_G(\dot{w} + pv - qu) - z_G(\dot{v} + ru - pw)] = K \qquad \text{M1.8}$$

$$I_y\dot{q} + (I_x - I_z)rp + m[z_G(\dot{u} + qw - rv) - x_G(\dot{w} + pv - qu)] = M \qquad \text{M1.9}$$

$$I_z\dot{r} + (I_y - I_x)pq + m[x_G(\dot{v} + ru - pw) - y_G(\dot{u} + qw - rv)] = N \qquad \text{M1.10}$$

For coordinate axes parallel to the principal axes of inertia, these equations satisfy Newton's Law for the motion of the vehicle as a rigid body in space with six degrees of freedom and represent, in a way, half of the structure of the mathematical model differential equations, or the left side of equation M1.1.

M1.3 HYDRODYNAMIC STRUCTURE - THE RIGHT SIDE OF EQUATION M1.1

The overall motion of an ocean vehicle through a fluid results in and from forces and moments functionally related to the properties of the vehicle body, the vehicle motion, and the fluid (A-1). The hydrodynamic structure has been defined here to represent all of the forces and moments acting upon the vehicle with the exception of the rigid body or Newton's Law forces. For the purposes of this thesis, the hydrodynamic structure will be broken into the sum of three component parts: hydrodynamic forces, gravity forces, and effector forces as in equations M1.11 and M1.12.

It must be emphasized at this point that the reduction of equation M1.11 to M1.12 is a significant one and must be justified for the particular ocean vehicle being modeled. This reduction says in effect that there is a way to linearly uncouple the effector forces from the hydrodynamic forces. This may be generally expected to be true as long as the motion and effector limitations which are implied by the uncoupling are determined for that vehicle.

Hydrodynamic Structure = f(Body, Motion, Fluid) M1.11

Hyd. Structure = Hyd. Forces + Gravity Forces + Effector Forces M1.12

Where:

Hydrodynamic Forces = f(Motion)

Gravity Forces = f(Body, Motion, Fluid)

Effector Forces = f(Body, Motion, Fluid)

The hydrodynamic forces are those resulting from the dynamics of the vehicle, not including the vehicle effectors. These forces are the result of hydrodynamic inertial efforts which produce "added" masses and of skin friction, separation, circulation, and cross-flow (secondary) drag effects. The hydrodynamic forces are here considered to be functions of the vehicle velocities and accelerations, but more general formulations can be made and used in the identification procedures (see Chapter M3). The fact that these forces depend upon a limited and specific number of variables means that a general structure for the hydrodynamic forces can be developed and applied to a large class of ocean vehicles using a Taylor series expansion.

The effector forces are generally functions which are specific to the vehicle under examination and only limited and specific structure can be developed. This means that the effectors for a given vehicle are best modeled for that vehicle or class of vehicles rather than by trying to find an all-encompassing system of equations which describes a large number of effectors for a large number of different vehicles.

The gravity forces are produced by the creation of a buoyancy force B through the center of buoyancy $\underline{x}_B$ caused by the hydrostatic pressure of the fluid. If the fluid properties are constant, the vehicle volume and buoyancy are constant, and the vehicle mass is constant; then the gravity equations depend primarily upon the vehicle

orientation. These equations are relatively simple and standard and are presented in Chapter D3.2, in references (T-1, p. 21) (B-2, p. 27), and are included in the equations of motion in reference (B-9, p. 4.2.1). For the purposes of this thesis the gravity terms are included in the effector equations.

The hydrodynamic structure can now be summarized in equations M1.13 through M1.18 which represent the right side of equation M1.1 and of equations M1.5 through M1.10. At this point the only equation

<u>Hydrodynamic Structure</u>

$$X = X_{hyd}\,(u\ v\ w\ p\ q\ r\ \dot{u}\ \dot{v}\ \dot{w}\ \dot{p}\ \dot{q}\ \dot{r}) + X_{eff}(\text{Body, Motion, Fluid}) \qquad \text{M1.13}$$

$$Y = Y_{hyd}\,(u\ v\ w\ p\ q\ r\ \dot{u}\ \dot{v}\ \dot{w}\ \dot{p}\ \dot{q}\ \dot{r}) + Y_{eff}(\text{Body, Motion, Fluid}) \qquad \text{M1.14}$$

$$Z = Z_{hyd}\,(u\ v\ w\ p\ q\ r\ \dot{u}\ \dot{v}\ \dot{w}\ \dot{p}\ \dot{q}\ \dot{r}) + Z_{eff}(\text{Body, Motion, Fluid}) \qquad \text{M1.15}$$

$$K = K_{hyd}\,(u\ v\ w\ p\ q\ r\ \dot{u}\ \dot{v}\ \dot{w}\ \dot{p}\ \dot{q}\ \dot{r}) + K_{eff}(\text{Body, Motion, Fluid}) \qquad \text{M1.16}$$

$$M = M_{hyd}\,(u\ v\ w\ p\ q\ r\ \dot{u}\ \dot{v}\ \dot{w}\ \dot{p}\ \dot{q}\ \dot{r}) + M_{eff}(\text{Body, Motion, Fluid}) \qquad \text{M1.17}$$

$$N = N_{hyd}\,(u\ v\ w\ p\ q\ r\ \dot{u}\ \dot{v}\ \dot{w}\ \dot{p}\ \dot{q}\ \dot{r}) + N_{eff}(\text{Body, Motion, Fluid}) \qquad \text{M1.18}$$

structure really known is the rigid body structure; the functions in equations M1.13 through M1.18 are still unknown. The structure of the hydrodynamic forces is usually set by expanding these functions in a truncated Taylor series. The structure of the effector forces is usually determined by theoretical and experimental analysis for a particular effector and vehicle. Once the variables have been specified in the effector force terms, they can also be expanded in a Taylor series.

This chapter has presented and referenced the development of the equation structure for mathematically modeling a general ocean vehicle. This model structure has been shown to consist of a known set of terms which make up the rigid body structure and an undetermined set of terms which make up the hydrodynamic structure. The next chapter develops the state space form for these equations and presents the Taylor series expansion equations for the hydrodynamic forces and moments.

CHAPTER M2

STATE SPACE REPRESENTATION FOR OCEAN VEHICLES

State space representations for multivariable, multidimensional systems are easier to write down and are easier to compute then explicit variable representations. For ocean vehicle dynamic equations the state space format is nothing more than calling the primary explicit variables by different, ordered, and indexed names. Once these equations have been placed in the state space form the wealth of recent, powerful, organized, and practical results from modern control theory can be applied to the understanding of the ocean vehicle.

The purposes of this chapter are to provide the background necessary to understand what the state of a system means, to present the dynamic equations of Chapter M1 in state space form, and to develop the structure of the hydrodynamic forces and moments by Taylor series expansion. The equations and coefficients presented here are the general forms to be coupled into the model reference and extended Kalman filtering equations presented in Section 3 (N).

M2.1 THE STATE OF A DYNAMIC SYSTEM

The state of a dynamic system is the minimum set of numbers $x_1(t_o)$, $x_2(t_o)$, $\cdots$, $x_n(t_o)$ which, in combination with the input to the system $\underline{u}(t)$ for time $t \geq t_o$, is sufficient to determine the behavior of the system for all time $t \geq t_o$. This means that the state is the minimum amount of information about the system needed to determine its future behavior without reference to its past inputs. In essence, the concept of the state of a system divorces the system

from its past behavior. In this thesis, as in most of the literature, the states and inputs are represented as column vectors $\underline{x}$ and $\underline{u}$ as in equation M2.1.

$$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \\ x_n(t) \end{bmatrix} \qquad \underline{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \\ u_m(t) \end{bmatrix} \qquad \text{M2.1}$$

A dynamic system which can be represented by states and state equations is called a state determined dynamic system. The equations for representing such systems usually consist of n first-order differential or difference equations, and the form of these equations is called the mathematical structure of the system. A non-state-determined system would be one which requires an infinite set or continuous set of numbers $\underline{x}(n, t)$ to specify its behavior at any time. Such systems are usually represented by sets of partial differential equations.

A dynamic system is one which changes state as a function of time and is represented by equation M2.2 and diagrammed in Figure M2.1. The structure of the dynamic system is expressed by the time derivatives of the individual states, and a time invariant structure means that the values of the derivatives in the structure do not depend explicitly upon time. The essence of equation M2.2 is that knowing $\dot{\underline{x}}$ at time t means that we can determine $\underline{x}$ at an infinitesimal time later by adding $\underline{x}(t) + \dot{\underline{x}}(t)\, dt$.

Most ocean vehicle dynamic structure is intended to be time invariant and so almost all of the equations in this thesis are

of time invarient structure. After all, if we have a system of differential equations which model a vehicle on one day, we hope that the same equations will be valid the next day. Some examples of time varying behavior for a ship might be the growth of barnacles on the hull causing the ship's drag to increase with time or the wearing out of machinery causing its properties to vary with time. In both of these examples the time variance can be eliminated by making the parameters which vary into states in the system of equations modeling the vehicle. The time invariant form of equation M2.2 is given by M2.3.

$$\dot{\underline{x}} = \underline{f}\,(\underline{x},\ \underline{u},\ t) \qquad \text{M2.2}$$

$$\dot{\underline{x}} = \underline{f}\,(\underline{x},\ \underline{u}) \qquad \text{M2.3}$$

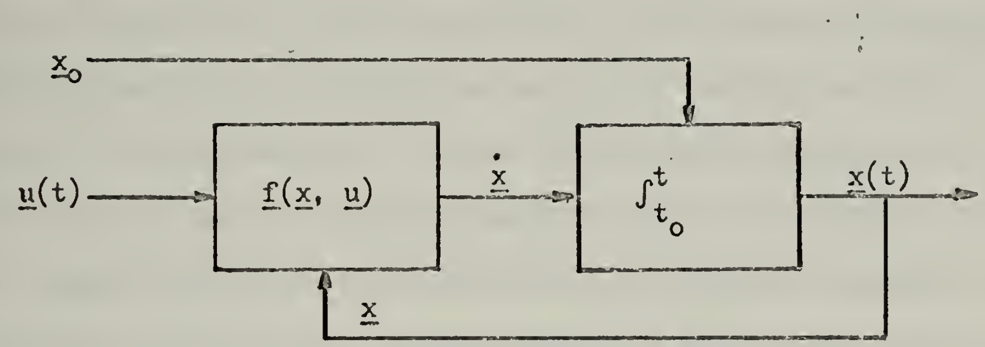


Figure M2.1 BLOCK DIAGRAM OF A DYNAMIC SYSTEM

M2.2 THE STATE OF AN OCEAN VEHICLE

Given a vehicle underway in the ocean environment, what are n numbers $\underline{x}_o$ which, in combination with the inputs $\underline{u}(t)$ to the vehicle for all future time, will completely specify the dynamic behavior of

the vehicle for all future time? These n numbers are the sum of the n_1 properties of the body of the vehicle, the n_2 properties of the vehicle motion, and the n_3 properties of the fluid (ocean). If the fluid properties and the body properties are constant, then the motion properties specify the vehicle behavior and are designated the states of the system. A partial listing of motion, fluid, and body properties is given in Chapter T2 and in reference (A-1).

The properties necessary to specify the state of the DSRV, the example vehicle analyzed in this thesis, are the six vehicle velocities, three vehicle angles, three location variables for the center of gravity, the vehicle weight, the vehicle second-degree coefficients (126) and all of the vehicle parameters (587). This makes a total of 852 states which specify the dynamic behavior of the DSRV mathematical model at any point in time. The initial value of these states, the structure of the equations $\underline{f}$, and the input values $\underline{u}(t)$ completely specify the mathematical model for the DSRV and fit directly into equation M2.3. For an identification problem all of these must be considered states, but for a general simulation the C.G., weight, second-degree coefficients, and vehicle parameters are considered to be constant. In that case, the DSRV states reduce to the vehicle velocities and angles ($u\ v\ w\ p\ q\ r\ \phi\ \theta\ \psi$), and the state vector may be then defined as $\underline{x} = [u, v, w, p, q, r, \phi, \theta, \psi]^T$. The DSRV states are described here merely as an example of a few of the considerations necessary in defining the states of a vehicle.

For the remainder of this thesis, the vehicle velocities will be considered the primary states and will be defined by equation M2.4. The vehicle angles are included in the effector (gravity) functions.

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix} \qquad \text{M2.4}$$

This is a somewhat arbitrary and easily changed definition as will be seen in later sections where the vehicle coefficients and parameters are "augmented" or "stacked" into the general state vector $\underline{x}$. The state vector will generally consist of those variables which change as the vehicle mathematical model executes a maneuver.

M2.3 TAYLOR SERIES EXPANSION OF THE HYDRODYNAMIC FORCES AND MOMENTS

As mentioned previously, mathematical modeling begins with the statement or definition of state variables and inputs. The basic structure of the differential equations for the mathematical model is then developed in terms of these variables using theoretical and experimental techniques. Chapter M1 discusses the development of the rigid body structure for the primary states defined in equation M2.4 and presented it in equations M1.5 through M1.10. The next step is to determine some form for the hydrodynamic structure in equations M1.13 through M1.18.

It is convenient at this point to define a force vector $\underline{X}$ as in equation M2.5 and to utilize it to write the six structure equations as one vector, state space equation M2.6. This convention serves to simplify the writing of these equations and to simplify their computation since they must eventually be calculated using a digital computer program.

$$\underline{X} = \begin{bmatrix} X^1 \\ X^2 \\ X^3 \\ X^4 \\ X^5 \\ X^6 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \\ K \\ M \\ N \end{bmatrix} \qquad \text{M2.5}$$

$$\boxed{\underline{X} = \underline{X}_{hyd}\,(\underline{x},\ \underline{\dot{x}}) + \underline{X}_{eff}\,(\text{Body, Motion, Fluid}) \qquad \text{M2.6}}$$

One method of generally specifying the structure of the term $\underline{X}_{hyd}\,(\underline{x},\ \underline{\dot{x}})$ in equation M2.6 is by expanding it in a Taylor series about some nominal values of the state $\underline{x}_o$ and state derivative $\underline{\dot{x}}_o$ and then by some means specify the coefficients or nominally evaluated partial derivatives of the terms in the series. It must be emphasized that the Taylor series is by no means the only way to begin specifying structure at this point. Perhaps a Fourier, Bessel, abs. square, or other series expansion with coefficients to be determined by identification could be utilized here. If theoretical or experimental investigations for specific vehicles contain any results which will help specify the structure of $\underline{X}_{hyd}$, they should be used at this point in the mathematical modeling process (see also Chapter M3.3). In the literature, the structure of $\underline{X}_{hyd}$ is often specified by a Taylor series containing the linear terms; see for example Salvesen, et al, (S-10), Abkowitz (A-1), Mandel (C-4), Rees (R-8), and Goclowski (G-8).

The general form of $\underline{X}_{hyd}$ to be investigated in detail in later sections of this thesis is the Taylor series expansion about $\underline{x}_o = \underline{\dot{x}}_o = \underline{0}$ with the retention of only the second-degree terms in $\underline{x}$ and the

linear terms in $\underline{\dot{x}}$. The methods used in the study of the second-degree coefficients, however, apply generally to other terms in the Taylor series or to other coefficients in other types of series expansions. The Taylor series expansion of $\underline{X}_{hyd}\ (\underline{x},\ \underline{\dot{x}})$ for $\underline{x}$ being the vehicle velocities is given by equation M2.7.

$$
\begin{aligned}
\underline{X}_{hyd}\ (\underline{x},\ \underline{\dot{x}}) = {} & \underline{X}_{hyd}\ (\underline{0},\ \underline{0}) + \sum_{i=1}^{6} \left.\frac{\partial \underline{X}_{hyd}}{\partial x_i}\right|_{\underline{o}} x_i + \sum_{i=1}^{6} \left.\frac{\partial \underline{X}_{hyd}}{\partial \dot{x}_i}\right|_{\underline{o}} \dot{x}_i + \\
& \sum_{j=1}^{6} \sum_{i=1}^{6} \frac{1}{2} \left.\frac{\partial^2 \underline{X}_{hyd}}{\partial x_i \partial x_j}\right|_{\underline{o}} x_i\ x_j + \sum_{j=1}^{6} \sum_{i=1}^{6} \frac{1}{2} \left.\frac{\partial^2 \underline{X}_{hyd}}{\partial \dot{x}_i \partial \dot{x}_j}\right|_{\underline{o}} \dot{x}_i\ \dot{x}_j + \\
& \sum_{j=1}^{6} \sum_{i=1}^{6} \left.\frac{\partial^2 \underline{X}_{hyd}}{\partial \dot{x}_i \partial x_j}\right|_{\underline{o}} \dot{x}_i\ x_j + \sum_{n=1}^{6} \sum_{j=1}^{6} \sum_{i=1}^{6} \frac{1}{6} \left.\frac{\partial^3 \underline{X}_{hyd}}{\partial x_i \partial x_j \partial x_n}\right|_{\underline{o}} x_i\ x_j\ x_n + \\
& \sum_{n=1}^{6} \sum_{j=1}^{6} \sum_{i=1}^{6} \frac{1}{6} \left.\frac{\partial^3 \underline{X}_{hyd}}{\partial \dot{x}_i \partial \dot{x}_j \partial \dot{x}_n}\right|_{\underline{o}} \dot{x}_i\ \dot{x}_j\ \dot{x}_n + \\
& \sum_{n=1}^{6} \sum_{j=1}^{6} \sum_{i=1}^{6} \frac{1}{2} \left.\frac{\partial^3 \underline{X}_{hyd}}{\partial x_i \partial x_j \partial \dot{x}_n}\right|_{\underline{o}} x_i\ x_j\ \dot{x}_n + \\
& \sum_{n=1}^{6} \sum_{j=1}^{6} \sum_{i=1}^{6} \frac{1}{2} \left.\frac{\partial^3 \underline{X}_{hyd}}{\partial x_i \partial \dot{x}_j \partial \dot{x}_n}\right|_{\underline{o}} x_i\ \dot{x}_j\ \dot{x}_n + \ldots
\end{aligned}
\qquad \text{M2.7}
$$

The terms on the right hand side of equation M2.7 which involve the derivatives of $\underline{X}_{hyd}$ evaluated at $(\underline{x},\ \underline{\dot{x}})$ are called the coefficients of the hydrodynamic forces and moments. In the mathematical simulation of the vehicle, these coefficients are constant and are designated as vehicle parameters. In the identification equations of later sections, however, these coefficients are not constants but become variables and are designated as vehicle states. Equation M2.7 represents a completely

generalized and state space formulated version of equation (75) on page 543 in PNA, reference (C-4).

Without Symmetry	Type of Coefficient	With Symmetry
6	constant	6
36	linear $\underline{x}$	36
36	linear $\underline{\dot{x}}$	36
216	second degree $\underline{x}$	126
216	second degree $\underline{\dot{x}}$	126
216	second degree cross product $x\dot{x}$	216
1296	third degree $\underline{x}$	336
1296	third degree $\underline{\dot{x}}$	336
1296	third degree cross $xx\dot{x}$	756
1296	third degree cross $x\dot{x}\dot{x}$	756

Table M2.1 NUMBERS OF COEFFICIENTS THROUGH THIRD DEGREE

Many of the coefficients in equation M2.7 are repeated because of the continuity and symmetry of the expansion. For example, let the second degree coefficients be defined by equation M2.8 as $\underline{X}_{ij}$. In that case,

$$\underline{X}_{ij} = \left.\frac{\partial^2 \underline{X}_{hyd}}{\partial x_i \partial x_j}\right|_{\underline{o}} \qquad i,j = 1,6 \qquad \text{M2.8}$$

symmetry requires that $\underline{X}_{ij} = \underline{X}_{ji}$ and therefore the 216 possible $\underline{X}_{ij}$ coefficients reduce to 126 possible unique coefficients. The numbers of unique coefficients including symmetry are also listed in Table M2.1.

The total mathematical model for the dynamics of a general ocean vehicle consists of the rigid body structure of equations M1.5 through

M1.10 (in state space form), the hydrodynamic structure of equation M2.6, and the Taylor series structure for the hydrodynamic forces and moments in equation M2.7. The state space form for the rigid body structure in equations M1.5 through M1.10 is given by equations M2.9 through M2.14. In order to place the total mathematical model into state space form, the vehicle state derivative $\underline{\dot{x}}$ must be either explicitly or implicitly solved for in terms of the states $\underline{x}$ and vehicle parameters.

<u>State Space Form of Rigid Body Structure</u>

$$m\,[\dot{x}_1 + x_5 x_3 - x_6 x_2 - x_{1G}(x_5^2 + x_6^2) + x_{2G}(x_4 x_5 - \dot{x}_6) + x_{3G}(x_4 x_6 + \dot{x}_5)] = X^1 \qquad \text{M2.9}$$

$$m\,[\dot{x}_2 + x_6 x_1 - x_4 x_3 - x_{2G}(x_6^2 + x_4^2) + x_{3G}(x_5 x_6 - \dot{x}_4) + x_{1G}(x_5 x_4 + \dot{x}_6)] = X^2 \qquad \text{M2.10}$$

$$m\,[\dot{x}_3 + x_4 x_2 - x_5 x_1 - x_{3G}(x_4^2 + x_5^2) + x_{1G}(x_6 x_4 - \dot{x}_5) + x_{2G}(x_6 x_5 + \dot{x}_4)] = X^3 \qquad \text{M2.11}$$

$$I_{x_1}\,\dot{x}_4 + (I_{x_3} - I_{x_2})\,x_5 x_6 + m\,[x_{2G}\,(\dot{x}_3 + x_4 x_2 - x_5 x_1) - x_{3G}(\dot{x}_2 + x_6 x_1 - x_4 x_3)] = X^4 \qquad \text{M2.12}$$

$$I_{x_2}\,\dot{x}_5 + (I_{x_1} - I_{x_3})\,x_6 x_4 + m\,[x_{3G}(\dot{x}_1 + x_5 x_3 - x_6 x_2) - x_{1G}(\dot{x}_3 + x_4 x_2 - x_5 x_1)] = X^5 \qquad \text{M2.13}$$

$$I_{x_3}\,\dot{x}_6 + (I_{x_2} - I_{x_1})\,x_4 x_5 + m\,[x_{1G}(\dot{x}_2 + x_6 x_1 - x_4 x_3) - x_{2G}(\dot{x}_1 + x_5 x_3 - x_6 x_2)] = X^6 \qquad \text{M2.14}$$

M2.4 SOLVING THE GENERAL EQUATIONS FOR $\dot{\underline{x}}$

In order to mathematically simulate an ocean vehicle as the state determined dynamic system shown in Figure M2.1, the state derivative $\dot{\underline{x}}$ must be calculated for each increment of time the system runs. This means that the general ocean vehicle equations thus far developed either must be manipulated into a form so that an explicit solution for $\dot{\underline{x}}$ may be calculated or must be solved for $\dot{\underline{x}}$ implicitly at each time step by finding the solution to a nonlinear algebraic expression involving $\dot{\underline{x}}$.

The beginning steps in any explicit solution for $\dot{\underline{x}}$ are to expand the rigid body structure, collect the primed derivatives, and write the equations in vector form as in equations M2.15 through M2.23. This then allows the total ocean vehicle mathematical model to be

$$A_r \, \dot{\underline{x}} = \underline{f}_r \, (\underline{x}) + \underline{X} \qquad \text{M2.15}$$

$$A_r = \begin{bmatrix} m & 0 & 0 & 0 & mx_{3G} & -mx_{2G} \\ 0 & m & 0 & -mx_{3G} & 0 & mx_{1G} \\ 0 & 0 & m & mx_{2G} & -mx_{1G} & 0 \\ 0 & -mx_{3G} & mx_{2G} & I_{x_1} & 0 & 0 \\ mx_{3G} & 0 & -mx_{1G} & 0 & I_{x_2} & 0 \\ -mx_{2G} & mx_{1G} & 0 & 0 & 0 & I_{x_3} \end{bmatrix} \qquad \text{M2.16}$$

$$\underline{f}_r \, (\underline{x}) = [f_r^1, \; f_r^2, \; f_r^3, \; f_r^4, \; f_r^5, \; f_r^6]^T \qquad \text{M2.17}$$

$$f_r^1 = -m \, [x_5 \, x_3 - x_6 \, x_2 - x_{1G} \, (x_5^2 + x_6^2)$$

$$+ x_{2G} \, x_4 \, x_5 + x_{3G} \, x_4 \, x_6] \qquad \text{M2.18}$$

$$f_r^2 = -m\left[x_6 x_1 - x_4 x_3 - x_{2G}(x_6^2 + x_4^2)\right.$$

$$\left. + x_{3G} x_5 x_6 + x_{1G} x_5 x_4\right] \qquad \text{M2.19}$$

$$f_r^3 = -m\left[x_4 x_2 - x_5 x_1 - x_{3G}(x_4^2 + x_5^2)\right.$$

$$\left. + x_{1G} x_6 x_4 + x_{2G} x_6 x_5\right] \qquad \text{M2.20}$$

$$f_r^4 = -(I_{x_3} - I_{x_2}) x_5 x_6 - m\left[x_{2G}(x_4 x_2 - x_5 x_1)\right.$$

$$\left. - x_{3G}(x_6 x_1 - x_4 x_3)\right] \qquad \text{M2.21}$$

$$f_r^5 = -(I_{x_1} - I_{x_3}) x_6 x_4 - m\left[x_{3G}(x_5 x_3 - x_6 x_2)\right.$$

$$\left. - x_{1G}(x_4 x_2 - x_5 x_1)\right] \qquad \text{M2.22}$$

$$f_r^6 = -(I_{x_2} - I_{x_1}) x_4 x_5 - m\left[x_{1G}(x_6 x_1 - x_4 x_3)\right.$$

$$\left. - x_{2G}(x_5 x_3 - x_6 x_2)\right] \qquad \text{M2.23}$$

written as equation M2.24. If the vehicle effector structure $\underline{X}_{eff}$ contains terms involving $\dot{\underline{x}}$, these terms would be included in the left hand side of equation M2.24. For a given value of $\underline{x}$, the state

$$\boxed{A_r\, \dot{\underline{x}} - \underline{X}_{hyd}(\underline{x}, \dot{\underline{x}}) = \underline{f}_r(\underline{x}) + \underline{X}_{eff} \qquad \text{M2.24}}$$

derivative $\dot{\underline{x}}$ is now theoretically calculable by an implicit solution of equation M2.24. The calculation of $\dot{\underline{x}}$ then permits mathematical simulation of the ocean vehicle dynamic behavior directly as the system in Figure M2.1

An explicit solution of equation M2.24 is readily arrived at if $\underline{X}_{hyd}$ is represented by the Taylor series of equation M2.7 but with only the $\dot{\underline{x}}$ terms which are linear in $\dot{\underline{x}}$ retained. In that case equation M2.7 reduces to equation M2.25 with the coefficients represented by abbreviations of the form of equation M2.8. This series approximation generally holds when the values of the state derivative $\dot{\underline{x}}$ are assumed

$$\underline{X}_{hyd}(\underline{x}, \dot{\underline{x}}) = \sum_{i=1}^{6} \left.\frac{\partial \underline{X}_{hyd}}{\partial \dot{x}_i}\right|_{\underline{o}} \dot{x}_i + \underline{X}_o + \sum_{i=1}^{6} \underline{X}_i \, x_i + \sum_{j=1}^{6} \sum_{i=1}^{6} \frac{1}{2} \underline{X}_{ij} \, x_i \, x_j + \sum_{n=1}^{6} \sum_{j=1}^{6} \sum_{i=1}^{6} \frac{1}{6} \underline{X}_{ijn} \, x_i \, x_j \, x_n + \cdots \qquad \text{M2.25}$$

to be very small, but the validity of that assumption must be verified for the specific vehicle being simulated. The result of substituting equation M2.25 into equation M2.24 is given by equation M2.26 with the definitions in equations M2.27 through M2.29. Thus, for the cases in

$$A\dot{\underline{x}} = \underline{f}_r(\underline{x}) + \underline{X}_{eff} + \underline{X}_o + \sum_{i=1}^{6} \underline{X}_i \, x_i + \sum_{j=1}^{6} \sum_{i=1}^{6} \frac{1}{2} \underline{X}_{ij} \, x_i \, x_j + \cdots \qquad \text{M2.26}$$

$$A = [A_{ij}] = \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \vdots \\ A_{n1} & \cdots & A_{nn} \end{bmatrix} \qquad n = 6 \qquad \text{M2.27}$$

$$A_{ij} = A_{r_{ij}} - X^{i}_{aj} \qquad i, j = 1, 6 \qquad \text{M2.28}$$

$$X^{i}_{aj} = \left.\frac{\partial X^{i}_{hyd}}{\partial \dot{x}_j}\right|_{o} \qquad (a = \text{added mass}) \qquad \text{M2.29}$$

which the matrix A is of full rank (det $A \neq 0$), the state derivative $\dot{\underline{x}}$ is explicitly solvable by using the inverse of A.

There are several other simplifications which can be made with regard to equation M2.26. The equilibrium $(\underline{0}, \underline{0})$ solution $\underline{X}_o$ for most ocean vehicles is $\underline{X}_o = \underline{0}$. This means that with no inputs they are designed to stand motionless, and if they are moving and the inputs become zero, they eventually stop. If, however, the $\underline{X}_{hyd}$ Taylor series is expanded about a point other than zero $(\underline{0}, \underline{0})$, these constant terms $\underline{X}_o$ would not be expected to be zero.

Another simplification to equation M2.26 results from utilizing the symmetry of $\underline{X}_{ij}$ to reduce the summations necessary. The total mathematical model with linear, second, and third degree coefficients is then given by equation M2.30. This equation contains 6 states and 541 parameters, not counting the effector parameters or structure.

$$A\dot{\underline{x}} = \underline{f}_r(\underline{x}) + \underline{X}_{eff} + \sum_{i=1}^{6} \underline{X}_i x_i + \sum_{j=1}^{6} \sum_{i=1}^{j} \underline{X}_{ij} x_i x_j + \sum_{n=1}^{6} \sum_{j=1}^{n} \sum_{i=1}^{j} \underline{X}_{ijn} x_i x_j x_n \qquad \text{M2.30}$$

The parameters in equation M2.30 consist of the following:

7	parameters in A and $\underline{f}_r$; m, x_{1G}, x_{2G}, x_{3G}, I_{x_1}, I_{x_2}, I_{x_3}	
36	added mass coefficients $X_{\dot{i}}^k$	k, i = 1, 6
36	linear coefficients X_i^k	k, i = 1, 6
126	second degree coefficients X_{ij}^k	k, i, j = 1, 6, sym.
336	third degree coefficients X_{ijn}^k	k, i, j, n = 1, 6, sym.
541	total parameters, not including effector parameters.	

M2.5 EFFECTOR STRUCTURE AND PARAMETERS

This thesis has thus far implicitly considered the effector forces and moments to be that part of the hydrodynamic structure excluding the hydrodynamic forces and moments. However, for a general ocean vehicle, this definition becomes even less precise, and the dividing line between effector and hydrodynamic forces is determined by the particular vehicle and by which portions of its behavior fit into the summations in equation M2.30 and which don't.

This is an important point and can perhaps be illustrated by a brief reference to the DSRV mathematical model of Section 5 (D). The primary DSRV effectors are its movable shroud, thrusters, propellor, and tanks. Since each of these directly involves an input variable, it is placed in the effector structure. Directly beneath the DSRV is a transfer skirt and a splitter plate immediately aft of it. The effects of these appendages upon the motion of the vehicle are calculated using the hydrodynamic coefficients and so these are not considered to be part of the effector structure. The horizontal and vertical sidewise (secondary) motions of the DSRV are modeled by a structure (Appendix A3) which does not fit into the coefficient summations of equation M2.30. Therefore, for the purposes of modeling using equation M2.30, these forces and moments, even though they are strictly functions of the state $\underline{x}$ and not functions of the inputs, are considered to be effectors. Finally, both structural uncertainty (Chapter M4) and input disturbances to the vehicle, such as waves or currents, are placed in the effector structure.

In general, all structure which is functionally dependent upon the input variables is considered effector structure. In many cases

the effectors on ocean vehicles themselves must be modeled as dynamic systems with state variables and parameters. It may also be that for a particular vehicle, the effector states or the vehicle inputs cannot be uncoupled from the hydrodynamic forces and moments. In this case a more general relationship than equation M2.30 would be required with structure specifying such coupling included in the equation.

As mentioned in Chapter M1.3, the effector structure must be developed for the specific vehicle or class of vehicles. For the purposes of the investigations of this thesis, the effector structure will be assumed to be known with the exception of a finite number of parameters $\underline{p}$. The effector forces and moments will also be assumed to be time invariant and to depend upon the state $\underline{x}$, the input vector $\underline{u}$, and the general parameter vector $\underline{p}$ as in equation M2.31.

$$\underline{X}_{eff} = \underline{f}_{eff} (\underline{x}, \underline{u}, \underline{p}) \qquad \text{M2.31}$$

If the effectors are dynamic systems and have their own state variables and parameters, they should, if possible, be modeled as separate subsystems with only their input-output relationship apparent in M2.31. The fact that for a particular vehicle the effectors are dynamic systems does not necessarily mean that the identification of their parameters in $\underline{p}$ is impossible or even that its identification is more difficult than that for a vehicle with static system effectors.

M2.6 SUMMARY OF THIS CHAPTER

It is a fascinating question to ask oneself what single equation can be written which will model the states of anything which moves! (Equation M2.2) We then ask ourselves how this equation changes when we restrict ourselves to a vehicle which moves under Newton's Law.

(Equation M2.15) At that point we enter the ocean environment and begin to develop a further reduced state equation. (Equation M2.30) If we now take the 541 parameters of equation M2.30 and include them in the general parameter vector $\underline{p}$, we can write a final state space equation which will model a large class of ocean vehicles. (Equation M2.32)

$$\boxed{A\,(\underline{p})\,\dot{\underline{x}} = \underline{f}\,(\underline{x},\,\underline{p}) + \underline{f}_{eff}\,(\underline{x},\,\underline{u},\,\underline{p}) \qquad \text{M2.32}}$$

$$\underline{f}\,(\underline{x},\,\underline{p}) = \underline{f}_r\,(\underline{x},\,\underline{p}) + \underline{X}_{hyd}\,(\underline{x},\,\underline{p}) \qquad \text{M2.33}$$

The next chapter is a more extensive discussion of several of the peculiarities of ocean vehicle behavior which in some cases cannot be modeled directly by equation M2.30. The concept and form of a measurement function for the states of an ocean vehicle is presented and discussed. The fact that fluidic memory does not directly fit equation M2.30 is discussed, and a more detailed formulation of several types of nonlinear behavior is analyzed and the inclusion of such behavior in the model structure is considered.

CHAPTER M3

PECULIARITIES OF OCEAN VEHICLE MODELS

There are some dynamic system peculiarities exhibited by ocean vehicles which cannot be modeled directly by equation M2.30 or which require further comment regarding their inclusion into the general model. Several of the topics to be covered in this chapter are the measurement function, fluidic memory modeling, and nonlinear behavior. This chapter is not essential to the identification studies of later sections of this thesis but is included as an extension to the theory of modeling a general ocean vehicle.

M3.1 MEASUREMENT FUNCTIONS

For the purposes of the identification studies in this thesis, the deterministic measurement function (Chapter I4) for the general ocean vehicle has been assumed to be given by equation M3.1. This

$$\underline{z} = \underline{x} \qquad \text{M3.1}$$

assumption is not valid for most measurement systems used on such vehicles. The measurement function is a transformation relationship which expresses the system output quantities $\underline{z}$ in terms of the observable or measurable states $\underline{x}$ of the dynamic system and is usually a nonlinear function for ocean vehicle measurements.

Equation M3.1 is from a simulation and identification standpoint the simplest measurement function which could be used. If the states of an ocean vehicle were measured by linear sensors and amplifiers, the relationship would be as in equation M3.2, where the H matrix represents the cross coupling of the states and the amplifier gains.

This type of measurement function fits directly into the existing

$$\underline{z} = H\underline{x} \qquad \text{M3.2}$$

state space theory for dynamic systems (S-7) (S-3) (D-4) (A-6) (H-4) (B-7) (O-1), etc.; and assuming that the states are "observable" (see references), equation M3.2 is easily incorporated into the general model and into the identification procedures of later sections. There have also been procedures developed for linear systems which are not observable (D-1).

The general class of sensors and measurement systems used for ocean vehicles contains many different types of devices (pressure, velocity, and inertial sensors, doppler sensors, meters, computers, etc.) and, therefore, will be modeled by many differently structured measurement functions. The form of the measurement function for a general nonlinear system is given by equation M3.3. With this form

$$\underline{z} = \underline{h}\ (\underline{x}) \qquad \text{M3.3}$$

for the measurement function, the first consideration with regard to any total mathematical model or to any system identification must be to test the system for observability. Such tests are developed or referenced in the literature (G-9) (S-5), but these methods become prohibitively tedious for significant nonlinearities or for multiple degrees of freedom. A possible, but perhaps expensive and time consuming, alternative to this test is to assume observability and proceed to the identifiability studies of later sections. If trouble is encountered there, this assumption, along with many others, would have to be questioned.

The form of equation M3.3 used in the general parametric identification problem is given by equation M3.4 in which the structure $\underline{h}$ is assumed known and the parameter vector $\underline{p}$ is to be determined by identification. The nonlinear observability criteria must also be

$$\underline{z} = \underline{h}\ (\underline{x},\ \underline{p}) \qquad \text{M3.4}$$

analyzed for this equation as with the previous equations.

The next step upward in generality of measurement functions would be one which was dependent upon the input $\underline{u}$. This equation is given by M3.5 and would be required whenever the measured variables could not be uncoupled from the vehicle inputs. Equation M3.5, as with

$$\underline{z} = \underline{h}\ (\underline{x},\ \underline{u},\ \underline{p}) \qquad \text{M3.5}$$

previous equations of this chapter, is a static relationship. If the effectors for a given vehicle are dynamic, having their own state variables; then if possible, only the input-output relationship should feed into equation M3.5.

M3.2 FLUIDIC MEMORY

The essence of fluidic memory is that the future behavior of a vehicle in a fluid depends not only upon its present motions, but also upon all past motions. This in fact says that the general ocean vehicle is not a state-determined system (Chapter M2.1) and that equation M2.5 is incomplete and should be modified as in equation M3.6.

$$\underline{X} = \underline{X}_{hyd}\ (\text{Motion, Past Motion}) + \underline{X}_{eff}\ (\text{Body, Motion, Past Motion, Fluid}) \qquad \text{M3.6}$$

The fact that vehicles moving in fluids have been shown to depend upon their past motion is pointed out and referenced by Newman (N-5) (N-6) and Dogan (D-2). The models which must be used for inclusion of all past motion in the general ocean vehicle simulation would have to consist of partial differential equations representing the distributed or infitite-state time behavior. It is expected that for the general case, these equations would be extremely difficult to solve.

One method of having the general vehicle mathematical model take into account the past vehicle behavior is to increase the number of states $\underline{x}$ to include the present state and the past states at n increments of time into the past. This representation implicitly assumes that the fluidic memory is finite in length of time and that it can be modeled by n time-samples into the past ($\underline{x}_1$, $\underline{x}_2$, ..., $\underline{x}_n$). This assumption then allows the general ocean vehicle with fluidic memory to be modeled as a state determined system with n + 1 times as many states as the model without fluidic memory.

The modifications which must be made to equation M2.24 to include the finite memory are conceptually straightforward even though perhaps computationally prohibitive. The first step is to define a new state vector $\underline{x}_{fm}$ to include all of the present and past states as in equation M3.7. In this equation the variable $\underline{x}$ is actually $\underline{x}(t)$, where t is the

$$\underline{x}_{fm} = \begin{bmatrix} \underline{x} \\ \underline{x}_1 \\ \vdots \\ \underline{x}_n \end{bmatrix} = \begin{bmatrix} \underline{x}(t) \\ \underline{x}(t - \delta_1) \\ \vdots \\ \underline{x}(t - \delta_n) \end{bmatrix} \qquad \text{M3.7}$$

present time of consideration, and is the same as equation M2.4. The "memory" variables are then spaced at increments of time into the past ($\delta_1, \ldots, \delta_n$) and the variables are the vehicle velocities evaluated at those times $[\underline{x}(t-\delta_1), \ldots, \underline{x}(t-\delta_n)]$.

The matrix A_r and vector $\underline{f}_r(\underline{x})$ in equation M2.24 are independent of the memory effects and are thus merely repeated and evaluated at the past times to form the matrix A_{fm} and vector $\underline{f}_{fm}$ given in equations M3.8 and M3.9. The terms $\underline{X}_{hyd}$ and $\underline{X}_{eff}$ represent the coupling

$$A_{fm} = \begin{bmatrix} A_r & & 0 \\ & \ddots & \\ 0 & & A_r \end{bmatrix} \qquad \text{M3.8}$$

$$\underline{f}_{fm} = \begin{bmatrix} \underline{f}_r(\underline{x}) \\ \underline{f}_r(\underline{x}_1) \\ \vdots \\ \underline{f}_r(\underline{x}_n) \end{bmatrix} \qquad \text{M3.9}$$

of the past states into the present as well as representing the present behavior of the vehicle. These new functions are designated here as $\underline{X}_{fm,hyd}(\underline{x}_{fm}, \dot{\underline{x}}_{fm})$ and $\underline{X}_{fm,eff}(\underline{x}_{fm}, \underline{u}_{fm}, \underline{p})$. If the effector parameters $\underline{p}$ were time varying, then a fluidic memory version $\underline{p}_{fm}$ would have to be included in $\underline{X}_{fm,eff}$. The combination of these results permits the total mathematical model including fluidic memory to be written as equation M3.10.

$$\boxed{A_{fm}\dot{\underline{x}}_{fm} - \underline{X}_{fm,hyd}(\underline{x}_{fm}, \dot{\underline{x}}_{fm}) = \underline{f}_{fm}(\underline{x}_{fm}) + \underline{X}_{fm,eff}(\underline{x}_{fm}, \underline{u}_{fm}, \underline{p}) \qquad \text{M3.10}}$$

The same considerations now apply to equation M3.10 as previously applied to equation M2.24 except some of the very beneficial symmetry reductions in the latter equation cannot be applied to the Taylor series expansion of the former. The finite fluidic memory version of equation M2.30 without taking into account any possible symmetry is given here as equation M3.11. The fluidic memory A matrix is defined the same as in equations M2.27 through M2.29 except that the indices run as i, j = 1, 6 (n + 1), and the $X^{i}_{fm,hyd}$ terms are differentiated. The summations run over all of the finite memory states, and the number of these is given by equation M3.12. The fluidic memory A matrix is square and of dimension $n_{fm} * n_{fm}$.

$$A\dot{\underline{x}}_{fm} = \underline{f}_{fm}(\underline{x}_{fm}) + \underline{X}_{fm,eff}(\underline{x}_{fm}, \underline{u}_{fm}, \underline{p}) + \sum_{i=1}^{n_{fm}} \underline{X}_{fm,i}\, x_{fm,i} + \sum_{j=1}^{n_{fm}} \sum_{i=1}^{n_{fm}} \underline{X}_{fm,ij}\, x_{fm,i}\, x_{fm,j} + \sum_{n=1}^{n_{fm}} \sum_{j=1}^{n_{fm}} \sum_{i=1}^{n_{fm}} \underline{X}_{fm,ijn}\, x_{fm,i}\, x_{fm,j}\, x_{fm,n} \qquad \text{M3.11}$$

$$n_{fm} = 6\,(n + 1) \qquad \text{where } n = \text{number of past times in memory} \qquad \text{M3.12}$$

Solving equation M3.11 for many past times is prohibitive from a computation-time standpoint. However, it is felt that the complexity of equation M3.11 can be very significantly reduced by incorporating the specific characteristics of a given ocean vehicle to eliminate a large percentage of the terms and coefficients and still successfully model the vehicle including memory effects. The point to be made here is that fluidic memory seldom need apply to all past times in all past

degrees of freedom but in general is only significant in one or two degrees of freedom. The study of these limitations and the utilization of equation M3.11 represent extremely profitable potential areas for breakthroughs in ocean vehicle mathematical modeling. These vehicle limitations combined with symmetry reductions would reduce equation M3.11 to a computationally solvable system which could then be utilized in the identification equations and methods of later sections of this thesis.

M3.3 NONLINEAR BEHAVIOR

There are several instances in which the structure of equations M2.30 and M3.11 may need to be modified to take into account some very obvious nonlinearities in ocean vehicle behavior. Several of the considerations necessary to make these modifications are given by Abkowitz (A-11) (A-1) and Mandel (C-4, p. 544). The modifications are described and illustrated in the below examples, but the basic aim of structural modifications is to minimize the number of coefficients or parameters which must be evaluated. The equation structure is by far the most important part of the mathematical model; and if variations in the basic Taylor series form gives closer data fit with fewer parameters, then the structure should be modified to include those variations.

As a first example let us consider the hydrodynamic drag force X exerted upon an object being pulled through a fluid with a constant velocity u. Suppose that a drag coefficient X_{uu} can then be defined for the object as in equation M3.13. If X_{uu} has been shown to be

$$X = X_{uu} u^2 \qquad \text{M3.13}$$

constant for the given object, then X_{uu} corresponds perfectly to X_{11}^{1} in equation M2.30. Now consider that the same object is being pulled in the opposite direction with a constant velocity u, where u is negative. In this case equation M3.13 is no longer structurally valid because we know that a negative force must be required.

There are at least three possible ways to remedy this mathematical structure discrepancy: a larger number of Taylor series terms, foreward and reverse values for X_{uu}, or adoption of an absolute square law structure. The objective here is to have a structure which corresponds as closely as possible to the behavior of the object, but it is also desirable to have the smallest number of parameters if these parameters are to be identified using the procedures of this thesis. If we plot the behavior of the object, we get the curve given in Figure M3.1.

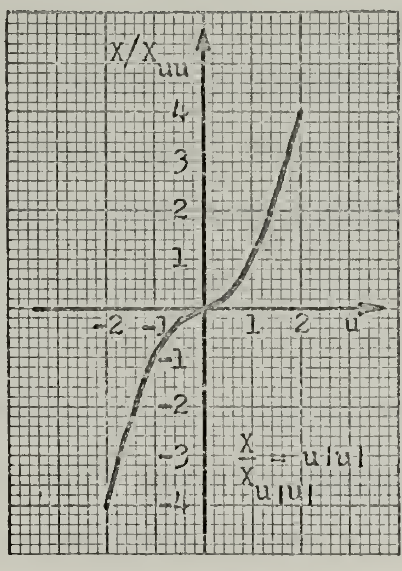


Figure M3.1 ABSOLUTE SQUARE LAW DRAG

Given a desired measure of accuracy which the model structure must have in order to be said to represent the behavior of the object, then certainly if enough odd degree terms in the Taylor series are utilized, the behavior of the object can be modeled. This structure, however accurate, must contain more parameters than simply X_{uu} and would therefore be a much more costly model in terms of later effort in identifying those parameters. All of this is, of course, for the case in which the above data plot is considered to be structurally proper, but numerically inaccurate, or in other words, is noisy.

Another way to take care of the reverse behavior in Figure M3.1 is to simply state that X_{uu} has one value for the foreward mode (namely X_{uu}) and another value for the reverse mode (namely $-X_{uu}$). This in effect gives two separate mathematical models for two separate regions of the object and requires two parameters for the complete model. There are several difficulties encountered with this representation used in the parametric identification procedures of later sections. First of all the regions of operation must be strictly observed for the specific model used. Secondly there are twice as many parameters to identify. Finally, when Kalman filtering is used and X_{uu} is considered a state variable, it must be modeled as a highly discontinuous function.

The best way to model the object behavior in Figure M3.1 is by recognizing that the basic curve shape is <u>absolute square</u>. This then permits the model to be written as equations M3.14 and M3.15 and requires only one parameter. This term would then replace the X^{1}_{11} term in the Taylor series model of equation M2.30, and its inclusion there would most likely contribute to the overall model parameter

identifiability from noisy data.

$$X = X_{u|u|}\, u|u| \qquad \text{M3.14}$$

$$X_{u|u|} = X_{uu} \qquad \text{M3.15}$$

Another specific instance in which structure has been developed which does not fit into equation M2.30 is with regard to the secondary or cross flow drag equations for the DSRV (Appendix A3). These equations are appended as effector functions and the corresponding terms in the Taylor series are deleted because the analysis of preliminary experimental and theoretical data resulted in a structure which did not fit directly into the series.

It is hoped that these examples may serve to point out the importance of proper model structure in later parameter identifiability studies and identifications. The lesson to be learned is that whenever a choice must be made between Taylor series terms and a different structural form resulting from preliminary investigations, the latter should always be included in the model.

This chapter has presented some extension and modification ideas which in some cases must be applied to the general mathematical models of Chapter M3 in order to properly simulate the vehicle or in order to be able to identify parameters using the techniques of this thesis. The first of these was the concept of a general nonlinear measurement function whose structure must be determined for a specific vehicle and whose parameters may need to be identified. Next the general ocean vehicle mathematical model was modified to include the effects of fluidic memory by assuming that the memory was finite and

that it could be represented by n past samples of the vehicle states. Finally, the general model was also modified to include significant structural nonlinearities which were not consistent with the Taylor series representation. The next chapter presents the modifications to the total mathematical model necessary to include the effects of uncertain structure, both in the basic vehicle equation and in the measurement function.

CHAPTER M4

OCEAN VEHICLE MODELS WITH UNCERTAIN STRUCTURE

The primary purpose of this chapter is to include in the general ocean vehicle dynamic mathematical model the fact that at this time neither theoretical nor experimental analysis can completely or perfectly determine the structure of the vehicle equations or of the measurement function. This fact introduces into the general model two forms of "uncertainty" or noise: process noise $\underline{w}$ and measurement noise $\underline{v}$. In both cases these uncertainties are modeled as stochastic processes (P-3) (S-6) (A-5) (B-8) (B-7) (C-3) (J-6) (L-6) (M-2) (R-4) (S-3) etc., and then these terms are included as random variables in the mathematical model used for simulating the real vehicle. The deterministic version (without $\underline{w}$ and $\underline{v}$) is used as the model to be varied in the identification procedures of later sections, and the stochastic version (with $\underline{w}$ and $\underline{v}$) is used in the sea trial data generation or real vehicle simulation.

The total ocean vehicle mathematical model may be summarized here by repeating equations M2.32 and M3.5 as equations M4.1 and M4.2.

Ocean Vehicle Dynamic Mathematical Model

$$A(\underline{p})\,\dot{\underline{x}} = \underline{f}\,(\underline{x}, \underline{p}) + \underline{f}_{eff}\,(\underline{x}, \underline{u}, \underline{p}) \qquad \text{M4.1}$$

$$\underline{z} = \underline{h}\,(\underline{x}, \underline{u}, \underline{p}) \qquad \text{M4.2}$$

This set of equations is a deterministic system of differential equations which contains in some sense all that is known about the dynamic behavior of an ocean vehicle. If, at this point, it is

desirable to also model the unknown aspects of the vehicle behavior by two stochastic processes $\underline{w}$ and $\underline{v}$, then the model becomes non-deterministic and is represented by equations M4.3 and M4.4. If

Ocean Vehicle Stochastic Mathematical Model

$$A(\underline{p})\ \dot{\underline{x}} = \underline{f}\ (\underline{x},\ \underline{p}) + \underline{f}_{eff}\ (\underline{x},\ \underline{u},\ \underline{p},\ \underline{w}) \qquad \text{M4.3}$$

$$\underline{z} = \underline{h}\ (\underline{x},\ \underline{u},\ \underline{p},\ \underline{v}) \qquad \text{M4.4}$$

the very strong assumption is then made that these noise processes are coupled linearly into the dynamics of the vehicle, then the equations M4.5 and M4.6 result. The validity of this assumption would have to be determined for any ocean vehicle under consideration by theoretical, experimental or identification studies.

Ocean Vehicle Stochastic Model With Linear Noises

$$A(\underline{p})\ \dot{\underline{x}} = \underline{f}\ (\underline{x},\ \underline{p}) + \underline{f}_{eff}\ (\underline{x},\ \underline{u},\ \underline{p}) + G\ \underline{w} \qquad \text{M4.5}$$

$$\underline{z} = \underline{h}\ (\underline{x},\ \underline{u},\ \underline{p}) + \underline{v} \qquad \text{M4.6}$$

In both cases the noise vectors $\underline{w}$ and $\underline{v}$ may be considered to be models for structural uncertainty or models for input function uncertainty. In equation M4.5 the noise vector $\underline{w}$ may represent a stochastic model of the remaining terms in the Taylor series expansion of $\underline{f}$ which were not included, or it may represent a model for the unknown aspects of the effector function $\underline{f}_{eff}$ structure. In

addition $\underline{w}$ may model unknown aspects of the input vector $\underline{u}$ (K-6, p. 166) or noise in the measurement system for determining $\underline{u}$ for the vehicle. Likewise, $\underline{v}$ may model the structural uncertainty of $\underline{h}$ or the noise in the input vector $\underline{u}$. In both cases the noise inputs change the equations from a time invariant to a time varying system.

For the purposes of the identification studies in this thesis, the noise vectors $\underline{w}_n$ and $\underline{v}_n$ will be assumed to be uncorrelated, discrete, zero-mean, gaussian white noise processes described by equations M4.7 through M4.10. This is done in some instances to simplify the techniques and in other instances to simplify the computations. Further

$$\bar{\underline{w}} = E[\underline{w}] = \bar{\underline{v}} = E[\underline{v}] = \underline{0} = \bar{\underline{w}}_n = \bar{\underline{v}}_n \qquad \text{M4.7}$$

$$E[\underline{w}(t)\ \underline{w}^T(t+\tau)] = Q_c\ \delta(\tau) \quad ; \quad Q \approx Q_c\delta t \text{ for discrete } \underline{w}_n \qquad \text{M4.8}$$

$$E[\underline{v}(t)\ \underline{v}^T(t+\tau)] = R_c\ \delta(\tau) \quad ; \quad R \approx R_c/\delta t \text{ for discrete } \underline{v}_n \qquad \text{M4.9}$$

$$E[\underline{w}\,\underline{v}^T] = [0] \qquad \text{M4.10}$$

studies can now be made using the techniques of this thesis and relaxing these assumptions through procedures in the literature to determine structure and identifiability for specific ocean vehicles.

As previously mentioned, the measurement function to be used in the analysis in this thesis is an identity between states and measurements plus linear noise. In addition, the process noise coupling matrix G is assumed to be the identity matrix. These two assumptions result in the model given by equations M4.11 and M4.12 and the system diagram given in Figure M4.1. Future studies should be made on the general vehicle model without these two assumptions to determine

their effect upon identifiability of parameters using the techniques of this thesis.

Specialized Ocean Vehicle Stochastic Model

$$A(\underline{p})\ \dot{\underline{x}} = \underline{f}\ (\underline{x},\ \underline{p}) + \underline{f}_{eff}\ (\underline{x},\ \underline{u},\ \underline{p}) + \underline{w} \qquad \text{M4.11}$$

$$\underline{z} = \underline{x} + \underline{v} \qquad \text{M4.12}$$

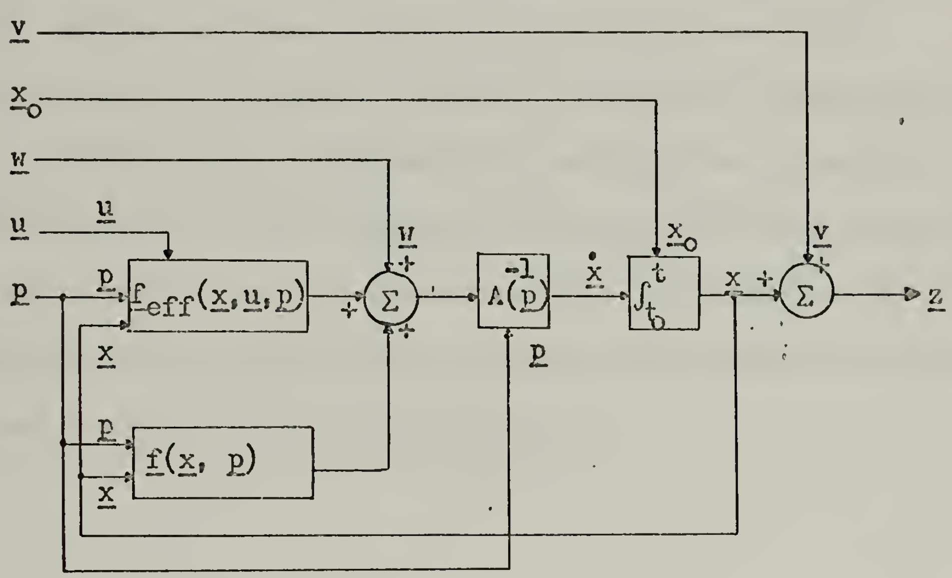


Figure M4.1 SYSTEM DIAGRAM OF A GENERAL OCEAN VEHICLE MATHEMATICAL MODEL WITH NOISE INPUTS

This chapter has presented the hierarchy of models which result from the inclusion of stochastic processes $\underline{w}$ and $\underline{v}$ to model structural uncertainty or input noise. These stochastic models are to be used

in this thesis for simulating the real ocean vehicle in a "noisy" environment. Their deterministic versions with variable parameters will then be used to process the data and identify the parameters.

This section has presented or referenced the complete development of general mathematical models for the dynamic behavior of ocean vehicles. These models have been presented in a hierarchial manner starting with the most general versions and then proceeding to the simpler versions by making assumptions with regard to the vehicle behavior. The models have been developed using a state space format to simplify writing them down and to simplify their computation. Several extensions to the general models in the form of measurement functions, fluidic memory, nonlinearities, and uncertain structure have been discussed and their equations presented. The next section presents the identification equations which are to be applied to these models and the simulated noisy data to evaluate the vehicle parameter vector $\underline{p}$ and to determine its identifiability.

SECTION 3

PARAMETRIC IDENTIFICATION OF NONLINEAR STOCHASTIC SYSTEMS (N)

N1 DEFINITION OF PARAMETRIC IDENTIFICATION

N2 MODEL REFERENCE IDENTIFICATION

N3 EXTENDED KALMAN FILTERING

N4 STUDYING IDENTIFIABILITY OF PARAMETERS

PERHAPS CONCERNING IDENTIFICATION:

" - - - ON A HIGH HILL
RAGGED AND STEEP, TRUTH DWELLS, AND HE THAT WILL
REACH HER, ABOUT MUST, AND ABOUT MUST GO;
AND WHAT TH' HILL'S SUDDENNESS RESISTS, WIN SO."

JOHN DONNE, SATIRE III OF RELIGION (1590)

THIS THESIS IS PRIMARILY CONCERNED WITH APPLYING MODEL REFERENCE AND EXTENDED KALMAN FILTERING IDENTIFICATION TECHNIQUES TO DYNAMIC MATHEMATICAL MODELS OF OCEAN VEHICLE MOTIONS, WITH THE DSRV AS AN EXAMPLE. THE LAST SECTION DEVELOPED GENERAL STATE SPACE OCEAN VEHICLE MODELS AND EXPLAINED THEIR LIMITATIONS AND POSSIBLE EXTENSIONS. THIS SECTION PRESENTS THE BASIC EQUATIONS FOR THE TWO IDENTIFICATION TECHNIQUES AND EXPLAINS HOW THEY ARE USED. THE NEXT SECTION THEN COMBINES THE GENERAL MODEL EQUATIONS AND THE IDENTIFICATION EQUATIONS AND SHOWS IN DEPTH HOW THEY APPLY TO SEVERAL SIMPLE EXAMPLE DSRV EQUATIONS.

CHAPTER N1

DEFINITION OF PARAMETRIC IDENTIFICATION

The purpose of this section is to present and discuss the equations for the general problem of identifying parameters in a nonlinear dynamic system using model reference and extended Kalman filtering techniques. To begin with, this chapter defines parametric identification for general and for more specialized cases and cites some of the literature applicable to these problems. The next two chapters then describe the two techniques to be utilized in this thesis to study the DSRV coefficients and parameters: model reference contouring and extended Kalman filtering. Finally, the last chapter presents a general discussion of the considerations and steps involved in determining quantitatively whether or not a given parameter is identifiable for a given vehicle and given type of maneuver.

N1.1 INTRODUCTION TO PARAMETRIC IDENTIFICATION

In Section 2 (M) the process of understanding the dynamic behavior of ocean vehicles using mathematical modeling was discussed. In this section the details of a group of techniques for determining certain unknown portions of those mathematical models from either full-scale or physical model data are presented. These techniques operate on mathematical models which are known except for the n - constants in a parameter vector $\underline{p}$ and on noisy data from a real vehicle or a simulated vehicle. The goal is then to determine a proper value of $\underline{p}$ and thereby complete the specification of the mathematical model.

The theory of parametric identification is by no means the only way to attack this problem. The history and literature of ocean

engineering are replete with valuable, practical, and proven techniques for modeling ocean vehicle dynamic motions. This thesis is proposing another set of techniques; however, this set of techniques is shown to be applicable to completely general, nonlinear, six-degree-of-freedom, noisy mathematical models with large numbers of undetermined parameters. These techniques have been proven extremely valuable and workable in aircraft, spacecraft, electrical and mechanical systems, economic systems, chemical processing systems, social systems, and many others. The combination of the present and past techniques from ocean engineering with the present and past techniques from modern control theory offers substantial breakthroughs--theoretical, experimental, and practical--in the area of ocean vehicle mathematical modeling.

N1.2 THE GENERAL PARAMETRIC IDENTIFICATION PROBLEM

Parametric identification is the determination of a set of parameters or coefficients of a dynamic system mathematical model of known structure using measurements of the actual system's dynamic behavior with the ultimate aim of having the model be the mathematical equivalent of the system. This chapter describes the general parametric identification problem for a nonlinear, state-determined dynamic system with noise and then specializes this problem to the point at which it can be shown to include the models of Section 2 (M).

The general nonlinear stochastic parametric identification problem is defined by equations N1.1, N1.2, and N1.3 in Figure N1.1. A block diagram of the dynamic system equations N1.1 and N1.2 is shown in Figure N1.2 (G-6) (C-2) (S-3) (L-6) (S-6) etc. This parametric identification problem has no completely general solution. Any

Given:

State Equation

$\dot{\underline{x}} = \underline{f}\ (\underline{x}, \underline{u}, \underline{p}, \underline{w}, t)$ N1.1

Where:

$\underline{x}$ = state vector (n * 1); $\underline{x}\ (t_o)$ known

$\underline{u}$ = control vector (m * 1); $\underline{u}$ (t) known

$\underline{p}$ = parameter vector (n_p * 1); $\dot{\underline{p}} = \underline{g}\ (\underline{p})$, $\underline{p}\ (t_o)$ known

$\underline{w}$ = process noise vector (r * 1)

$\underline{t}$ = time scalar; t_o known

$\underline{f}$ = system structure vector (n * 1); known

Measurement Equation

$\underline{z} = \underline{h}\ (\underline{x}, \underline{u}, \underline{p}, \underline{v}, t)$ N1.2

Where:

$\underline{z}$ = measurement vector (k * 1); $\underline{z}$ (t) known

$\underline{v}$ = measurement noise vector (j * 1)

$\underline{h}$ = measurement structure vector (k * 1); known

Cost Functional

$C = C\ (\underline{z}, \underline{z}_m)$; $C \geq 0$ N1.3

Where:

C = scalar cost functional representing a measure of closeness between the system output $\underline{z}$ and the mathematical model output $\underline{z}_m$; Known structure; $C \geq 0$

Using:

$\underline{u}$ (t), $\underline{z}$ (t), $\underline{f}$, $\underline{g}$, $\underline{h}$, $\underline{x}\ (t_o)$, $\underline{p}\ (t_o)$, t_o, C

Find:

$\underline{p}$ (t) to minimize C (or to maximize -C)

Figure N1.1 THE GENERAL PARAMETRIC IDENTIFICATION PROBLEM

solution techniques to be applied to problems of this nature must in general be tailored to the positive semidefinite cost functional and to the specific types of structural nonlinearities. In many cases the properties of the specific nonlinear system must be used to reduce the complexity or generality of equations N1.1, N1.2, and N1.3 to facilitate identification.

N1.3 THE OCEAN VEHICLE PARAMETRIC IDENTIFICATION PROBLEM

The specialized ocean vehicle stochastic model which was presented as equations M4.11 and M4.12 in Section 2 (M) can be seen to be a sub-case of equations N1.1 and N1.2 in this chapter. The assumptions which must be made on equations N1.1, N1.2, and N1.3 to reduce them in generality to the level of equations M4.11 and M4.12

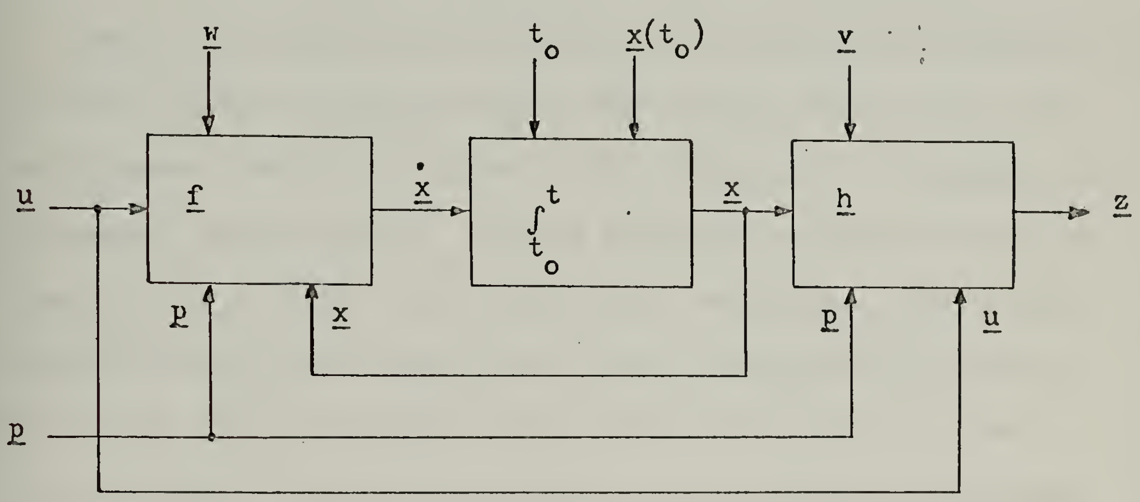


Figure N1.2 PARAMETRIC STOCHASTIC DYNAMIC SYSTEM BLOCK DIAGRAM

with a weighted quadratic cost functional are listed below. The weighted quadratic cost functional is designed to be a general distance measure between the model $\underline{z}_m$ and the system $\underline{z}$ for a given maneuver and set of

1. Model structure and measurement structure are time invariant.

2. Model and measurement noises are linear and enter the system directly.

3. The structure of the measurement function $\underline{h}$ is simply the vehicle states $\underline{x}$ with linear measurement noise.

4. The cost functional is a weighted integral of the square of the difference between the model and the system.

5. The parameters are not states but are constants to be evaluated.

model parameters. These assumptions reduce the general parametric identification problem of Figures N1.1 and N1.2 to the ocean vehicle parametric identification problem of Figure N1.3 and Figure M4.1. It is this latter problem with only the second degree coefficients and with the DSRV values and effectors which will be investigated in detail in Section 4 (P) and 6 (C) of this thesis.

Much of the applicable literature (see Chapter B1) is devoted to the linear state estimation problem (Chapter N3) which results when the parameter vector $\underline{p}$ in Figure N1.3 is "augmented" or "stacked" into the general state vector $\underline{x}$. A great many results are available with regard to the solution of the linear state estimation problem (A-2) (A-6) (A-8) (B-5) (B-8) (D-1) (D-4) (E-1) (F-2) (H-3) (J-1 to 5) (M-4) (M-5) (N-1) (R-5) (R-7) (S-3) (S-6) (S-7) (T-2) etc., but these results do not in general apply to the nonlinear models which must be used for ocean vehicle behavior. Instead, they give a very specialized and often incomplete picture of the actual nonlinear behavior involved.

Given:

State Equation

$$\underline{\dot{x}} = \underline{f}\ (\underline{x},\ \underline{u},\ \underline{p}) + \underline{w} \qquad \text{N1.4}$$

Where:

$$\underline{f}\ (\underline{x},\ \underline{u},\ \underline{p}) = A^{-1}(\underline{p})\ [\underline{f}\ (\underline{x},\ \underline{p}) + \underline{f}_{eff}\ (\underline{x},\ \underline{u},\ \underline{p})]$$

in equation M4.11

$\underline{w} = A^{-1}(\underline{p})\ \underline{w}$ in equation M4.11

$$\underline{\dot{p}} = \underline{0} \quad ; \quad \underline{g} = \underline{0}$$

Measurement Equation

$$\underline{z} = \underline{x} + \underline{v} \qquad \text{N1.5}$$

Cost Functional

$$C = \int_{t_o}^{t_f} (\underline{z} - \underline{z}_m)^T\ R_n^{-1}\ (\underline{z} - \underline{z}_m)\ dt \qquad \text{N1.6}$$

$C = C\ (\underline{u},\ \underline{p})$; $C \geq 0$; R_n = Weighting Matrix

Using:

$\underline{u}\ (t),\ \underline{z}\ (t),\ \underline{f},\ \underline{x}\ (t_o),\ \underline{p}\ (t_o),\ t_o,\ C$

Find:

$\underline{p}$ to minimize C

Figure N1.3 THE OCEAN VEHICLE PARAMETRIC IDENTIFICATION PROBLEM

As mentioned previously, there are no general methods with which to solve nonlinear stochastic parametric identification problems. The best that can usually be done is to take a proven method for linear systems and extend it in some manner to cover the specific nonlinear system of interest. If the extension can then be shown to be valid in some manner, then it can indeed be useful for that specific nonlinear system.

This chapter has presented both a general and a more specific definition of the problem of identifying parameters in a nonlinear stochastic dynamic system. The assumptions leading from the general to the specific versions have been presented, and the ocean vehicle stochastic model of Chapter M4 has been shown to be equivalent to the specific model equations N1.4 and N1.5. The next chapter presents one solution technique, namely model reference contouring, which is applicable to the ocean vehicle parametric identification problem of Figure N1.3.

CHAPTER N2

MODEL REFERENCE IDENTIFICATION

The problem of finding a set of numbers or parameters $\underline{p}$ in a mathematical model in such a way that the same input $\underline{u}$ to the system and to the mathematical model produces the same output $\underline{z}$ from both has been defined in Chapter N1 as parametric identification. This chapter describes what might be considered the most "obvious" or the "brute force" method for attacking this very general problem, model reference identification. As a general procedure, model reference identification runs the model, with the same inputs as to the system, for a large number of different parameter vectors $\underline{p}$ and then selects the specific $\underline{p}$ which results in the model output $\underline{z}_m$ which is "closest" to the original system output $\underline{z}$. This technique is described here by first listing its detailed computational steps, then by discussing the problems and possible variations of each, and finally by describing cost function contouring, the specific variation to be used in this thesis.

It is not intended that an extensive presentation of model reference identification be given in this chapter. This chapter presents, as a background, the specific configuration and computation steps necessary to understand the general utilization of the method and the specific computations and studies employed in this thesis. The literature (Chapter B1) abounds with other variations of the model reference approach to parametric identification, many of which offer promise for application to ocean vehicle studies.

[illegible]

N2.1 MODEL REFERENCE CONFIGURATION

One basic configuration for the model reference identification technique is shown in Figure N2.1. In this configuration the SYSTEM may be the input-output data from one of the following: a noisy full-scale sea trial, a noisy scale-model towing tank or self-propelled test, or, as in this thesis, a simulated ocean vehicle using Figure M4.1 and equations N1.4 and N1.5 with a fixed set of parameters $\underline{p}$. The most general version of the SYSTEM is given by equations N1.1 and N1.2 and Figure N1.2.

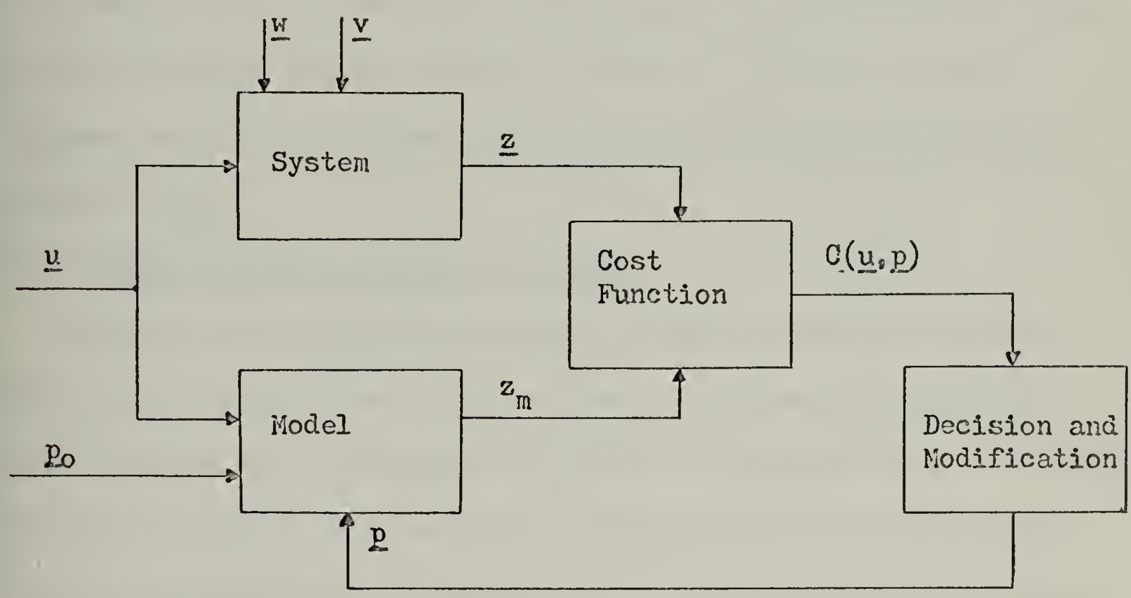


Figure N2.1 MODEL REFERENCE CONFIGURATION

The MODEL in Figure N2.1 is generally the expected equations of motion of the vehicle, without noise. The general MODEL is given by equations N1.1 and N1.2 with $\underline{v}$ and $\underline{w}$ set to zero. The modeling process is, in that case, the fitting of a deterministic structure to a noisy system. The specialized MODEL used in this thesis is given by

equations N1.4 and N1.5 with $\underline{v} = \underline{0}$ and $\underline{w} = \underline{0}$. The COST FUNCTION is given by equation N1.6 for the integral error-squared case and is computed or incremented at each time step of the model and system outputs.

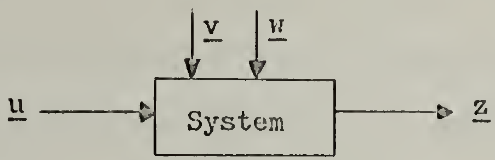
The DECISION AND MODIFICATION block in Figure N2.1 takes the present and past values of C ($\underline{u}$, $\underline{p}$) and calculates a new value of the model parameter vector $\underline{p}$. There are many algorithms which may be used for determining the new value of $\underline{p}$ (B-7) (B-1) (R-12) etc., and the specific algorithm used determines to a great extent the characteristics and value of the model reference method for a particular system. For a given input function $\underline{u}$, the cost function becomes solely a function of the parameter vector $\underline{p}$. In this case the "optimum" or the "identified" value or values of $\underline{p}$ are those which minimize C ($\underline{p}$).

N2.2 MODEL REFERENCE COMPUTATION STEPS

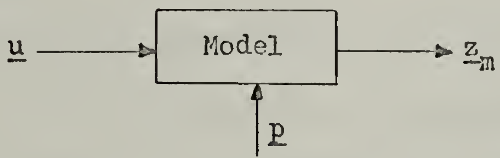
Now that the basic configuration of model reference identification in Figure N2.1 has been explained, the detailed computation steps involved can be presented as Steps N2.1 through N2.5. The first step in this method of parametric identification is to collect the data from the system to be modeled. In the case of an ocean vehicle this step would be a full-scale sea trial with a set of maneuvers $\underline{u}$ and a set of measurements of the vehicle behavior $\underline{z}$. In the case of a physical model this would be an experiment in a towing tank with data recordings of the vehicle inputs $\underline{u}$ and the vehicle responses $\underline{z}$. In the case of a simulated ocean vehicle, as in this thesis, step N2.1 would be the computed solution to equations N1.4 and N1.5 for a fixed, assumed, predetermined, or starting set of parameters $\underline{p}$. In all of

Model Reference Computation Steps

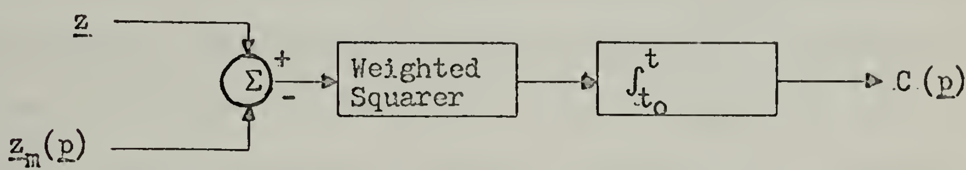
Step N2.1 Collect or generate noisy data $\underline{z}$ and inputs $\underline{u}$



Step N2.2 Using the inputs $\underline{u}$ run the model for a fixed set of parameters $\underline{p}$



Step N2.3 Calculate the cost function C ($\underline{p}$)



Step N2.4 Calculate a new value of $\underline{p}$ by some decision and modification algorithm

Step N2.5 Branch to Step N2.2 and continue until complete or until C ($\underline{p}$) is minimum

these cases the data which results is the time history, either continuous or discrete, of the inputs $\underline{u}$ and outputs $\underline{z}$ of the system.

Once the data in step N2.1 has been collected, the process of analyzing it to determine the model parameters can begin. The identification procedure may be an on-line, real-time process in which an on-board computer calculates and updates the parameters at each system

time increment. If this is the case, the identification computations must be completed between the process time steps, which may require a fast computer or general model simplifications. The identification procedure may also be an off-line or non-real-time process in which the data is collected during the running of the system, and then processed at a later time. The off-line process is more realistic for ocean vehicle identification applications and is the one to be utilized in this thesis.

The computation steps for processing the data generated in step N2.1 to determine the model parameter vector $\underline{p}$ using the model reference configuration appear here as steps N2.2 through N2.5. Step N2.2 essentially says "try the system inputs $\underline{u}$ on the model with one parameter vector $\underline{p}$ and generate the model outputs $\underline{z}_m$." Step N2.3 then says "calculate how close the model was to the system for that $\underline{p}$." Steps N2.4 and N2.5 then say "using our knowledge of the system and our experience at trying different values of $\underline{p}$ and getting a larger or smaller C ($\underline{p}$), we'll decide on another value of $\underline{p}$ to use and try again until we can't get C ($\underline{p}$) any smaller." Thus, the model reference identification technique can be viewed as the system shown in Figure N2.2 in which the inputs are $\underline{u}$, $\underline{v}$, $\underline{w}$, and $\underline{p}_o$ while the outputs are sets of $\underline{p}$'s and their corresponding C ($\underline{u}$, $\underline{p}$)'s.

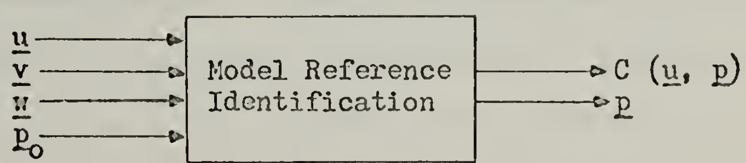


Figure N2.2 MODEL REFERENCE IDENTIFICATION SYSTEM

N2.3 FURTHER COMPUTATIONAL CONSIDERATIONS

Several things can happen during the model reference identification process which may make its results useless, make the technique inapplicable to a given system, or cause erroneous conclusions to be drawn from the results. In the first phase, certain values of $\underline{p}$ in the mathematical model may result in unstable solutions and thereby meaningless or infinite values of both $\underline{z}_m$ and C ($\underline{p}$). The values and ranges of $\underline{p}$ must be specified to avoid unstable solutions or perhaps singular solutions.

Another possibility is that the mathematical model structure could be erroneous or inadequate. In that case the model reference technique would still give values for $\underline{p}$ and C ($\underline{p}$), but the minimum value of C ($\underline{p}$) might be very large, indicating that no value of $\underline{p}$ really brought the model close to the system. This kind of behavior could also be expected when the input noises or uncertainties were very large in comparison to the system variables. In the case of improper structure, the mathematical model structural changes which resulted in lower values of C ($\underline{p}$) for a given input should probably be retained in the model. It could be expected that different values of the optimum $\underline{p}$ would result from these changes.

A similar situation derives from a possible misunderstanding of the meaning of identification. Parametric identification as defined in Chapter N1 does not imply that a single value of $\underline{p}$ which perfectly models the system exists or is desirable. No specification was made as to "how close" the minimum value of C ($\underline{p}$) had to be to zero in order to call the system "modeled." In general, the desired closeness

of modeling depends upon the purpose for which the model is to be used (A-13). Therefore, if a given system results in a large number of $\underline{p}$'s which minimize C ($\underline{p}$), the statement that $\underline{p}$ is "unidentifiable" because a single value cannot be found to model the system is erroneous. The stated criterion was to minimize C ($\underline{p}$) and any $\underline{p}$ which does so is an "identified" value. However, suppose that for a given system and input every value of $\underline{p}$ resulted in the same C ($\underline{p}$), then by the parametric identification problem definition of Figure N1.1, $\underline{p}$ would be completely identifiable since <u>any</u> value of $\underline{p}$ would minimize C ($\underline{p}$). In this case considerations external to that defined problem would enter and say whether or not the system was adequately "modeled" by any value of $\underline{p}$. Model structure or input function changes would probably be indicated at that point.

A final possibility is that the inputs to the system and model might be of such type that variations in $\underline{p}$ might be meaningless, as in the previous example. To further illustrate this point, consider a noiseless vehicle initially at rest and with all inputs set at zero. In that case, all outputs would be zero and C ($\underline{0}$, $\underline{p}$) would be a constant C for all $\underline{p}$. If the input were changed, however, the values of C ($\underline{p}$) would probably change. Thus, the object of modeling is to find a structure and set of parameters $\underline{p}$ which, in some limited sense, result in a minimum set of C ($\underline{u}$, $\underline{p}$) which are valid for a number of different inputs $\underline{u}$ and produce the same optimum set of parameters $\underline{p}$.

N2.4 <u>COST FUNCTION PLOTTING</u>

The model reference identification technique has been previously described as an evaluation of C ($\underline{p}$) over a limited space of parameters $\underline{p}$ in such a way that a specific $\underline{p}$ is found which minimizes C ($\underline{p}$).

The outputs of the model reference system in Figure N2.2 are the spatial values of p and their corresponding cost functions C (p). If the optimum value of p is designated p^*, and its corresponding minimum value of C (p^*) is designated C^* ; then for a scalar parameter p a typical cost function curve might be as shown in Figure N2.3.

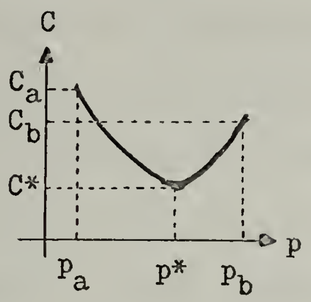


Figure N2.3 EXAMPLE SINGLE PARAMETER COST FUNCTION CURVE

If a predetermined range for p were set $[p_a \leq p \leq p_b]$, and then the values of C (p) were computed at increments of p within that range; then the plotting of the values of C (p) vs. p would allow the observer to minimize C (p) simply by looking at the plot. The actual minimization is in that case performed by the person reading the plot and not by a decision and modification role, such as a gradient algorithm (B-7).

This type of model reference identification has both advantages and disadvantages. The primary advantage of the plotting technique is that it provides the plot reader with more information about the parametric behavior of the system than just the values p^* and C^*. By observing the shape and magnitude of the curve, the plot reader

can instantly see such facets as multiple minima, absolute and relative minima, sharpeness or well defined shape of the minimum, and the overall slope and value of the cost function over the specified range of p. In effect, the plot provides not only identification but also identifiability information about the behavior of a specific parameter.

Another advantage to the technique is that it is a well specified computation problem. This means that it does not require any extensive computation in the decision and modification algorithm, since the new value of p is merely the old one plus an increment.

The primary disadvantage of this form of plotting technique is that, in general, a larger number of parameter values and corresponding cost function values must be computed than for a technique using a computed decision and modification routine. This becomes very significant when the amount of computation involved in each evaluation of C (p) is large. For the case in which p* was located in the center of the range $[p_a \leq p \leq p_b]$ and in which the increments were set equal, then the curve of Figure N2.3 would require about twice as many C (p) calculations as for a gradient algorithm started at p_a or at p_b.

Another disadvantage of this technique is that it only permits observation of the behavior of one parameter at a time. If the general case of the vector parameter $\underline{p}$ were to be analyzed with this technique, there would be n_p plots required in order to look at the selected behavior of the n_p parameters. However, it is important to note here that the $\underline{p}^*$ formed by taking the individual minimum values of each of these curves would NOT in general be the optimum $\underline{p}^*$ for the system when all of the parameters were varied at once. In order

for this to be true, the individual parameter must be shown to be "uncoupled" in the system equations in some manner.

In the case of ocean vehicle coefficients and parameters which enter as products of states in a linear fashion, it is expected that theoretical investigations could show which are coupled to which. Certainly one would not expect a coefficient such as X_{uu} to be coupled to Y_{vv}, but one would expect X_{uu} to be coupled to X_{uv}. Thus, in some cases these independent, individual plots would be very useful, but in other cases they would be inapplicable.

With the initial study of an ocean vehicle's coefficients and parameters in mind, an overview of the above advantages and disadvantages leads the system modeler to the conclusion that the knowledge of identifiability is worth its cost. However, it is more efficient from the standpoint of the human visual interpretation process and from the standpoint of practical decoupling of ocean vehicle models to look at the behavior of two parameters at once. This leads to cost function contouring.

N2.5 COST FUNCTION CONTOURING

Suppose there are two parameters p_1 and p_2 in $\underline{p}$ which are of specific interest. One way of applying the previously discussed visual identification procedure is to attempt to make a three-dimensional plot using contours of the cost function over the defined space of p_1 and p_2 on a two-dimensional plot. Such a set of contours would appear as Figure N2.4 for the case in which the ranges of the parameters are $[p_a \le p_1 \le p_b]$ and $[p_c \le p_2 \le p_d]$. The curves in Figure N2.4 represent lines of constant values of $C\ (p_1,\ p_2)$ with the minimum values being at the center and the maximum values along the edges of the plot.

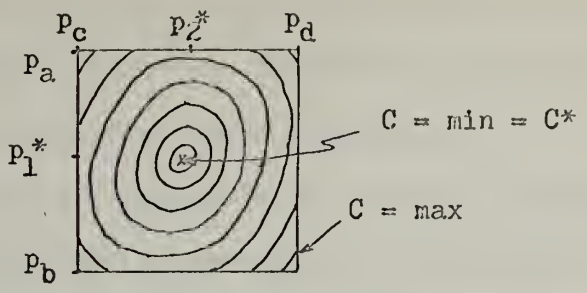


Figure N2.4 EXAMPLE DUAL PARAMETER COST FUNCTION CONTOURS

Visual inspection of this set of contours shows that in this case $C(p_1, p_2)$ is shaped like a bowl with a clearly defined minimum at (p_1^*, p_2^*).

In some cases it may be desirable or necessary to greatly accentuate the minimum value of C* in Figure N2.4. This can be accomplished by contouring the natural logarithm $\log_e C(\underline{p})$ vs. $\underline{p}$ or by using some other form of weighting curve for the cost function.

All of the considerations, advantages, and disadvantages which were discussed concerning cost function plotting in Chapter N2.4 apply also to cost function contouring. The big advantage of contouring vs. plotting is that the coupling of two parameters can be easily observed, and thus coupling ties amoung parameters can be established in pairs. However, even with the pairs and their associated coupling, the set of individual optimal pairs may not be the totally optimal $\underline{p}^*$ because there may be higher dimensional coupling. Another significant consideration is that whereas the cost function plotting was estimated to require about twice as many calculations as a gradient algorithm, the cost function contouring is estimated to require about $2 * n_i$ times as many calculations as a gradient algorithm, where n_i is the number of parameter incremental values. However, this method offers great

promise for ocean vehicle coefficients through second-degree and is the model reference technique employed in this thesis for studying coefficient and parameter identifiability.

This chapter has presented an overview of model reference identification and the specific techniques of cost function plotting and cost function contouring. The techniques of this chapter are applied to simple ocean vehicle models in Section 4 (P) and to the complete DSRV dynamic model in Section 6 (C). The primary benefit deriving from the application of this method is a knowledge of the identifiability characteristics of the parameters, especially in the presence of noise. The next chapter presents the equations and a discussion for the method of extended Kalman filtering. This method, unlike model reference, takes into account the characteristics of the system noises $\underline{v}$ and $\underline{w}$ and uses a decision and modification algorithm to calculate the successive values of $\underline{p}$ to minimize C ($\underline{p}$).

CHAPTER N3

EXTENDED KALMAN FILTERING

Another method which can be applied to the parametric identification problems of Chapter N1 is that of extended Kalman filtering. Kalman filtering is essentially a linear technique with a firm theoretical foundation which, when extended to specific nonlinear systems, loses its theoretical foundation but sometimes works extremely well. This chapter presents the linear Kalman filter equations from the literature and then presents the extended equations for nonlinear systems. These equations will be used in the DSRV example coefficient studies of later sections of this thesis. Kalman filtering is a technique for estimating the state of a linear dynamic system. Therefore, the first step in applying such a technique to parametric identification problems is to convert them to state estimation problems.

N3.1 CONVERTING PARAMETRIC IDENTIFICATION TO STATE ESTIMATION

Identifying parameters can be made equivalent to estimating states simply by treating the parameters as states in the dynamic system. In effect this is merely a redefinition of the state vector $\underline{x}$ in equations N1.1 and N1.2 to include the parameter $\underline{p}$ as in equations N3.1, N3.2, and N3.3. The structure vectors $\underline{f}$ and $\underline{h}$ are

$$x = \begin{bmatrix} \underline{x} \\ \underline{p} \end{bmatrix} \longleftarrow \text{From equations N1.1 and N1.2} \qquad \text{N3.1}$$

$$\dot{\underline{x}} = \underline{f}\,(\underline{x}, \underline{u}, \underline{w}, t) \;;\; \underline{f} = [\underline{f}, \underline{g}]^T \quad \text{Eq. N1.1} \qquad \text{N3.2}$$

$$\underline{z} = \underline{h}\,(\underline{x}, \underline{u}, \underline{v}, t) \;;\; \underline{h} = \underline{h} \quad \text{Eq. N1.2} \qquad \text{N3.3}$$

similarly augmented with the structure relationships for the parameters (i.e. $\dot{\underline{p}} = \underline{g}$). With this definition the parametric identification problem of Figure N1.1 becomes the state estimation problem of Figure N3.1.

Given:

State Equation

$$\dot{\underline{x}} = \underline{f}\ (\underline{x},\ \underline{u},\ \underline{w},\ t) \qquad \text{N3.2}$$

Measurement Equation

$$\underline{z} = \underline{h}\ (\underline{x},\ \underline{u},\ \underline{v},\ t) \qquad \text{N3.3}$$

Cost Functional

$$C = C\ (\underline{z},\ \underline{z}_m)\ \ ;\ C \geq 0 \qquad \text{N3.4}$$

Using:

$\underline{u}$ (t), $\underline{z}$ (t), $\underline{f}$, $\underline{g}$, $\underline{h}$, $\underline{x}$ (t_o), t_o, C

Find:

$\underline{x}$ (t) to minimize C (or to maximize -C)

Figure N3.1 A GENERAL STATE ESTIMATION PROBLEM

The ocean vehicle parametric identification problem is converted to an ocean vehicle state estimation problem in exactly the same manner. If, in addition, it is recognized that the input functions $\underline{u}$ (t) are known vehicle maneuvers (functions of time) and are not optimal control inputs which might depend upon the state, then the effect of $\underline{u}$ (t) is simply to make $\underline{f}$ behave as a time-varying system. With these considerations, the ocean vehicle parametric identification problem of Figure N1.3 becomes the corresponding state estimation problem of Figure N3.2. As mentioned in previous chapters, the model

equations for calculating $\underline{z}_m$ are the state and measurement equations with the noises $\underline{v}$ and $\underline{w}$ set to zero.

Given:

State Equation

$$\dot{\underline{x}} = \underline{f}\,(\underline{x},\ t) + \underline{w} \qquad \text{N3.5}$$

Measurement Equation

$$\underline{z} = \underline{x} + \underline{v} \quad ; \quad \underline{x} = \text{primary states only} \qquad \text{N3.6}$$

Cost Functional

$$C = \int_{t_o}^{t_f} (\underline{z} - \underline{z}_m)^T R_n^{-1} (\underline{z} - \underline{z}_m)\, dt \qquad \text{N3.7}$$

Using:

$\underline{z}\,(t),\ \underline{f},\ \underline{x}\,(t_o),\ t_o,\ C,\ \underline{g} = \underline{0}$

Find:

$\underline{x}\,(t)$ to minimize C (or to maximize -C)

Figure N3.2 THE OCEAN VEHICLE STATE ESTIMATION PROBLEM

Neither of the state estimation problems in Figures N3.1 and N3.2 has a general solution or a directly applicable general solution technique. The complexity of these very general problems must be significantly reduced before any general techniques are applicable. The next steps in the equation developments of this chapter are, therefore, to present a general problem which is solvable, namely the linear problem, and then to extend it to a reduced version of the ocean vehicle state estimation problem and present the corresponding equations.

N3.2 KALMAN FILTERING FOR LINEAR SYSTEMS

The Kalman Filter equations, their derivatives, their applications, and their theoretical foundations are presented in many different forms throughout the literature. The basic form to be used in this thesis is given by Brock (B-8), and similar forms appear in Bryson and Ho (B-7), Ho and Lee (H-3) and Sage (S-3). Brock's development and explanation of these filtering equations and of the overall filtering process is somewhat unusual in that both continuous and discrete equation forms are used. However, this seems to allow simpler notation and more straightforeward computational implementation than either completely continuous or completely discrete forms.

In the general parametric identification problems of Chapter N1 and in the model reference identification technique of Chapter N2 no assumptions were made concerning the characteristics of the stochastic processes (or noises) $\underline{v}$ and $\underline{w}$. The basic Kalman filtering technique for linear systems requires rigid assumptions of the form of $\underline{v}$ and $\underline{w}$ and known numerical characteristics of these two noises. Specifically, $\underline{v}_n$ and $\underline{w}_n$ are assumed to be zero mean, uncorrelated, gaussian white noise processes described in equations M4.7 through M4.10 with assumed or known process noise covariance Q and measurement noise covariance R. The Kalman filter can then be shown (B-8) (H-3) to be the optimum estimator for the state of the given linear system. Discrete noises $\underline{w}_n$, $\underline{v}_n$ are used in simulations (Chap. T2).

Developments in the literature also permit the application of the linear Kalman filtering technique to systems with relaxed constraints on the noise characteristics. A large number of these

extensions with regard to the characteristics of $\underline{v}$ and $\underline{w}$ such as non-zero mean, correlated, time structured, and non-gaussian $\underline{v}$ and $\underline{w}$ are discussed by Schweppe (S-6). The system in which the matrices Q and R are unknown and must be determined is discussed by Abramson (A-2).

The Kalman filtering technique is presented here by describing its computational steps, as was done with the model reference technique in Chapter N2. These steps and their corresponding equations are presented as steps N3.1 through N3.8. In both the model reference and the Kalman filtering techniques the first step is to collect or generate the noisy data $\underline{z}$ and inputs $\underline{u}$. For the purposes of this thesis those values will be discrete time values of the system inputs and outputs, although the procedure is somewhat the same for continuous-time system data (B-7). The Kalman filtering technique implicitly assumes that the data is produced by a linear system, and so in the case of simulated data, a linear system would be used.

Steps N3.2 through N3.8 provide an iterative procedure for processing the noisy data generated in Step N3.1 in such a way as to get optimal estimates of the system states at each time step t_i. The continuous differential equations in N3.1, N3.2, and N3.3 are solved in this thesis by a digital computer using an Euler integrator as given by Equation N3.8. However, more sophisticated integration

$$\underline{x}_{i+1} = \underline{x}_i + \dot{\underline{x}}_{i*}(t_{i+1} - t_i) \qquad \text{N3.8}$$

techniques, both implicit and explicit, such as Runge-Kutta or Adams-Moulton, could be employed here (H-5).

Kalman Filtering Computation Steps

Step N3.1 Collect or generate noisy data $\underline{z}$ and inputs $\underline{u}$

$$\left.\begin{array}{l} \dot{\underline{x}} = F(t)\,\underline{x} + \underline{w} \\ \underline{z} = H\,\underline{x} + \underline{v} \\ \overline{\underline{w}} = E[\underline{w}] = \underline{0} = \overline{\underline{v}} = E[\underline{v}] \\ Q = E[\underline{w}_n \underline{w}_n^T] \;;\; R = E[\underline{v}_n \underline{v}_n^T] \\ \underline{x}(t_o) = \underline{x}_o \end{array}\right\}$$

(solved discretely by computer for $\underline{z}_n$ at time t_n)

N3.1

(uncorrelated $\underline{w}_n$ and $\underline{v}_n$)

Step N3.2 Propagate the estimated state $\hat{\underline{x}}$ one time increment t_i

$$\dot{\hat{\underline{x}}} = F(t)\,\hat{\underline{x}} \quad ; \quad \hat{\underline{x}}(t_o) = \hat{\underline{x}}_o = \underline{x}_o$$

$$\underline{z}_m = H\,\hat{\underline{x}} \qquad \text{N3.2}$$

Step N3.3 Propagate the error covariance matrix E

$$\dot{E} = FE + EF^T + Q \quad ; \quad E(t_o) = E[\underline{x}_o\,\underline{x}_o^{\;T}]$$

$$E = E[\underline{e}\,\underline{e}^T] = \overline{\underline{e}\,\underline{e}^T} \quad ; \quad \underline{e} = \hat{\underline{x}} - \underline{x} \qquad \text{N3.3}$$

$\underline{e}$ = state estimate error; carat denotes estimate

Step N3.4 Calculate the Kalman filter gain matrix K at t_n

$$K = EH^T\,(HEH^T + R)^{-1} \qquad \text{N3.4}$$

Step N3.5 Update the estimated state $\hat{\underline{x}}$ to $\hat{\underline{x}}'$ at t_n

$$\hat{\underline{x}}' = \hat{\underline{x}} + K\,(\underline{z} - \underline{z}_m) \quad ; \quad \underline{z}_n = \underline{z}(t_n) \qquad \text{N3.5}$$

Step N3.6 Update the error covariance matrix E to E' at t_n

$$E' = E - KHE \qquad \text{N3.6}$$

Step N3.7 Set $\hat{\underline{x}}'$ and E' as initial conditions for Steps N3.2 and N3.3 at t_n

$$\hat{\underline{x}}\,(t_n) = \hat{\underline{x}}'\,(t_n)$$

$$E\,(t_n) = E'\,(t_n) \qquad \text{N3.7}$$

Step N3.8 Branch to Step N3.2 and continue until the end of the process.

Many of the problems encountered in the model reference technique may also be encountered with Kalman filtering, both system problems and computation problems. Most of these are treated in the same manner for Kalman filtering as they were for model reference in Chapter N2. One computation problem which may be encountered is that of the error covariance matrix E going negative at some point in the process. This may result in the process becoming completely unstable at that point because of the nature of equation N3.6. The causes for negative E may lie in the violation of some of the basic theoretical rules for Kalman filtering, or the causes may be that the computer has insufficient accuracy or that the time steps were chosen too large.

N3.3 KALMAN FILTERING EXTENDED TO NONLINEAR SYSTEMS

The linear Kalman filter is also valid for nonlinear systems as long as it can be shown that the errors in the estimate of the state can be approximated by a linear system. The detailed steps and theoretical considerations for this extension are presented by Brock (B-8) for the nonlinear system in equations N3.9 (slightly different notation).

$$\begin{aligned} \dot{\underline{x}} &= \underline{f}\,(\underline{x},\ \underline{w},\ t) \\ \underline{z} &= \underline{h}\,(\underline{x},\ \underline{v},\ t) \end{aligned} \qquad \text{N3.9}$$

The equations N3.9 are inserted into Step N3.1 in order to generate the data using a general nonlinear stochastic system. The equations N3.2 in Step N3.2 are replaced by equations N3.10 which have the same structure as equations N3.9 with $\underline{w}$ and $\underline{v}$ set to zero.

$$\dot{\hat{\underline{x}}} = \underline{f}\ (\hat{\underline{x}},\ t)$$

$$\underline{z}_m = \underline{h}\ (\hat{\underline{x}},\ t) \qquad \text{N3.10}$$

The error covariance matrix equation in Step N3.3 is replaced by equations N3.11, and the Kalman filter gain equation in Step N3.4

$$\dot{E} = FE + EF^T + Q_n$$

$$F = \frac{\partial \underline{f}\ (\hat{\underline{x}},\ t)}{\partial \hat{\underline{x}}} \qquad \text{N3.11}$$

$$Q_n = \frac{\partial \underline{f}}{\partial \underline{w}}\ Q\ \frac{\partial f^T}{\partial \underline{w}}$$

is replaced by equations N3.12. With these substitutions the

$$K = EH^T\ (HEH^T + R_n)^{-1}$$

$$H = \frac{\partial \underline{h}\ (\hat{\underline{x}},\ t)}{\partial \hat{\underline{x}}} \qquad \text{N3.12}$$

$$R_n = \frac{\partial \underline{h}}{\partial \underline{v}}\ R\ \frac{\partial \underline{h}^T}{\partial \underline{v}}$$

computation steps N3.1 through N3.8 become those of the general technique of extended Kalman filtering.

N3.4 EXTENDED KALMAN FILTERING FOR OCEAN VEHICLES

The extended Kalman filtering steps for the ocean vehicle state estimation problem can now be derived and presented as Steps N3.9 through N3.16 and as equations N3.13 through N3.19. There are several very significant simplifications which result from the symmetry of the E matrix and from the fact that the parameters $\underline{p}$ in equation N3.1 are assumed to be constants (i.e. $\underline{g} = \underline{0}$).

Extended Kalman Filtering Steps for Ocean Vehicles

Step N3.9 Collect or generate noisy data $\underline{z}$ and inputs $\underline{u}$

$$\dot{\underline{x}} = \underline{f}(\underline{x}, t) + \underline{w} \quad ; \quad \underline{f} \text{ from equation N1.4}$$

$$\underline{z} = H\underline{x} + \underline{v} \qquad \text{(solved discretely for } \underline{z}_n) \tag{N3.13}$$

Step N3.10 Propagate the estimated state $\hat{x}$ to t_n

$$\dot{\hat{\underline{x}}} = \underline{f}(\hat{\underline{x}}, t) \quad ; \quad \underline{f} \text{ from equation N1.4}$$

$$\underline{z}_m = H\hat{\underline{x}} \tag{N3.14}$$

Step N3.11 Propagate the error covariance matrix E to t_n

$$\dot{E} = FE + EF^T + Q$$

$$F = \frac{\partial \underline{f}(\hat{\underline{x}}, t)}{\partial \hat{\underline{x}}} \tag{N3.15}$$

Step N3.12 Calculate the Kalman filter gain matrix K at t_n

$$K = EH^T (HEH^T + R)^{-1} \tag{N3.16}$$

Step N3.13 Update $\hat{\underline{x}}$ to $\hat{\underline{x}}'$ at t_n

$$\hat{\underline{x}}' = \hat{\underline{x}} + K(\underline{z}_n - \underline{z}_m) \quad ; \quad \underline{z}_n = \underline{z}(t_n) \tag{N3.17}$$

Step N3.14 Update E to E' at t_n

$$E' = E - KHE \tag{N3.18}$$

Step N3.15 Set $\hat{\underline{x}}'$ and E' as initial conditions for propagation equations at t_n

$$\hat{\underline{x}}(t_n) = \hat{\underline{x}}'(t_n)$$

$$E(t_n) = E'(t_n) \tag{N3.19}$$

Step N3.16 Branch to Step N3.10 and continue until the end of the process

Suppose that these are n_s states and n_p parameters in the ocean vehicle model. This means that the state estimation problem has $n = n_s + n_p$ states and that equations M3.14 are actually structured

as in equations N3.20 and N3.21. Therefore, instead of integrating n

$$\dot{\hat{\underline{x}}} = \begin{bmatrix} \dot{\hat{\underline{x}}} \\ \hdashline \dot{\hat{\underline{p}}} \end{bmatrix} = \begin{bmatrix} \underline{f}\ (\hat{\underline{x}}, \hat{\underline{p}}, t) \\ \hdashline \underline{0} \end{bmatrix} \quad \text{where } \underline{f} \text{ is } n_s{*}1 \quad \underline{0} \text{ is } n_p{*}1 \qquad \text{N3.20}$$

$$\underline{z}_m = \begin{bmatrix} I_{n*n} & \vdots & 0_{n*np} \end{bmatrix} \begin{bmatrix} \hat{\underline{x}} \\ \hdashline \hat{\underline{p}} \end{bmatrix} = H\hat{\underline{x}} \qquad \text{N3.21}$$

differential equations in equation N3.20, only n_s equations need to be propagated. In addition, the H matrix which has dimension n*n actually has only n_s nonzero elements which are the $H_{ii} = 1$ identity matrix diagonal elements.

Further reductions occur in Step N3.11 because of the symmetry of E ($E = E^T$) and the form of equation N3.20 applied to equations N3.15. Normally the E matrix is n*n, but the fact that it is symmetric means that there are n (n + 1)/2 unique elements. The gradient matrix F is also of a special form described by equation N3.22; and when included in the error covariance matrix propagation equation, the

$$F = \begin{bmatrix} \dfrac{\partial \underline{f}\ (\hat{\underline{x}}, \hat{\underline{p}}, t)}{\partial \hat{\underline{x}}} & \vdots & \dfrac{\partial \underline{f}\ (\hat{\underline{x}}, \hat{\underline{p}}, t)}{\partial \hat{\underline{x}}} \\ \hdashline 0 & \vdots & 0 \end{bmatrix} \text{ of dimensions } \begin{bmatrix} n_s{*}n_s & \vdots & n_s{*}n_p \\ \hdashline n_p{*}n_s & \vdots & n_p{*}n_p \end{bmatrix} \qquad \text{N3.22}$$

form of equation N3.23 results. The fact that there are significant numbers of zeroes in F reduces the number of nonzero elements in the $\dot{E}$ equation N3.23 to a total of n_s (n_s + $2n_p$). The symmetry of E and of Q further reduces the total number of propagation equations to the n_e values of equation N3.24.

$$\begin{array}{ccccc} \dot{E} & = & F & & E \\ \left[\begin{array}{c|c} n_s * n_s & n_s * n_p \\ \hline n_p * n_s & n_p * n_p \end{array}\right] & = & \left[\begin{array}{c|c} n_s * n_s & n_s * n_p \\ \hline 0 & 0 \end{array}\right] & & \left[\begin{array}{c} \ddots \; E_{sym.} \\ \ddots \end{array}\right] \end{array}$$

$$\begin{array}{ccc} & + \, E & F^T \\ + & \left[\begin{array}{c} \ddots \; E_{sym.} \\ \ddots \end{array}\right] & \left[\begin{array}{c|c} n_s * n_s & 0 \\ \hline n_p * n_s & 0 \end{array}\right] \end{array}$$

$$\begin{array}{cc} & + \, Q \\ + & \left[\begin{array}{c|c} n_s * n_s & 0 \\ \hline 0 & 0 \end{array}\right] \end{array} \qquad \text{N3.23}$$

$$n_e = n_s \left[(n_s + 1)/2 + n_p\right] \qquad \text{N3.24}$$

These reductions are highly significant from a computational standpoint and lead to the possibility of solving equations with large numbers of ocean vehicle parameters. The total number of differential equations which must be propagated in Steps N3.10 and N3.11 is $n_s + n_e$. Similar considerations to the above lead to simplifications in the Kalman filter gain matrix calculation and in the updating equations. The K matrix is of dimension $n*n_s$ but only requires the inversion of an n_s*n_s matrix because of the HEH^T structure and the fact that the matrix R is of dimension n_s*n_s.

The form of H in equation N3.21 greatly simplifies the computation of the K matrix. The updating equations require n updates for the states and $n \cdot (n + 1)/2$ updates for the error covariance matrix.

Consider an example case in which the number of primary states is 6 and the number of parameters is 194 for a total of 200 states in the estimation problem. In this case $n_s = 6$ and $n_e = 1185$ for a total of 1191 differential equations which must be propagated. The K matrix calculation requires the inversion of a 6*6 matrix and is of dimension 200 * 6. There are 200 updates for the states and 20100 updates for the error covariance matrix for a total of 20300 updates. These example numbers are really not intended to present an accurate measure of the computation effort involved, since this primarily depends upon the complexity of the basic system structure $\underline{f}$.

This chapter has presented the extended Kalman filter equations for application to ocean vehicle parametric identification. The parametric identification problem was first converted to a nonlinear state estimation problem, and then the linear Kalman filter equations were extended to apply to that nonlinear problem. The detailed computational steps were listed for the linear, nonlinear, and ocean vehicle state estimation problems. In addition, several computational simplifications were derived for the ocean vehicle problem. An overall system view of extended Kalman filtering is presented in Figure N3.3, where the $\underline{x}$ (t) contains the parameters $\underline{p}$ (t) in Figure N2.2, and the error covariance matrix E (t) is a measure of confidence in the values of $\underline{x}$.

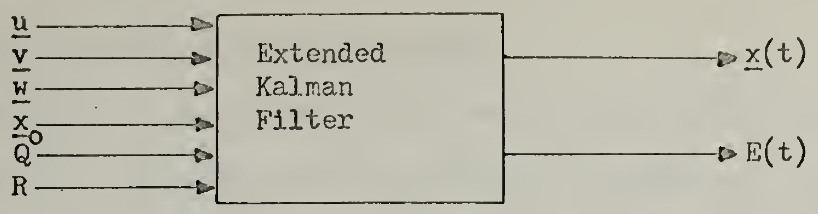


Figure N3.3 EXTENDED KALMAN FILTERING SYSTEM

The next chapter presents the basic equations and considerations for studying the identifiability of specific parameters in nonlinear dynamic system mathematical models. Both model reference and extended Kalman filtering can be used to study identifiability, and the next chapter describes in detail how each technique can be used for this purpose.

CHAPTER N4

STUDYING IDENTIFIABILITY OF PARAMETERS

One primary reason for using identification techniques on ocean vehicles is to determine their mathematical models for the purpose of simulating the motion of the vehicle. Identifiability must therefore be viewed from the standpoint of the overall mathematical modeling objective or system modelability rather than from the standpoint of solving a specific parametric identification problem. This represents a somewhat more general view of identifiability than the view presented in the discussions of Chapter N2.3 in that not only must $\underline{p}^*$ minimize C $(\underline{p})$, but C $(\underline{p}^*)$ must also satisfy the system modeler's criteria for a good mathematical model. If the cost function $C(\underline{p})$ is truely designed to represent the closeness between the model and the system, then one of the modelability or system modeler's criteria might be interpreted as minimum C $(\underline{p}^*)$.

In this thesis the parameters for the mathematical model of the DSRV are studied for their identifiability characteristics by using a known set of parameters and a computer simulation with added noises to generate the data for use in the model reference and Kalman filtering techniques. If these identification techniques result in the true or known set of parameters, then the system modeler's confidence in the identifiability of those same parameters when found from real vehicle data is greatly increased. However, if the true or known set of parameters are not found, then the system modeler must question many of the original steps used in the modeling and identification processes. In this case the conclusion might be reached by the system modeler that the original parameters are

unidentifiable either with or without noise. This conclusion, if well founded, would be valuable information from the standpoint of the time saved trying to find unidentifiable parameters from the real vehicle data. If the true or known set of parameters are shown to be identifiable with multiple values which minimize C ($\underline{p}$), then the system modeler can use this information in interpreting the parameters found from identification passes on real vehicle data. In summary, identifiability studies provide the system modeler with a "bridge" between the mathematical model parameters found from simulated data and those found from real vehicle data. In this thesis identifiability studies will be concerned with finding the original set of parameters used in the vehicle simulation rather than with the single problem of minimizing C ($\underline{p}$).

There is at present no absolute or universally accepted definition of identifiability for parameters in mathematical models (S-6, Chap. 18). First of all, there is no guarantee that the general parametric identification problem is solvable or that there exists a $\underline{p}^*$ which will minimize a general nonlinear cost function C ($\underline{p}$). Then if C ($\underline{p}$) can be shown to have a minimum value, there is no guarantee that such a minimum is unique or that the actual minimizing value(s) of $\underline{p}^*$ could ever be found. Identifiability, therefore, might refer to the capability or theoretical possibility of finding a $\underline{p}^*$ to minimize C ($\underline{p}$), or identifiability could be some measure of the computational difficulty involved in finding one or more values of $\underline{p}^*$ accurately for a given form of C ($\underline{p}$).

This chapter presents a general view of the concepts of modelability and identifiability. For use in this thesis a general

definition of modelability is given, and then identifiability is defined and shown to be one of the factors affecting modelability. These definitions are then interpreted in terms of both quantitative and qualitative measures, and the direct applicability of these measures to the identification studies of the DSRV and of other vehicles is discussed.

N4.1 THE CONCEPTS AND DETERMINING FACTORS OF MODELABILITY AND IDENTIFIABILITY

In Chapter M1.1 the concept of understanding the behavior of a physical system, namely an ocean vehicle, was discussed. There are two approaches which can be taken with regard to gaining such an understanding: microscopic (theoretical) and macroscopic (pragmatic). In the microscopic approach, one begins with the known basic physical laws of matter and the axioms of mathematics and develops a mathematical model for the behavior of the system with a rationally understood and connected set of steps. In the macroscopic approach, one begins with the characteristics of the input-output behavior of the system and attempts to deduce the mathematical model for such behavior. Both of these approaches contribute to the total understanding of the system, but any true or complete understanding of the physical system must include the microscopic approach. However, system modelability might apply to the microscopic or macroscopic approaches.

Modelability is the capability of mathematically modeling or simulating a physical system. In this thesis the physical systems to be modeled are ocean vehicles or physical models of ocean vehicles, but the concept can be applied to any physically existing dynamic system. Note, however, that modelability does not imply or require

a complete understanding of the system, but that such an understanding would imply modelability. This thesis is concerned with modelability as applied to the macroscopic approach for modeling physical systems as state determined dynamic systems (Section M).

Let the modelability of a physical system be designated by the scalar functional M and the identifiability of a set of parameters in a general parameter vector $\underline{p}$ by the scalar functional I. The modelability and identifiability functionals can then be described in terms of the factors in the modeling process upon which they are dependent as in equations N4.1 and N4.2. The dividing line between

$$M = M\,(\underline{f},\ \underline{g},\ \underline{h},\ C,\ Id,\ Int,\ \underline{v},\ \underline{w},\ I,\ \cdots) \qquad \text{N4.1}$$

$$I = I\,(\underline{u},\ \underline{x}_o,\ \underline{p}_o,\ \delta t,\ Q,\ R,\ \cdots) \qquad \text{N4.2}$$

Where: $\underline{f}$ = system structure vector

$\underline{g}$ = parameter structure vector

$\underline{h}$ = measurement structure vector

C = cost functional structure

Id = identification method

Int = integration method or solution technique

$\underline{v}$ = measurement noise structure

$\underline{w}$ = process noise structure

$\underline{u}$ = vector of vehicle inputs

$\underline{x}_o$ = starting state vector

$\underline{p}_o$ = starting parameter vector

δt = time increment in discrete process

Q = process noise parameters

R = measurement noise parameters

M = mathematical modelability measure

I = parametric identifiability measure

D = dependent variables of I (D)

S = dependent structure of M (S, C, I);

S = all except C and I

M and I need not be as specific as in equations M4.1 and M4.2, and in fact many of the dependents in these two equations could apply to both functionals. The modelability definition used here has M primarily dependent upon the structure of the system and of the modeling process, and the identifiability definition is set up to have I be dependent upon the numerical values in the parametric identification process for a fixed structure and modeling procedure. There are many other factors which might be included as dependent functionals in M and I such as the size and speed of the computational facility to be used in the modeling and identification processes, the available manpower to formulate the problems, the available computation time, desired model accuracy, etc.

The system modeler begins by specifying the structures of the dependent functionals in M, and then proceeds to specify the dependent variables in I. Then the parametric identification problem is investigated to determine its parametric identifiability; and if the parameters are identifiable, then one or more values of $\underline{p}^*$ which minimize $C(\underline{p})$ are determined. If the parameters in the parametric identification problem are not identifiable or if the value of $C(\underline{p}^*)$ is too large to allow the system to be called "modeled," then the structure of the functional dependents in M is varied and the entire process repeated iteratively until satisfactory modeling is achieved.

A similar iteration process can be described for the solution to the general parametric identification problem. The dependent variables in I are specified and attempts are made to find $\underline{p}^*$ to minimize C ($\underline{p}^*$). If this is impossible or too difficult, then the dependent variables are changed in some manner; and the identification process is repeated iteratively until satisfactory identification is achieved or until the parameters are shown to be unidentifiable. A flowchart for the modeling and identification ideas discussed above is presented in Figure N4.1. The overall goal of the system modeler in Figure N4.1 might be to find the S* which would minimize C ($\underline{p}^*$, S).

The studies in this thesis are especially relevant to the identifying process and specifically to blocks 4 through 9 in Figure N4.1. In Sections 2 (M) and 3 (N), and 5 (D) the structure of the modeling process is specified as to the structure of the equation and the identification procedures to be used. Sections 4 (P) and 6 (C) represent the theoretical and DSRV example steps to be followed in blocks 5 and 6 of Figure N4.1. Another system modeler would have to tailor the blocks in Figure N4.1 to his specific dynamic system and his facilities for studying it. Several explicit quantitative and qualitative methods for studying identifiability for a specific ocean vehicle's parameters can now be developed. These identifiability measures are discussed in the remainder of this chapter and applied to the DSRV mathematical model in Sections 4 (P) and 6 (C).

N4.2 IDENTIFIABILITY FOR OCEAN VEHICLE PARAMETERS

It is now desirable to specialize the general procedure of Figure N4.1 to the models of Section M and the identification techniques of this section. This means that the forms of S and C in

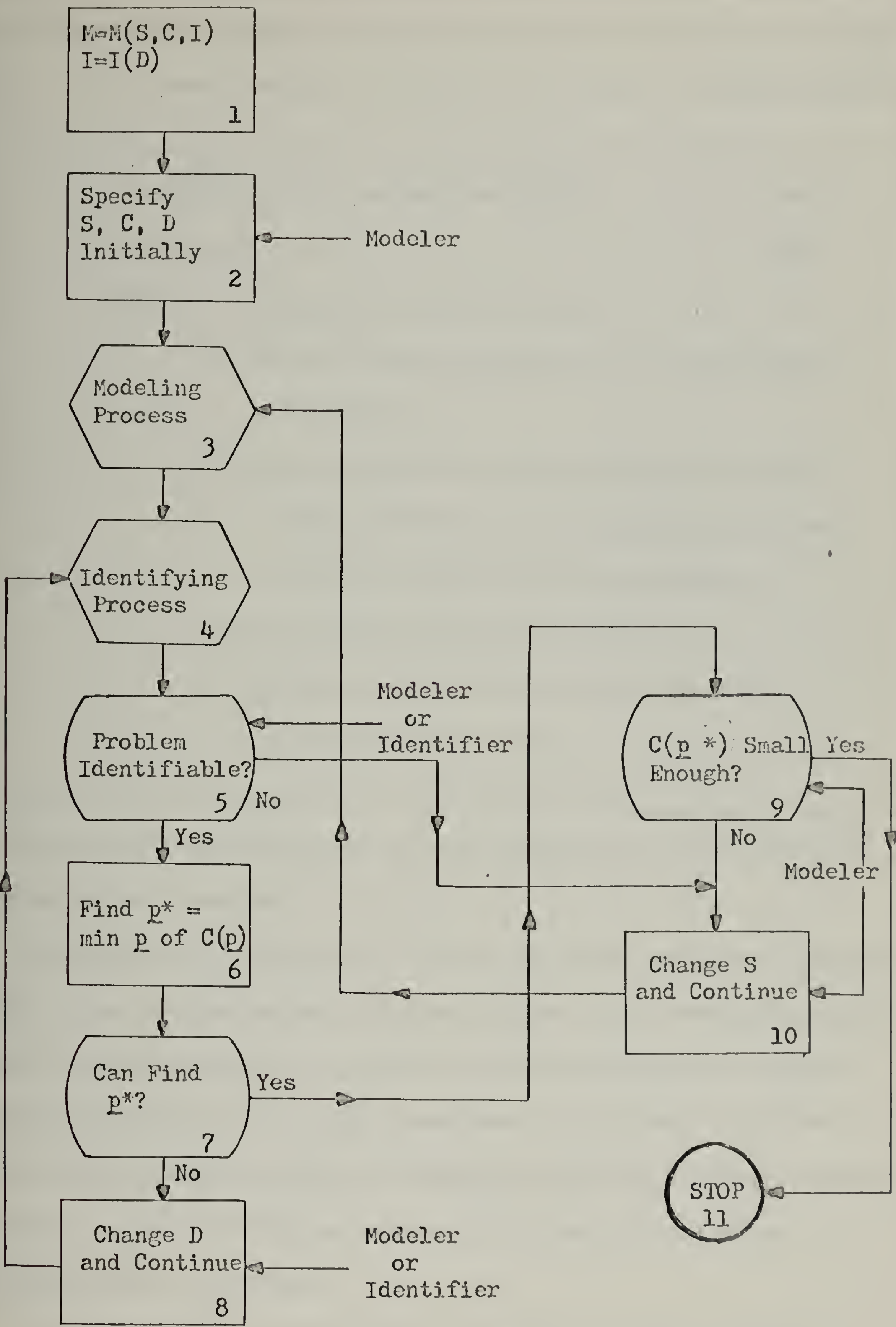


Figure N4.1 FLOWCHART FOR MODELING AND IDENTIFICATION

equation N4.1 are fixed to those equations and identification techniques previously presented and are summarized in equations N4.3 and N4.4. With these structural functions now fixed, the identifiability

M = fixed S and C; assumed modelable	N4.3
I = I ($\underline{u}$, $\underline{p}_o$, Q, R)	N4.4
Where: S = $\underline{f}$, $\underline{g}$, $\underline{h}$, C, Id, Int, $\underline{v}$, $\underline{w}$	
$\underline{f}$, $\underline{g}$, $\underline{h}$, C given in Figure N1.3; ocean vehicle structure	
Id given by equations N3.13 through N3.19 for Kalman filtering and by steps N2.1 through N2.5 for model reference contouring	
Int given by equation N3.8; Euler	
$\underline{v}$, $\underline{w}$ given by equations M4.7 through M4.10, gaussian white noises	

of the specific parameters of an ocean vehicle can now be defined, discussed, and measured.

A parameter p_i belonging to the vector $\underline{p}$ will be termed <u>identifiable</u> if one or more values of p_i^* may be found from simulated vehicle data by model reference contouring or by extended Kalman filtering. The identifiability of p_i will then refer to the ease with which one or more values of p_i^* may be found or "seen" in the model reference contours and to the accuracy with which it may be determined by extended Kalman filtering.

It is important at this point to ensure that this definition be interpreted properly, and not be placed at odds with the discussion

in the introduction to this chapter or in Chapter N2.3. Equation N4.3 assumes modelability or that C ($\underline{p}^*$) will be small enough that the vehicle will be adequately modeled by the structure and value of $\underline{p}^*$. This also implies that C ($\underline{p}$) has some structure to it which will preclude the possibility of all values of $\underline{p}$ minimizing C ($\underline{p}$). Therefore, somewhere within the space of all possible values of $\underline{p}$ [i.e. ($-\underline{\infty} < \underline{p} < +\underline{\infty}$)] there must be a subspace of values which will minimize C ($\underline{p}$). It is therefore possible to talk about the difficulties involved in finding that subspace.

In the identification studies of this thesis an ocean vehicle (DSRV) is simulated using the model structure, gaussian white noise processes, and a fixed set of parameters $\underline{p}^*$. Noisy input-output data is collected from this simulation and processed using the identification techniques to determine $\underline{p}^*$ once again. The closeness with which these procedures give the true value of $\underline{p}^*$ then determines how well the identification procedure works, how easily and accurately the parameter is identified, and to some extent whether, if the parameter $\underline{p}^*$ were unknown for a real vehicle, the identified value would adequately model the real vehicle. It is for these considerations in this thesis, that identifiability is concerned with the difficulties involved in finding a specific and unique minimizing value of $\underline{p}^*$ accurately, in finding if multiple values of $\underline{p}^*$ exist, or in finding if $\underline{p}^*$ is unidentifiable. There are both quantitative and qualitative measures of identifiability which can be applied to the problem of finding $\underline{p}^*$ from simulated vehicle data.

N4.3 MEASURES OF IDENTIFIABILITY

The identifiability I of a parameter vector $\underline{p}$ for the ocean

vehicle mathematical models studied in this thesis depends, as in equation N4.4, upon the input function $\underline{u}$, the starting values of the parameters $\underline{p}_o$, and the noise parameters Q and R. In order for I to be a measure of the difficulties involved in finding $\underline{p}^*$, it must in some manner be a function of the characteristics of C ($\underline{p}$), since the minimization of C is the goal for the identification of $\underline{p}^*$. In the Kalman filtering technique, I must be dependent upon those final states which are parameters and upon their final error covariances, since these determine how accurately the parameters have been identified.

The identifiability of a parameter is a measure function which falls into the general category of sensitivity functions or sensitivity analysis in the area of control theory (S-3) (R-10) (D-10). Many quantitative sensitivity measures may be found in this area which can be applied to identifiability studies for ocean vehicle models, but such formulations have not been investigated and are not applied in this thesis. The identifiability studies made here represent, for the most part, qualitative analyses of the model reference contours for selected parameters and a qualitative (percentage) judgement of closeness of the estimated parameters and their error covariances from the Kalman filtering technique.

N4.4 MODEL REFERENCE IDENTIFIABILITY

The model reference contouring technique is the primary tool for studying identifiability in this thesis, and the system modeler's interpretation of those contours leads to his qualitative judgement of the difficulty involved in identifying the parameters being varied using a more efficient technique. The basic considerations applied

by the system modeler to the model reference contours are judgements with regard to the slopes, shapes, and minimum values around the known or true values of the parameters used to generate the data. These considerations are described here by presenting example contours of the type shown in Figure N2.4 and then describing the identifiability characteristics of these examples.

As discussed in Chapter N2, many identification techniques represent some form of stepping down the contours of the cost function to the minimum value of C ($\underline{p}$) and then designating that point as $\underline{p}$*. Gradient methods step down these contours directly, whereas extended Kalman filtering steps down these contours using the known or estimated noise characteristics to decide upon the best direction and amount to step at each contour. Qualitatively then, identifiability (as defined in Chapter N4.2) is determined by the ease, speed, or final accuracy with which these methods can step down the cost function contours generated by the model reference contouring studies. Most of the qualitative measures discussed here apply most directly to the gradient and contouring methods, but these also represent comparable measures for identifiability using extended Kalman filtering.

The applicable identifiability measure will usually depend upon the identification method used. In the case of a first order gradient method, the most identifiable form of cost function is given by equation N4.5 and represents a perfect absolute linear cost function. The plot of this cost function is shown in Figure N4.2 and would appear as a cone for the same form of cost function contoured in two dimensions.

$C(p) = |p|$ N4.5

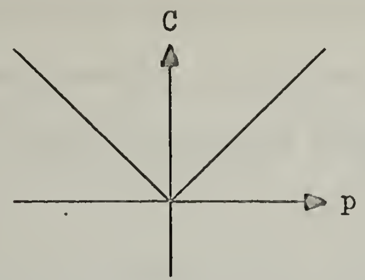


Figure N4.2 AN ABSOLUTE LINEAR COST FUNCTION

The reason that this absolute linear cost function is considered the most identifiable form for a first order gradient technique is that convergence is generated in one iteration step. The first order gradient iteration technique for this cost function is given by equation N4.6, and the applicable gradient is given by equation N4.7. When these two equations are combined, the next iteration in p turns out to be p*, which is given by equation N4.8. Thus, no matter what initial value p_o of the parameter is used, the method identifies p* in one iteration.

$$p_{n+1} = p_n - C\,(p_n)/[d\,C\,(p_n)/d\,p_n] \qquad \text{N4.6}$$

$$d\,C\,(p_n)/d\,p_n = \text{Sign}\,(p_n) = \begin{cases} 1 & p_n \geq 0 \\ -1 & p_n < 0 \end{cases} \qquad \text{N4.7}$$

$$p_{n+1} = 0 = p^* \qquad \text{N4.8}$$

The primary identification method used in this thesis is model reference contouring, where the actual identification is done by the observer who visualizes the contours and sees the minimum value of C and the corresponding optimum values of p_1 and p_2. For this

method, the most identifiable cost function is a "hole" or point displayed exactly at the optimum as given by equation N4.9 and shown in Figure N4.3. This cost function is the most identifiable because it is the easiest one to "see" the optimum C ($\underline{p}$*) and the minimizing values of p_1 and p_2.

$$C\,(\underline{p}) = C_o \quad \text{for } \underline{p} \neq \underline{p}^* \qquad \text{N4.9}$$
$$C\,(\underline{p}) = 0 \quad \text{for } \underline{p} = \underline{p}^*$$

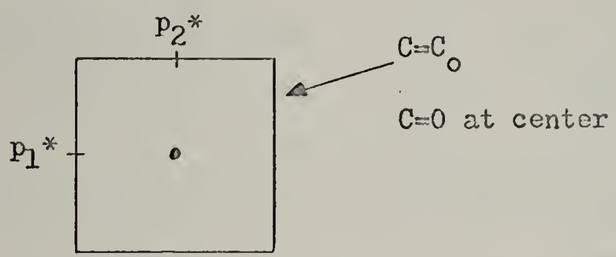


Figure N4.3 MOST IDENTIFIABLE C ($\underline{p}$) FOR MODEL REFERENCE CONTOURING

There are several types of contour configurations which may typically be encountered in the use of model reference contouring on an ocean vehicle cost function C (p_1, p_2) and simulated vehicle data. The identifiability of the parameters is reflected, in one way or another by these configurations, and the system modeler can use this identifiability knowledge to interpret the results of identification passes on real vehicle data. Several of these configurations are listed below and shown in Figure N4.4.

1. both p_1 and p_2 identifiable and independent
2. both p_1 and p_2 identifiable with multiple values
3. p_2 unidentifiable; C (p_1, p_2) = C (p_1)
4. p_1 unidentifiable; C (p_1, p_2) = C (p_2)

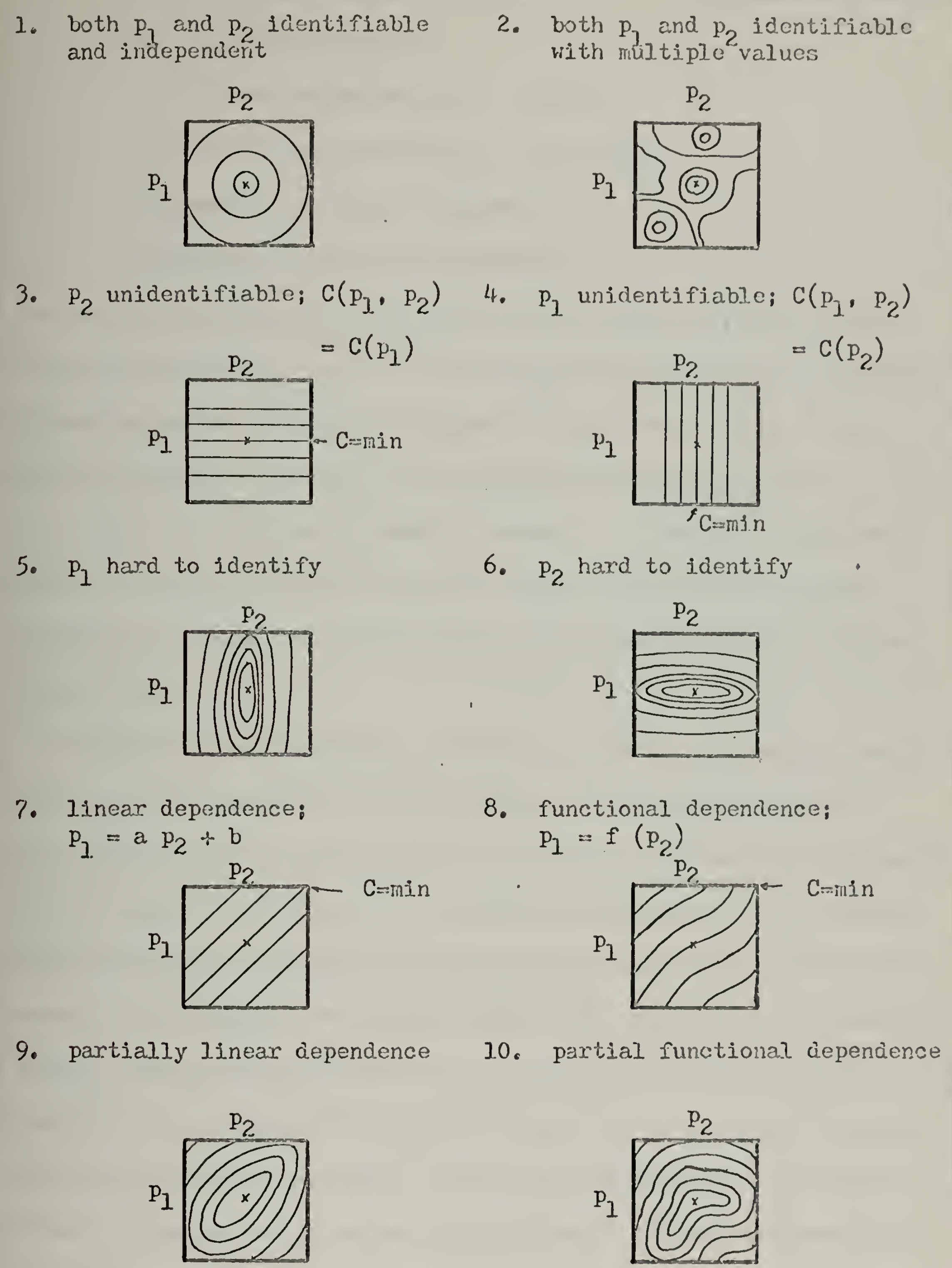


Figure N4.4 <u>TYPICAL MODEL REFERENCE CONTOUR CONFIGURATIONS</u>

5. p_1 hard to identify

6. p_2 hard to identify

7. linear dependence; $p_1 = a\ p_2 + b$

8. functional dependence; $p_1 = f\ (p_2)$;

9. partially linear dependence

10. partial functional dependence

The case of neither p_1 nor p_2 identifiable would be a blank contour map (not shown) or a map with completely random contours (not shown). In some cases the contours of Figure N4.4 may become somewhat uncertain or "fuzzy" because of the stochastic processes $\underline{v}$ and $\underline{w}$. If the shape of the contours remains somewhat the same for significant amounts of $\underline{v}$ and $\underline{w}$ noises, then the system modeler may be fairly certain to be able to identify those same parameters from noisy real vehicle data.

Qualitative identifiability studies may now be carried out using these contours and their interpretations. Variations are made in the input function $\underline{u}$, the ranges of p_1 and p_2, and the noise characteristics Q and R. In each case the shape of the contours is examined and the identifiability of the parameters determined for that variation. In this manner the system modeler may eventually learn which inputs $\underline{u}$ make the most identifiable contours, what the necessary ranges for p_1 and p_2 and for the contours must be in order to determine $\underline{p}^*$ to a desired accuracy, what types and amplitudes of noises cause the parameters to become unidentifiable or hard to identify, and which parameters are functionally dependent for a given input $\underline{u}$. All of this information can then be used to design the inputs and to interpret the results of identification studies for real vehicles.

Most of the information obtained by the system modeler concerning identifiability should come from model reference contouring, and then the actual identifications made on real vehicle data should be done using extended Kalman filtering. Model reference contouring requires a great deal more computation (Chapter N2) in many cases than does extended Kalman filtering. In addition, the accuracy of the optimal parameters or their steady-state estimates can generally be expected to be better from the extended Kalman filtering technique than from model reference contouring.

N4.5 EXTENDED KALMAN FILTER IDENTIFIABILITY

There is some identifiability information which can be gained from the use of the extended Kalman filtering technique, but this information is more of a "go or no go" type than that obtained from model reference contours. Extended Kalman filtering results in the state trajectories (including parameters) $\underline{x}(t)$ and their error covariances E (t) as described in Chapter N3. In the case of unidentifiable or functionally dependent parameters, the parametric states in $\underline{x}(t)$ may not converge to steady-state values or may become unstable and increase exponentially. In some cases the states may reach steady-state values which are "biased" away from the true values of the parameters; and at the same time, the corresponding covariances E (t) may be very small, saying that the filter has a high degree of confidence in an erroneous value of a parameter. However, many of the cases in which the extended Kalman filter will not provide proper answers can be explained by using the corresponding model reference contours.

In later sections of this thesis the outputs of the extended Kalman filtering process are displayed as plots of the parameters $\underline{p}$ (t) and their error covariances E (t). Example plots of a parameter p (t) and its error covariance E (t) are shown in Figure N4.5. The final

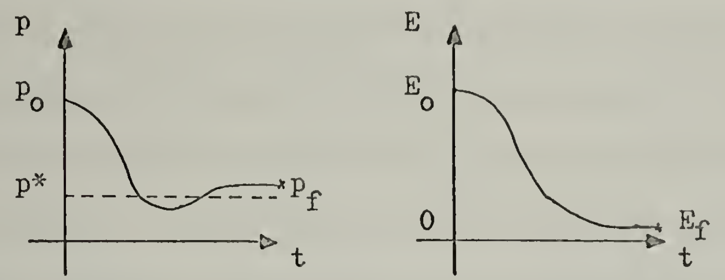


Figure N4.5 TYPICAL OUTPUT PLOTS FROM A KALMAN FILTER

values p_f and E_f on these plots are the points of primary interest from the standpoint of parametric identification. The final parameter value is interpreted as a gaussian random variable with mean p_f and variance E_f or a normal random variable of the form N (p_f, E_f).

The identifiability of the parameter p may be judged from Figure N4.5 by how closely the random variable p = N (p_f, E_f) corresponds to the known value of p* used in the vehicle simulation. There are both quantitative and qualitative judgements which can be made relative to this correspondence. Qualitatively, if p* falls within one standard deviation σ_f of p_f as given by equation N4.9,

$$p^* \in \left[p \,\middle|\, p_f - \sigma_f \leq p \leq p_f + \sigma_f \right]$$
$$\sigma_f = \sqrt{E_f} \qquad \text{N4.9}$$

then the parameter p may in some instances be said to have been identified. The difference between p_f and p* is termed the parameter bias. This bias will certainly have a significant effect upon

any judgement of identifiability for the parameter p and will usually be expressed as a percentage of p^*. The identification of p is highly dependent upon the initial values $N(p_o, E_o)$ used in the Kalman filter, and some qualitative judgements of identifiability may be made based upon how much closer $N(p_f, E_f)$ is to the true value p^* than $N(p_o, E_o)$ was prior to the Kalman filtering. Often in extended Kalman filtering the value E_f may become very small and indicate a much greater confidence in p_f than should be accepted by the system modeler. In this case the identifiability judgement would be more dependent upon the bias than upon the one-standard-deviation measure of equation N4.9. In all of these qualitative judgements, the system modeler must know how close, percentagewise, he needs to identify the parameters in order to have the model represent the system. This information is provided in many cases by the model reference contours.

Quantitative judgements or measures of identifiability of parameters estimated as states by an extended Kalman filter must reflect the closeness of the state estimates p to the known value p^*. One such measure is suggested here as a possibility and is presented in equations N4.10 through N4.14. This measure is the

$$I^* = p^{*2} + \sigma^{*2} \qquad \text{N4.10}$$

$$\sigma^{*2} = 0 \qquad \text{N4.11}$$

$$I_p = (p_f - p^*)^2 + k\,(\sigma_f - \sigma^*)^2 \qquad \text{N4.12}$$

$$I = I_p / I^* \qquad \text{N4.13}$$

$$k = 1 \qquad \text{N4.14}$$

normalized vector distance (for $k = 1$) between the true parameter

$p^* = N\ (p^*, o)$ and the identified value $p = N\ (p_f, E_f)$, considering the random variable as a vector with elements p_f and E_f. The identifiability I in equation N4.13 corresponds to equation N4.4 and could be computed using different inputs, starting states, and noise properties in an attempt to minimize it for the ocean vehicle structure and the extended Kalman filtering technique.

Another possible form for the identifiability measure could be the integral of the square of the difference between the extended Kalman filter estimate and the true value p* over the time of the filter pass as given in equation N4.15. This measure has the

$$I = \int_{t_o}^{t_f} [p(t) - p^*]^2 \, dt \qquad \text{N4.15}$$

advantage of reflecting the complete process history but has the disadvantage of not including any information or measure concerning the error covariance E (t). Variations of the dependent variables in equation N4.4 could also be made to minimize this identifiability measure.

The extended Kalman filter does not provide as much identifiability information as do the model reference contours, but it is in many cases a more efficient technique to use for the actual parameter identifications. The extended Kalman filter uses the noise characteristics in its estimation of the parameters and is in this sense a much better data processor than model reference contouring. In fact, in many instances, the Kalman filter will give better estimates of the parameters than one would expect possible from looking at very noisy model reference contours.

Two methods for identification of parameters and for studying parametric identifiability have been presented and discussed throughout this section. There are significant difficulties encountered when model reference contouring and extended Kalman filtering are applied to the ocean vehicle mathematical models developed in Section M. The difficulties encountered in applying these methods and the techniques for overcoming these difficulties remain to be discussed before the techniques can actually be applied to general ocean vehicle models or to the DSRV model used in this thesis.

N4.6 APPLICATION OF IDENTIFIABILITY STUDIES TO OCEAN VEHICLE PARAMETERS

When the ocean vehicle mathematical modeler is faced with a system of state equations as complex as those presented in the general models of Section M, the solution of the equations, the identification of the parameters, and the making of judgements as to whether the mathematical model really represents the system may appear to be impossible tasks. The dimensionality of the equations, the large numbers of parameters, the highly nonlinear form of the equations, and the significant uncertainties of the process and measurement noises all make the mathematical modeling of a specific ocean vehicle a formidable problem unless some way can be found to simplify the problem. For sophisticated vehicles the equations will be nonlinear and will have noise inputs regardless of the simplifications made, but the equations, the effectors, and the parameters may be capable of being decoupled in several different ways for a specific vehicle model.

This decoupling is a key step in making the modeling problem solvable and must be utilized whenever 6 degree of freedom models

with hundreds of parameters are analyzed with model reference contouring techniques. The extended Kalman filtering technique will apply to the general problem but may require an unnecessarily larger amount of computation time than that required for decoupled, reduced problems. The manner in which the equations of motion for a specific vehicle decouple must be determined by the system modeler through analysis, experiment, or simulation. The decoupling of the equations will facilitate the design of the input functions, both for the simulated vehicle and for the real vehicle sea trials.

Model simplification by degree of freedom, effector, or parameter decouplings requires digital computer programming for either a set of many different simplified and decoupled mathematical models or for one total mathematical model which is structure selective or capable of being automatically decoupled. The single degree of freedom equations of Section P fall into the first type of model set and represent one type of model which would be required. This set of models would have to have many two, three, four, and five degree of freedom models in order to represent totally the ocean vehicle behavior (specifically for DSRV). For small numbers of parameters and for modeling applications in which great precision is not required, it may be advantageous to develop sets of decoupled models and study the parameters of each individually. Then the identified parameters could be placed in the total model and, if the decouplings were valid, would adequately model the vehicle. The single degree of freedom DSRV equations are the only ones of this type developed for use in this thesis (Section P).

The structure selective approach is the one taken in this thesis and is the one suggested for any 6 degree of freedom, multi-effector, multi-parameter ocean vehicle whose mathematical model must be identified somewhat precisely. Structure selectivity is little more than a programming technique in which the mathematical equations are solved only for those degrees of freedom, effectors, and parameters previously selected by the system modeler. In this technique the system modeler specifies the number KB and the indexes $\underline{L}$ of each of the degrees of freedom to be used in the model (B-19). Then the digital computer program for solving an equation such as M2.30 reduces to one for solving only KB equations, and the summations run only over the KB indexes in the vector $\underline{L}$ instead of over all 6 degrees of freedom. Similarly, the effectors are selected as to their number KE and their indexes $\underline{LE}$; and the parameters are selected as to their number KP and their indexes $\underline{LP}$. The use of structure selectivity requires that a significant amount of computation time be spent selecting the degrees of freedom, effectors, and parameters; but the method pays for itself many times over when reduced (decoupled) models are used in the system identification techniques.

As examples of structure selectivity and for use in later sections Table N4.1 presents several reduced degree of freedom models, their indexes, and their applicable effectors for the DSRV. It is rather obvious and highly fortunate that reducing the degrees of freedom of the model also reduces the applicable effectors and parameters very significantly.

The system modeler begins the study of the identifiability characteristics of the particular ocean vehicle of interest by using

single degree of freedom models (equation M2.30) and the applicable effectors and parameters (Section P for DSRV). Simulations are made with known or assumed values of $\underline{p}^*$, and then the identification techniques are used to find $\underline{p}^*$ again from the noisy simulated vehicle data, to develop the best vehicle inputs for finding these parameters, and to learn about the identifiability characteristics of the parameters. This process is then continued with two, three, four, and five degree of freedom selected models, selected effectors, selected maneuvers, and selected parameters (equation M2.30). At some point the system modeler will become confident enough in the behavior of the model structure and identification techniques to specify inputs for a full scale vehicle or physical model sea trial. Then the data from the physical system is processed using the same structure-selected models as were used on the simulated vehicle data. The parameters identified from the real vehicle data interpreted in the light of the identifiability studies made on simulated vehicle data should determine the proper mathematical model for the real vehicle. The system modeler must be certain that the measurement structure, both input and output, used on the real vehicle is the same as that used in the identifiability studies. The total modeling process thus follows the flowchart given in Figure N4.1 and is demonstrated in this thesis for the DSRV simulated model in Sections P, D, and C.

This chapter has presented the details involved in using model reference contours and extended Kalman filtering plots to study the identifiability of specific ocean vehicle parameters. These details have been presented in the framework of completely general definitions of both modelability and identifiability and in the framework

Name	Variables	KB	$\underline{L}^T$	Comments relative to DSRV eff.
surge	u	1	[1]	propellor
sway	v	1	[2]	thrusters, shroud
heave	w	1	[3]	thrusters, ballast tanks, shroud, transfer skirt
roll	p	1	[4]	propellor torque, roll tanks
pitch	q	1	[5]	trim tanks, ballast tanks, shroud, transfer skirt
yaw	r	1	[6]	shroud, thrusters
surge,roll	u,p	2	[14]	propellor torque coupling
surge,heave	u,w	2	[13]	transfer skirt coupling
surge,pitch	u,q	2	[15]	transfer skirt coupling
sway,roll	v,p	2	[24]	transfer skirt coupling, splitter plate coupling
sway,yaw	v,r	2	[26]	splitter plate coupling
heave,pitch	w,q	2	[35]	secondary drag coupling
roll,yaw	p,r	2	[46]	transfer skirt coupling, splitter plate coupling
horizontal plane	u,v,r	3	[126]	all effectors, primarily propellor, shroud
vertical plane	u,w,q	3	[135]	all effectors, primarily propellor, shroud
horizontal roll coupling	u,v,p	3	[124]	all effectors, tanks, prop.
horizontal plane with roll	u,v,p,r	4	[1246]	all effectors
vertical plane with roll	u,w,p,q	4	[1345]	all effectors

Table N4.1 SELECTED REDUCED DEGREE OF FREEDOM MODELS FOR THE DSRV

of a flowchart for both modeling and identification. Most of the qualitative identifiability information has been shown to come from model reference contours of the cost function changes under parameter variations, whereas the quantitative identifiability information resulting from extended Kalman filtering has been shown to be measures of the closeness between the true parameters $\underline{p}^*$ and the identified parameters $\underline{p}_f$. Finally, the complete modeling steps for ocean vehicles using these two techniques and structure selective models have been described in detail and several example reduced degree of freedom DSRV models were given.

This section has presented general and specific definitions of parametric identification problems and the equations of model reference contouring and extended Kalman filtering for the solution of these problems. The application of these techniques to ocean vehicle mathematical modeling and to the equations of Section M has been explored, and several of the difficulties involved in this application have been discussed. The next section begins the actual use of these identification techniques on single degree of freedom equations from the DSRV mathematical model.

SECTION 4

PARAMETRIC IDENTIFICATION OF OCEAN VEHICLE DYNAMICS (P)

"A BAD BEGINNING MAKES A BAD ENDING." EURIPIDES (484-406 B.C.)

SECTION M PRESENTED THE STATE SPACE DIFFERENTIAL EQUATIONS FOR MODELING THE DYNAMIC MOTIONS OF OCEAN VEHICLES. SECTION N THEN PRESENTED THE MATHEMATICS FOR IDENTIFYING PARAMETERS IN THE NONLINEAR OCEAN VEHICLE EQUATIONS USING MODEL REFERENCE CONTOURING AND EXTENDED KALMAN FILTERING. THIS SECTION DESCRIBES IN DETAIL THE PROCESS OF APPLYING THE MODEL REFERENCE AND KALMAN FILTERING TECHNIQUES TO OCEAN VEHICLE MODELS. THIS APPLICATION IS DESCRIBED BY USING, AS EXAMPLES, THE SINGLE DEGREE OF FREEDOM DYNAMIC EQUATIONS FOR THE DSRV. MANY RESULTS OF COEFFICIENT AND PARAMETER IDENTIFICATIONS BY BOTH METHODS ARE PRESENTED AND DISCUSSED. THIS SECTION IS INTENDED TO BE AN INTRODUCTION BY EXAMPLE TO THE MORE SIGNIFICANT INDENTIFICATION STUDIES IN SECTION 6 (C).

CHAPTER P1

THE NATURE OF OCEAN VEHICLE PARAMETRIC IDENTIFICATION

Having established the detailed equation structure for mathematically modeling ocean vehicle dynamics in Section M and the detailed equations for identifying parameters in that structure in Section N, the foundation has been laid for the development of total ocean vehicle mathematical models. This section presents the detailed steps in that development in the simplified format of the DSRV single degree of freedom equations; and at the same time, this section serves as a detailed study of the behavior of the DSRV coefficients and parameters in the six uncoupled degrees of freedom. The next section (D) then presents the structure, simulation set of parameters, and digital computer programming description of the DSRV dynamic mathematical model in six degrees of freedom with structure selectivity, complete coupling, effector equations, and second degree coefficients. The following section (C) then applies the identification techniques to some selected multiple degree of freedom DSRV models and presents qualitative descriptions of the identifiability characteristics of selected coefficients and parameters.

P1.1 OVERALL MODELING STEPS

The nature and specific characteristics of ocean vehicle parametric identification are best described by discussing each of the steps in the general mathematical modeling process for such vehicles. The system modeler begins with a vehicle or a set of vehicle characteristics and proceeds to develop the structure and initial parameter values for the mathematical model as in block 2 of

Figure N4.1. The structure may consist of an equation of the form of either equation M2.30 or equation M3.11 with the effector structure having been developed specifically for the given vehicle. The effector structure may or may not include the structure and parameters of all of the vehicle control systems, depending upon whether or not the effects of such control systems can be decoupled from the vehicle or effector structure. These beginning structure and parameter specification steps are very important to the overall modeling and identification steps which follow.

Once a set of structural equations and initial parameters have been specified, simulation studies may be carried out. These studies consist of Steps P1.1 through P1.7 and represent parts or variations of the steps in blocks 3 through 10 in Figure N4.1.

<u>Overall Modeling Steps</u>

Step P1.1	Specify the vehicle
Step P1.2	Design and run a sea trial
Step P1.3	Collect input-output data from the sea trial
Step P1.4	Process sea trial data to determine vehicle parameters
Step P1.5	Decide upon adequacy of the parameters and structure
Step P1.6	Decide upon changes to be made in the structure or parameters
Step P1.7	Go to Step P1.2 and continue until satisfactory modeling is achieved

These steps can now be discussed in more detail relative to the specific problems of ocean vehicle parametric identification rather than the total problem of overall mathematical modeling.

P1.2 SPECIFYING THE VEHICLE TO BE STUDIED

In Step P1.1 the vehicle is specified in detail as to its structure and parameters and can be one of the following: the actual full-scale vehicle, a physical model of the real vehicle, or a previously developed mathematical model for simulating the real vehicle. The full-scale vehicle is the most expensive specification which can be made at this point, it is the hardest to vary structure, inputs, and parameters upon, but it undoubtedly gives the best or most realistic data from a sea trial.

A physical model of the vehicle may be used at this point and, in general, will be less expensive than the full-scale vehicle. For overall dynamic mathematical models the physical model will most likely need to be self-propelled and will probably need an elaborate measurement system for the acquisition of useful and realistic data from the model sea trials. If it is desired to mathematically model the effectors or control system structure, then these could be physically modeled in this step also. The data acquired from physical models can, in general, be expected to be less realistic than that obtained from the full scale vehicle, and some procedures for extrapolating or scaling the results of physical model identifications to the full scale vehicle must be developed and employed. One of the main advantages of using a physical model is that the vehicle and mathematical structure and the input functions are more easily changed than on full scale vehicles so that many more runs may be made with the physical model for the same expenditures.

In the case of the specific studies in this thesis, the DSRV mathematical model is specified as the vehicle in this step. In

general whenever extensive identifiability studies are to be made in order for the system modeler to increase his overall understanding of the specified structure, the mathematical model is used to simulate the vehicle in Step Pl.1. When structure selectively programmed for solution on a digital computer, the mathematical model is generally the least expensive and the most flexible way of specifying the vehicle and of actually making simulated sea trial maneuvers. The main disadvantage of a simulated model is that the data generated may be far different from the real vehicle behavior; however, if the initial structure of the model has been properly specified, the primary behavior difference will be due to different parameters in the real vehicle than in the simulation. If identifiability studies on simulated vehicles show that the parameters are identifiable, then later real vehicle data may be analyzed and the correct parameters found by identification. If the parameters are not identifiable, their input or structural variations may be indicated along with further simulation studies.

Pl.3 THE SEA TRIAL

The overall characteristics of the sea trial in Step Pl.2 will, of course, depend upon the vehicle specified in Step Pl.1. The primary consideration in designing a sea trial to be used in sophisticated model parametric identification is the fact that the mathematical model must be decoupled or broken down into manageable sub-models. Parameters are, in general, only identifiable if they are in some way excited by the vehicle effectors or are in some manner coupled into the vehicle equations of motion for a specific maneuver. Thus, the sea trial maneuvers must be designed to excite specific parameters of

interest, and the model structure must be selected specifically for the pertinent effectors, inputs, degrees of freedom, and parameters being utilized in order to have any chance of identifying the true parameters from noisy data. As more knowledge is gained concerning parameter identifiability, and as more confidence is gained in the values of the parameters; then more sophisticated models and maneuvers are employed. In this manner the system modeler "bootstraps" his way upward in complexity, understanding, and accuracy until the full-scale vehicle is adequately modeled.

For preliminary identifiability studies the sea trial should be divided into blocks of time during which specific inputs and selected degree of freedom models are used. The data generated during these blocks is then used in the model reference contouring studies with a model of the same degrees of freedom and with only the parameters of interest being varied. As an example of the block breakdown for a sea trial, Figure P1.1 shows several DSRV sea trial blocks for the simulated vehicle using some of the models in Figure N4.1. These blocks need not be run in this order, and they need not be run all at once, especially in the case of a simulated vehicle. For other types of vehicles the decouplings to single and double degree of freedom models may or may not give valuable information relative to parameter identifiability.

Once the sea trial blocks have been specified, then the inputs to the vehicle may be designed either as a set of maneuvers for the real vehicle or its physical model, or as a set of mathematical input functions for the simulated vehicle. The primary consideration in specifying an input for the purpose of identifying a parameter is

Block 1	Block 2
t_o | Surge (u) model ▷ | t_1	Sway (v) model ▷ | t_2 ▷
Propellor input, Identify X_{uu} and Propellor thrust coefficient	Horizontal thrusters input, Identify Y_{vv} and H-Thruster thruster coefficient

Block 3	Block 4
Heave (z) model ▷ | t_3	Roll (p) model ▷ | t_4 ▷
Vertical thrusters input, Identify Z_{ww} and V-Thruster thrust coefficient	Propellor input, Identify K_{pp} and Propellor torque coefficient

Block 5	Block 6
Pitch (q) model ▷ | t_5	Yaw (r) model ▷ | t_6 ▷
Vertical thrusters input, Identify M_{qq} and V-Thruster moment coefficient, $M_{qq}=C_{15}$ in secondary drag	Horizontal thrusters input, Identify N_{rr} and H-Thruster moment coefficient, $N_{rr}=C_{16}$ in secondary drag

Block 14	Block 1345
Surge, roll (u,p) model ▷ | t_7	Vertical plane with roll model (u, w, p, q) | t_8
Propellor input, Identify X_{uu}, K_{pp}, K_{up}, and propellor coefficients	Propellor, shroud inputs, Identify all primary and cross-coupled vehicle coeff., propellor and shroud coeff.

Figure P1.1 EXAMPLE SEA TRIAL BLOCKS FOR DSRV

that the input should excite the system dynamics involving that parameter as much as possible. In the literature (A-13) (A-4) (A-16) etc., an input of this type is designated "persistently exciting," has a precise definition, and is shown to be a necessary type of input for certain cases of linear system parametric identification. For nonlinear systems this all translates somewhat imprecisely as an input function whose Fourier spectrum contains all frequencies, especially those frequencies at or near the natural frequencies influenced by the parameter of interest. Step functions, impulse functions, and white noise are examples of this type of input, whereas single and multiple frequency sinusoidal functions are not.

Specifying proper inputs is extremely important to the process of parametric identification. The kinds of inputs which provide the best identifiability for parameters of interest may be determined from simulated vehicle studies and model reference contouring. In general, the proper input specification is more critical to the extended Kalman filtering technique than to model reference contouring. For a given ocean vehicle and its effectors, the kinds of inputs which may be used will be constrained by the physical limitations of the vehicle, its operators, and its effectors and by sea states. Thus, inputs designed for simulated vehicle studies should reflect these limitations.

P1.4 INPUT-OUTPUT DATA COLLECTION

Once the sea trial blocks and input functions have been designated, the actual sea trial may be conducted, with the real vehicle, its physical model, or its mathematical model; and the input-output data ($\underline{u}$, $\underline{z}$) for the parametric identification processes may be collected.

The sea trial data for the real vehicle or for its physical model is usually collected by some form of measurement system. It is important that the structure of this measurement system be specified (Chapter M3.1) and that the same structure be used in the simulated sea trials and identifiability studies as is used in the real vehicle or physical model sea trials.

The parameters for the measurement system may theoretically be identified in the same process as the sea trial parameters. However, if it is at all possible to treat the measuring process as a different system and to identify its parameters separately, then some effort should be spent in this form of parameter decoupling. For some vehicles it may be possible to transform the sea trial data into a form directly usable by the parameter identification techniques without affecting the identification process or the true values of the parameters. In this case the measurement system would not have to be included in the simulation studies.

When sea trials are conducted with full scale vehicles or with physical models, the input-output data is usually recorded on digital or analog magnetic tapes. For the case of the simulated vehicle sea trial this data may be recorded on magnetic tape, punched cards, punched paper tape, magnetic disc, high speed printer, or many other devices. In either case care must be taken to insure that the recorded input-output data is compatible with the inputs to the identification processes and with the computer programs used.

P1.5 PROCESSING THE SEA TRIAL DATA

In Step P1.4 the sea trial data is used as the input data for either the extended Kalman filtering or the model reference contouring

techniques. When the digital computer programs are set up to include the model and measurement structure and the initial or "best guess" values of the vehicle parameters, then the sea trial data can be processed to determine the parameters which were used in its generation. Both of the identification techniques utilize multiple passes over the sea trial data. The model reference contours are generated by making many simulated sea trials with different parameter values and comparing the outputs at each pass with the sea trial data. The extended Kalman filter may be run many times over the same sea trial data by using the previous-pass best estimates of the states and error covariances as the initial states and error covariances for the next pass. The outputs of these processes are contours, plots, and numerical values which may be analyzed by the system modeler.

P1.6 ANALYZING IDENTIFICATION RESULTS

The results of the identification processes are analyzed in Steps P1.5, P1.6, and P1.7 by the system modeler to determine whether or not the parameters have been identified, whether or not the identified parameters adequately model the system, and what changes are to be made in the modeling and identification processes before more sea trial runs are made. The determination as to whether the parameters have or have not been identified is made based upon the system modeler's understanding of the basic characteristics of the vehicle behavior and upon his knowledge of the identifiability characteristics of the specific parameters of interest. Whether or not the identified parameters adequately model the vehicle is determined primarily by the cost function or closeness value for that set of parameters and that set of maneuvers and by the system modeler's

criterion of that value of closeness for which he may call the vehicle adequately modeled. The changes which must be made in the modeling and identification processes are in many cases indicated by the characteristics of the output contours and plots as interpreted by the system modeler. Many of the detailed considerations for analyzing identification results are presented by example in the DSRV studies of this section and of Section C.

This chapter has presented an overall view of the steps used in developing mathematical models for ocean vehicles. The considerations used by the system modeler for specifying the vehicle, breaking the sea trial into blocks, designing the input functions, and utilizing and interpreting results from the identification techniques have been briefly described. The next chapter begins the detailed studies of the DSRV by specifying its single degree of freedom equations and conducting preliminary analytical analyses of their behavior.

CHAPTER P2

EXAMPLE SINGLE DEGREE OF FREEDOM DSRV EQUATIONS

When five of the six degrees of freedom in the mathematical models of Section M are set to their equilibrium values, the resulting six separate models are called the single degree of freedom equations for the vehicle. These separate equations no longer truely model the behavior of the vehicle, but a great deal of information about the vehicle behavior may be acquired from studying them. This chapter presents the detailed DSRV single degree of freedom equations and many of the basic considerations which may be applied to their study.

P2.1 DSRV SINGLE DEGREE OF FREEDOM EQUATION STRUCTURE AND PARAMETERS

When any combination of five of the six primary states (u, v, w, p, q, r) are set to the equilibrium values 0.0, the resulting equation form is essentially the same for all cases of the DSRV and is given by equation P2.1. For the case in which positive velocities are used,

$$\dot{x} = a\,x\,|x| + b\,\dot{u}\,|u| \qquad \text{P2.1}$$

this equation reduces to equation P2.2. Note that the u in equations

$$\dot{x} = ax^2 + bu^2 \qquad \text{P2.2}$$

P2.1 and P2.2 is the appropriate propellor or thruster revolutions per second and not the vehicle surge velocity and that the gravity terms in all of these equations have been set to zero. The essential structure of equation P2.1 is that of an object of unit mass being driven through a fluid with absolute-square drag behavior (Chapter M3.3). A more detailed surge equation with ux coupling is given by equation D3.13.

The specific DSRV characteristics for these six single degree of freedom models are presented in Table P2.1. The values of the parameters a and b in Table P2.1 are those values used in the sea trial data generation and the values of u (inputs) represent maximum values of these particular effectors used in the sea trial. The steady state value of x is calculated by solving equation P2.3 which represents the equilibrium solution to equation P2.2 when the inputs are set to their constant step-function values.

$$0 = a\, x_{ss}^2 + b\, u^2 \qquad \text{P2.3}$$

The value of an approximate equation time constant τ may be derived by linearizing equation P2.1, with the input u set at its step-function value using the points $(\dot{x}, x) = (0, x_{ss})$ and $(\dot{x}, x) = (bu|u|, 0)$. The value of the time constant for the linearized system becomes $1/(ax_{ss})$, and the linearized equation is given by equation P2.4. The value of δt is somewhat arbitrarily chosen as one-tenth of

$$\dot{x} = ax_{ss}\, x + b\, u^2 \qquad \text{P2.4}$$

a time constant. The large number of trajectory calculations in model reference contouring requires that the time steps be as large as possible to limit computation time.

P2.2 EXAMPLE OF AN EXTENDED SINGLE DEGREE OF FREEDOM EQUATION

The equation presented in equation P2.1 is the simplest structure possible which exhibits the essential motion behavior of the DSRV. There are numerous extensions, still in a single degree of freedom, which can be made and studied regarding this structure. An example is the DSRV roll equation which is extended to include the gravity

Variable	u	v	w
a	$X_{uu}/(m-X_{\dot{u}})$	$Y_{vv}/(m-Y_{\dot{v}})$	$Z_{ww}/(m-Z_{\dot{w}})$
X^i_{ii}	-16.7	-346.	-207.
i, name	1, surge	2, sway	3, heave
$(m-X^i_{\dot{x}_i})$	4507.	8683.	7963.
a	-0.0037	-0.04	-0.026
b	0.168	0.0011	0.0012
u-step	1 rps.	8 rps.	8 rps.
$x_{ss}=u\sqrt{-\frac{b}{a}}$	6.74 ft/sec	1.33 ft/sec	1.72 ft/sec
$\tau \approx 1/a\ x_{ss}$	40 sec	19 sec	23 sec
3τ	120 sec	57 sec	69 sec
δt	4 sec	2 sec	2 sec
u-type	propellor	thrusters	thrusters
Variable	**p**	**q**	**r**
a	$K_{pp}/(I_x-K_{\dot{p}})$	$M_{qq}/(I_y-M_{\dot{q}})$	$N_{rr}/(I_z-N_{\dot{r}})$
X^i_{ii}	$-3.3*10^4$	$-6.7*10^5$	$-1.1*10^6$
i, name	4, roll	5, pitch	6, yaw
$(m-X^i_{\dot{x}_i})$	$4.23*10^4$	$8.62*10^5$	$8.6*10^5$
a	-0.78	-0.78	-1.3
b	0.0126	0.00023	0.00021
u-step	1 rps.	8 rps.	8 rps.
$x_{ss}=u\sqrt{-\frac{b}{a}}$	.126 rad/sec	.136 rad/sec	.102 rad/sec
$\tau \approx 1/a\ x_{ss}$	10 sec	9 sec	8 sec
3τ	30 sec	27 sec	24 sec
δt	.5 sec	.5 sec	.5 sec
u-type	propellor	thrusters	thrusters

Table P2.1 CHARACTERISTICS OF DSRV SINGLE DEGREE OF FREEDOM MODELS

and angular effects. When the vehicle angles are included as state variables, then the DSRV roll equation appears as the two-state system given by equations P2.5 and P2.6. If these states are combined, the second-order nonlinear differential equation P2.7 in the roll angle φ results (for a simplified, 2-parameter system).

$$\left.\begin{aligned} \dot{\varphi} &= p \\ \dot{p} &= a\, p\, |p| + c \sin\varphi + b\, u\, |u| \end{aligned}\right\} \quad \begin{aligned} &\text{P2.5} \\ &\text{P2.6} \end{aligned}$$

$$\ddot{\varphi} - a\, \dot{\varphi}\, |\dot{\varphi}| + c \sin\varphi = b\, u\, |u| \qquad \text{P2.7}$$

Where: $c = 0.444$ for DSRV

The system in the above equations is a perfect example of what is called Newtonian Damping. The characteristics of this nonlinear differential equation are discussed in detail by Struble (S-8, p. 20), where the properties of solution existence, uniqueness, continuity, and stability are all shown to be valid for this set of equations. This extended single degree of freedom model is presented to show some of the considerations which might be necessary in studying the single degree of freedom equations of a general ocean vehicle. The parameters a, b, and c could now be studied for their identifiability characteristics using model reference contouring and could also be identified using the extended Kalman filtering technique.

P2.3 ANALYTICAL STUDIES OF SINGLE DEGREE OF FREEDOM EQUATIONS

If the system modeler can show that the single degree of freedom equations for his specific ocean vehicle are well behaved in a mathematical sense, then he has some degree of confidence in the proper behavior of multiple degree of freedom models. Precise generalizations

cannot be made in this respect because of the nonlinear nature of the equations, but the verification of the analytical properties of these multiple degree of freedom models is usually much more difficult than for the single degree of freedom equations.

The homogeneous portion of equation P2.2 is a subcase ($a = a_2$) of the more general homogeneous nonlinear differential equation P2.8. The general solution to this equation is given by equation P2.9, and the more specialized solution for $a_1 = 0$ is given by equation P2.10.

$$\left.\begin{aligned} \dot{x} &= a_1 x + a_2 x^2 \\ x(t_o) &= x_o \end{aligned}\right\} \qquad \text{P2.8}$$

$$x(t) = \frac{x_o \, e^{a_1(t-t_o)}}{1 + x_o \, a_2 \left[1 - e^{a_1(t-t_o)}\right]/a_1} \qquad \text{P2.9}$$

$$x(t) = \frac{x_o}{1 - x_o \, a_2 \, (t-t_o)} \quad \text{for } a_1 = 0 \qquad \text{P2.10}$$

The fact that the derivative $d\dot{x}/dx$ is continuous means that equation P2.8 satisfies the Lipschitz condition (S-8) (H-7). This guarantees that a solution exists which is unique and continuous (Equation P2.9). The equilibrium solutions to equation P2.8 are $x = 0$ and $x = -a_1/a_2$. For the cases in which $a_1 < 0$, the equilibrium solution $x = 0$ is stable, and the equilibrium solution $x = -a_1/a_2$ is unstable.

Another analytical consideration is one which applies to the nonhomogeneous equation P2.2 and concerns the nonlinear observability of this equation when expressed as a state estimation problem. This test is discussed in Chapter M3.1, is developed in detail by Schoenwandt (S-5), and is applied to equation P2.2 in Appendix All.

The results in Appendix A11 show that the nonlinear state estimation problem for equation P2.2 is observable and, therefore, that the states may be estimated based upon input-output information. This is very important information relative to the utilization of this equation in an extended Kalman filter. The nonlinear observability test also gives useful information concerning certain characteristics which the input function must have in order that observability be possible. In the case of equation P2.2, the input cannot be zero or the steady-state step value of u for the states to be observable.

As one might expect, the application of the nonlinear observability criterion to any but the simplest equations is extremely difficult. When applicable, however, the information gained relative to the observability of the system and to the characteristics of the input is very useful in modeling and identification studies.

P2.4 STOCHASTIC SINGLE DEGREE OF FREEDOM MODELS

The single degree of freedom models discussed thus far in this chapter have been deterministic and have contained none of the structural, input, and measurement uncertainties discussed in Chapter M4. The structure of the stochastic models for the DSRV in single degrees of freedom is given by equations P2.11 and P2.12. Note that

$$\dot{x} = a\, x^2 + b\, u^2 + w \qquad \text{P2.11}$$

$$z = x + v \qquad \text{P2.12}$$

the w and v in these equations represent zero mean gaussian white process and measurement noises and not the heave and sway velocities

of the vehicle. Equations P2.11 & P2.12 are solved discretely using v_n & w_n.

The process noise w appears to affect the system in a completely linear fashion, but in actuality, the fact that the state variable x is squared at each time increment causes the process noise to enter the system nonlinearly. The measurement noise v enters the measurements of the states in a linear fashion; but when the model reference cost function is calculated, the v noise is essentially squared at each time increment. The fact that both of these zero-mean gaussian processes are squared can cause biases in both the system outputs and the identification technique outputs. The reason for this is that the square of a zero-mean gaussian random variable is no longer of zero mean as shown in Appendix A10.

In later studies of single and multiple degree of freedom DSRV parameters, the amounts of w_n and v_n noises are expressed as percentage values %w and %v. The process noise percentage %w means that the standard deviation ($\sigma_w = \sqrt{Q}$) of w_n is that percentage of the maximum value of $\dot{x}$ in equation P2.11 for a step function maneuver u. The measurement noise percentage %v means that the standard deviation ($\sigma_v = \sqrt{R}$) of v_n is that percentage of the maximum value of x being measured in equation P2.12 for a step function maneuver u. These are convenient definitions for simulation studies, but they must be interpreted properly for a given maneuver. If $\dot{x}$ or x is at a small value for most of the maneuver and then assumes its maximum value only for a short period during the maneuver, then the associated w or v noise has a much greater effect upon the overall system uncertainty than it does if $\dot{x}$ or x is at its maximum value for most of the maneuver.

This chapter has presented the structure, numerical values of the parameters, and several analytical considerations for the DSRV single degree of freedom equations with and without process and measurement noises. These equations may now be used to simulate sea trials for the simulated vehicle input-output data. This data may then be processed for parameter identifiability studies using model reference contouring in the next chapter (P3) or for actual parameter identifications using extended Kalman filtering in Chapter P4.

MODEL REFERENCE STUDIES OF SINGLE DEGREE OF FREEDOM DSRV EQUATIONS

This chapter presents an in-depth study of the DSRV surge equation using model reference contouring and varying the types of inputs, the amount of process noise w, and the amount of measurement noise v. The next chapter presents the results of similar studies, with many of the same inputs and noise levels as this chapter, using extended Kalman filtering. In this manner the detailed application of these techniques to simple ocean vehicle models is explained, and the surge parameters for the DSRV are examined to determine their identifiability characteristics relative to these two techniques, different inputs, and different noise levels.

The exact numerical values of the two DSRV surge parameters and the specific vehicle and model equations to be studied in this chapter are presented in Figure P3.1. The reader should remember the values $p_1^* = -16.7$ and $p_2^* = 755.$ because these are the "true" or optimal parameters used throughout this chapter and the next. The results of the identification studies are classified and presented in this chapter by the propellor input function types: step, staircase, crashback full astern, impulse, and sinusoidal of different periods. One version of the Fortran IV computer program is presented in Appendix A12, and the contouring and plotting subroutines used are presented in Appendix A5.

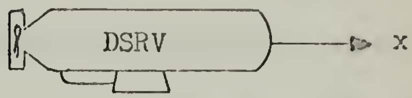
P3.1 MODEL REFERENCE SURGE CONTOURS USING A STEP FUNCTION INPUT

Given an ocean vehicle to test, the most obvious thing to do is to "turn it on and see what happens." Let the DSRV propellor

<u>Vehicle</u> (Simplified as a 2 parameter system, see Equation D3.13)

$$m\dot{x} = X^*_{uu}\ x\,|x| + C^*_p\ u\,|u| + w$$

$$z = x + v \quad ; \quad x(t_o) = 0$$



numerically:

$$4507.\ \dot{x} = -16.7\ x\,|x| + 755.\ u\,|u| + w$$

$$z = x + v \quad ; \quad \text{solved discretely for } z_n = z(t_n)$$

Where: $p^*_1 = X^*_{uu} = -16.7$ = Surge Drag Coefficient

$p^*_2 = C^*_p = 755.$ = Propellor Thrust Coefficient

$m = 4507.$ slugs = mass of the DSRV

w_n = gaussian white noise = N (0,Q) ; discrete

v_n = gaussian white noise = N (0,R) ; discrete

x = surge velocity in ft/sec

u = propellor rotation in rev/sec

<u>Math. Model</u>

$$m\dot{x} = p_1\ x\,|x| + p_2\ u\,|u|$$

$$z_m = x \quad ; \quad x(t_o) = 0 \quad ; \quad \text{solved discretely at times } t_n$$

Where: p_i is in the domain $[p^*_i - \delta \le p_i \le p^*_i + \delta]$,

with $\delta \approx \frac{1}{2}\ p^*_i$;

In other words, vary p_i approximately $\pm$ 50% of p^*_i

$m = 4507.$ slugs

Figure P3.1 <u>DSRV SURGE EQUATIONS AND PARAMETER VALUES</u>

input u(t) be given by the step function in Figure P3.2 for the sea trial length of 470 seconds. With this input and the computer program of Appendix A12 a simulated sea trial may be computed for

the DSRV in surge. A plot of this simulated sea trial without any process or measurement noise is computed by SUBROUTINE PLOT in Appendix A5 and shown in Figure P3.3. Note that the plot is read using the left side as the time axis and placing the origin in the upper left corner of Figure P3.3. A similar plot for the step function input with 10% v and 10% w noises (See Chapter P2.4) is presented in Figure P3.4.

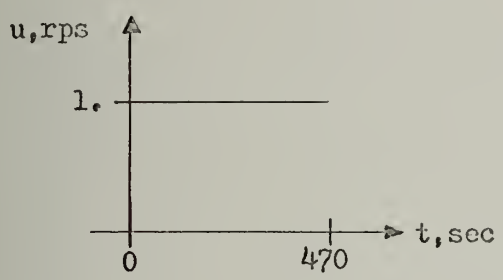


Figure P3.2 PROPELLOR STEP FUNCTION INPUT

With a step function input the DSRV accelerates from 0 to 6.7 ft/sec exponentially in about 90 seconds for a drag coefficient of -16.7 and a propellor coefficient of 755. If the mathematical model is now used with the same input and different parameters, a mathematical model sea trial can be generated. If the output of this sea trial is subtracted from the vehicle sea trial at each instant of time, and the result is squared and summed over the sea trial; then the resulting value C(p) (See Chapter N2) is generated and represents the closeness of this model to the vehicle over that sea trial. If these C(p) values are generated for many different values of the parameters, then the resulting set of C(p)'s may be contoured to

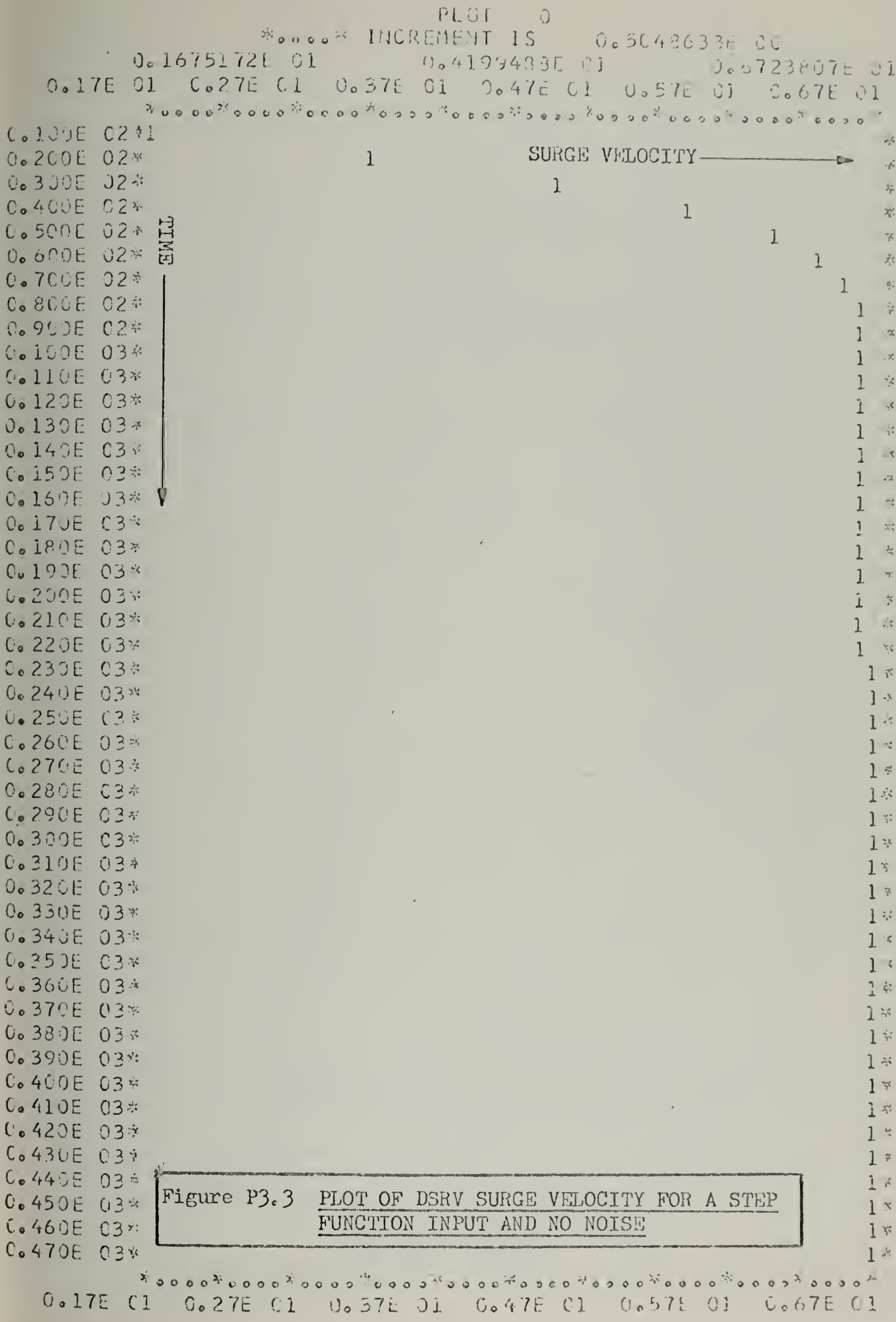


Figure P3.3 PLOT OF DSRV SURGE VELOCITY FOR A STEP FUNCTION INPUT AND NO NOISE

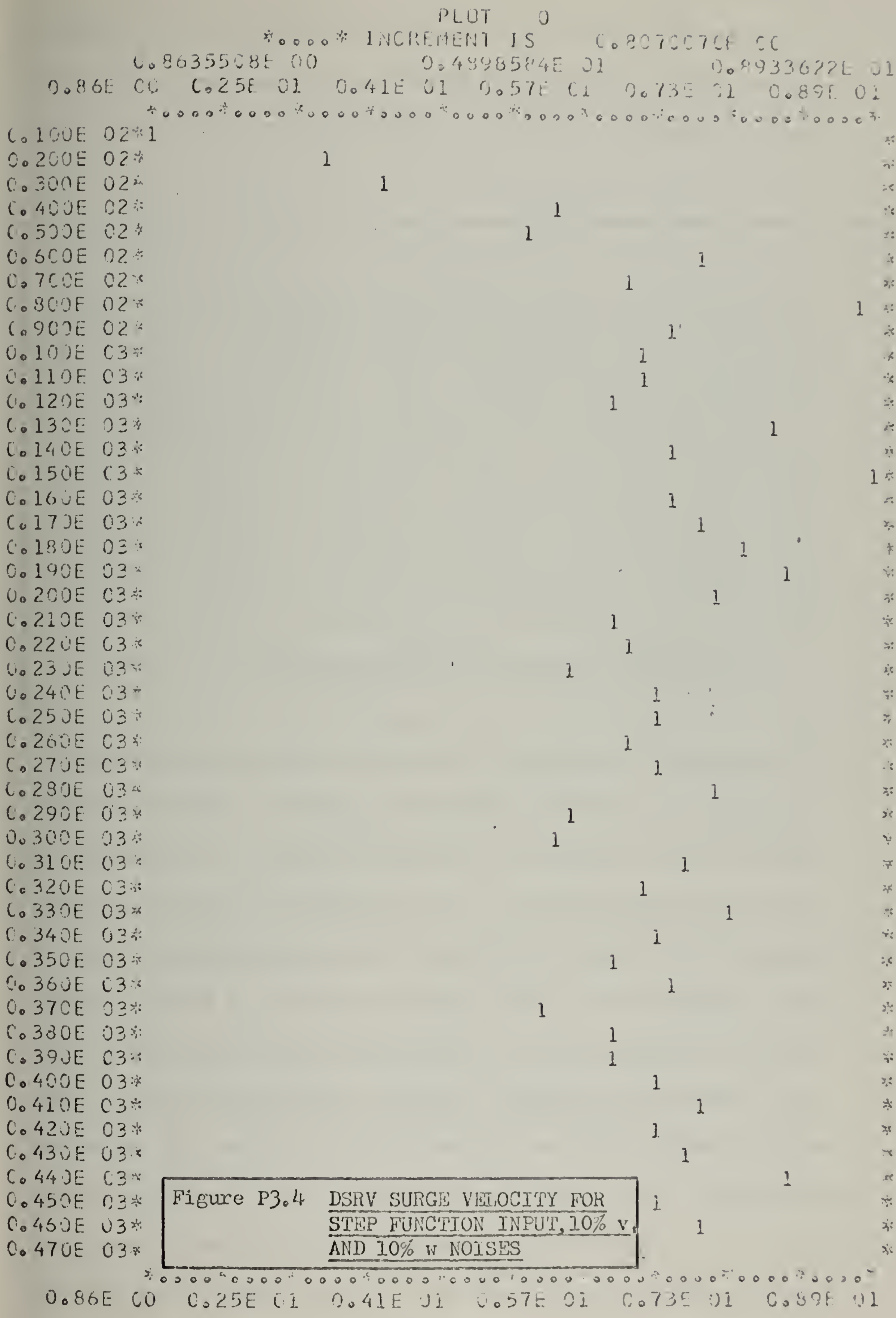


Figure P3.4 DSRV SURGE VELOCITY FOR STEP FUNCTION INPUT, 10% v, AND 10% w NOISES

determine the identifiability characteristics of $\underline{p}$. The contours for the step function input of Figure P3.2 and for no noise using the sea trial data in Figure P3.3 is presented in Figures P3.5 and P3.6.

```
****************************************************************
*                 CONTOUR    1   PARAMETERS                    *
* X RANGE :-0.2220E 02 TO -0.1070E 02 DX= 0.2500E 00*
* Y RANGE : 0.5550E 03 TO  0.1055E 04 DY= 0.1000E 02*
* Z DOMAIN: 0.3729E-09 TO  0.4021E 04 DZ= 0.2011E 03*
* Z DOMAINS FOR THE CONTOURS :MAX VALUES FOR EACH   *
* NO. 1 0.2011E 01 NO. 8 0.1409E 04 NO.15 0.2817E 04*
* NO. 2 0.2031E 03 NO. 9 0.1611E 04 NO.16 0.3018E 04*
* NO. 3 0.4041E 03 NO.10 0.1812E 04 NO.17 0.3219E 04*
* NO. 4 0.6052E 03 NO.11 0.2013E 04 NO.18 0.3420E 04*
* NO. 5 0.8063E 03 NO.12 0.2214E 04 NO.19 0.3621E 04*
* NO. 6 0.1007E 04 NO.13 0.2415E 04 NO.20 0.3822E 04*
* NO. 7 0.1208E 04 NO.14 0.2616E 04 NO.21 0.4021E 04*
****************************************************************
```

Figure P3.5 NUMERICAL VALUES OF CONTOURS IN FIGURE P3.6

The detailed numerical values are automatically printed out by SUBROUTINE CONTUR in Appendix A5 whenever a contour is generated. In Figure P3.5 the X-RANGE corresponds to the range of p_1, and the Y-RANGE corresponds to the range of p_2. These detailed values of p_1 and p_2 are printed along the edges of the contour with X running vertically and Y horizontally in Figure P3.6. The horizontal values of p_2 are set such that the last number in the exponential corresponds to the numerical value for that * location on the axis. At the top of the contour, the left, center, and right values of p_2^* are given with greater accuracy than the axis values. The Z-DOMAIN values correspond to the minimum and maximum values of $C(\underline{p})$ over the contour. The contours in Figure P3.6 run from the minimum or 1-value to the

```
                       CONTOUR    1
              *....* INCREMENT IS     0.5000000E 02
      0.5550000E 03          0.8050000E 03           0.1055000E 04
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
              *....*....*....*....*....*....*....*....*....*....*
-0.222E 02*8887776666555544443333332222222222222222222222222*
-0.219E 02*8887776666655554444433333322222222222222222222222*
-0.217E 02*8877766665555444443333333222222222222222222222222*
-0.214E 02*8877766655555444443333333222222222222222222222222*
-0.212E 02*7777666655555444443333333222222222222222222222222*
-0.209E 02*7777666555544444333333322222222222222222222222222*
-0.207E 02*7776665555544444333333322222222222222222222222222*
-0.204E 02*7766665555544443333333222222222222222222222222222*
-0.202E 02*7766655555444443333332222222222222222222222222222*
-0.199E 02*6666555554444433333322222222222222222222222222222*
-0.197E 02*6666555554444333333222222222222222222222222222222*
-0.194E 02*6665555444433333332222222222222222222222222222222*
-0.192E 02*6655554444433333322222222222222222222222222222233*
-0.189E 02*6655544444333333222222222222222222222222222222333*
-0.187E 02*5555344443333332222222222222222222222222222223333*
-0.184E 02*5555444433333222222222222222222222222222222233333*
-0.182E 02*5554444443333322222222222222222222222222222333333*
-0.179E 02*5544444333333222222222222222222222222222223333333*
-0.177E 02*5544444333333222222222222221222222222222222333334*
-0.174E 02*4444433333322222222222222122222222222222233333344*
-0.172E 02*4444433333222222222222221112222222222222233333344444*
-0.169E 02*44443333322222222222222111222222222222233333334444*
-0.167E 02*444333333222222222222221012222222222222233333344444 5*
-0.164E 02*4443333322222222222222211122222222222222333333444444 55*
-0.162E 02*44333332222222222222211122222222222222333333344445555*
-0.159E 02*33333322222222222222111222222222222223333333444445556*
-0.157E 02*33333222222222222221112222222222222333333444444555556*
-0.154E 02*3333322222222222222122222222222222333333444444555566666*
-0.152E 02*33332222222222222222222222222222222333333444445555666667*
-0.149E 02*33322222222222222222222222222222333333444444555566667 77*
-0.147E 02*3322222222222222222222222222222333333444445555666667778*
-0.144E 02*33222222222222222222222222222223333333444445555666677788 8*
-0.142E 02*22222222222222222222222222222233333344444555556666777888 9*
-0.139E 02*2222222222222222222222222222233333344444455566666777888999*
-0.137E 02*2222222222222222222222222222333333444455556666777788899AA*
-0.134E 02*222222222222222222222222233333344444555566677788899AAB*
-0.132E 02*2222222222222222222222233333334444455556667778889 99AAABB*
-0.129E 02*222222222222222222222233333334444455556667778888 99AAABBCC*
-0.127E 02*2222222222222222222233333344445555666777888999AABBBCCD*
-0.124E 02*222222222222222222223333333444445555666677788999AABBBCCDDE*
-0.122E 02*22222222222222222233333344444555556667778889 9AAABBCCCDDEEF*
-0.119E 02*2222222222222222333334444555566667788899 9AABBCCDDEEFFG*
-0.117E 02*222222222222223333344444555566667778899 9AABBCCCDDEEFGGH*
-0.114E 02*22222222222233333444445555666777888899AABBBCCDDEEFFGHHJ*
-0.112E 02*22222222223333334444455566667788899AAABBCCDDEEFFGHHJJK*
-0.109E 02*2222222223333334444455566667778899 9AABBCCDDEEFFGHHJJKLL*
-0.107E 02*22222223333334444555566677 88899AABBCCDDEEFFGHHJJKLLMN*
              *....*....*....*....*....*....*....*....*....*....*
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
```

Figure P3.6 SURGE, NO NOISE, STEP FUNCTION CONTOURS

maximum or M-value in linear increments of DZ. The Z-DOMAINS descriptions in Figure P3.5 correspond to the 21 numbers and letters 1 through M used in the contour.

The studies in this thesis required that large numbers of plots and contours be generated. There are, of course, much better methods for generating contours and plots than the printer method used here. However, the author's experience has been that the sub-routines of Appendix A5 provide a cost savings factor of approximately 10 when compared to the SC4020 or CALCOMP. Each of the contours and plots in this thesis along with many others not in this thesis were generated at a cost well under 5¢ each. The author hopes that the reader will not experience too much conceptual or visual trouble interpreting these plots and contours.

The reader can see from P3.6 that for a step function input the cost function looks somewhat like a "trough" running upward from left to right. The contours here look like those of contour 9 in Figure N4.4, and show that for a step function input p_1 and p_2 are almost linearly dependent in the form of equation P3.1. The reason

$$p_2 = -40\ p_1 + 88 \qquad \text{P3.1}$$

for this behavior is that the input (step) function causes $\dot{x}$ to be nearly zero for most of the sea trial at the steady-state value of $x = 6.7$ ft/sec. Substituting these values into the vehicle equation of Figure P3.1 gives equations P3.2 and P3.3. The difference between

$$0 = p_1\ (6.7)^2 + p_2\ (1)^2 \qquad \text{P3.2}$$

$$p_2 = -45\ p_1 \qquad \text{P3.3}$$

equation P3.1 and P3.3 is caused by the dynamics of the vehicle in the first 90 seconds of the sea trial in Figure P3.3 and is the only reason that the parameters are identifiable in Figure P3.6.

A better insight into the shape of the contours in Figure P3.6 may be gained by plotting 5 equally spaced vertical slices of these contours as in Figure P3.7. Here the numbers 1 through 5 represent these vertical slices running from the left side to the right side of Figure P3.6. The most obvious characteristic of these contours is the very flat minimum along the bottom of the trough. In the case of this differential equation, the flatness is caused by the fact that the input does not excite the dynamic behavior of the vehicle for much of the sea trial. This means either that the parameters are not easily identified in the basic model structure or that the input function is not exciting the vehicle dynamics enough to identify the parameters. It is highly probable that small amounts of noise introduced into the vehicle will obscure the parameters and prohibit their identification.

When the same contour calculation steps are used with the 10% w and 10% v noisy sea trial data in Figure P3.4, the resulting contour is shown in Figures P3.8 and P3.9. This contour has essentially the same shape as the noiseless one in Figure P3.6, but there are some very important differences. The most significant difference is that the minimum values of the noisy contour are shifted away from the known "true" values of -16.7 and 755 to the values of -14.9 and 700. This shift is most likely due to the biases generated by the two noise processes and has the effect of making the parameters unidentifiable if 85% or better accuracy is required. In addition, if 85% accuracy

```
                              PLOT    1
                  *ooooo* INCREMENT IS     0.4020002E 03
         0.1283484E 01          0.2011285E 04          0.4021288E 04
    0.13E 01  0.81E 03  0.16E 04  0.24E 04  0.32E 04  0.40E 04
          *oooo*oooo*oooo*oooo*oooo*oooo*oooo*oooo*oooo*oooo*
-0.222E 02*5 3    2        1                                *
-0.219E 02*5 3    2        1                                *
-0.217E 02*5 3    2       1                                 *
-0.214E 02*5 3   2        1                                 *
-0.212E 02*453   2       1                                  *
-0.209E 02*45    2       1                                  *
-0.207E 02*45   2       1                                   *
-0.204E 02*45   2       1                                   *
-0.202E 02*45   2      1                                    *
-0.199E 02*45  2       1                                    *
-0.197E 02*4 5 2      1                                     *
-0.194E 02*4 5 2      1                                     *
-0.192E 02*4  52     1                                      *
-0.189E 02*4  5      1                                      *
-0.187E 02*4  5     1                                       *
-0.184E 02*4 25     1                                       *
-0.182E 02*4 2 5   1                                        *
-0.179E 02*342 5   1                                        *
-0.177E 02*34   5 1                                         *
-0.174E 02*34   5 1                                         *
-0.172E 02*324   5                                          *
-0.169E 02*324   15                                         *
-0.167E 02*3 4   15                                         *
-0.164E 02*3  4 1   5                                       *
-0.162E 02*3  4 1    5                                      *
-0.159E 02*3   4     5                                      *
-0.157E 02*3   4      5                                     *
-0.154E 02*3  1 4      5                                    *
-0.152E 02*23 1 4       5                                   *
-0.149E 02*23 1  4       5                                  *
-0.147E 02*231    4       5                                 *
-0.144E 02*2 3     4       5                                *
-0.142E 02*2 3      4        5                              *
-0.139E 02*213       4        5                             *
-0.137E 02*21 3       4         5                           *
-0.134E 02*21  3       4         5                          *
-0.132E 02*21  3        4          5                        *
-0.129E 02*21   3        4           5                      *
-0.127E 02*12    3         4           5                    *
-0.124E 02*12     3         4            5                  *
-0.122E 02*12      3          4            5                *
-0.119E 02*1 2      3           4             5             *
-0.117E 02*1  2      3           4              5           *
-0.114E 02*1  2       3            4               5        *
-0.112E 02*1   2        3             4               5     *
-0.109E 02*1    2        3              4                5  *
-0.107E 02* 1    2         3               4               5*
          *oooo*oooo*oooo*oooo*oooo*oooo*oooo*oooo*oooo*oooo*
    0.13E 01  0.81E 03  0.16E 04  0.24E 04  0.32E 04  0.40E 04
```

Figure P3.7 PLOT OF 5 LEFT TO RIGHT VERTICAL SLICES OF FIGURE P3.6

```
**********************************************************************
*                 CONTOUR    4   PARAMETERS                          *
* X RANGE :-0.2220E 02 TO -0.1070E 02 DX= 0.2500E 00*
* Y RANGE : 0.5550E 03 TO  0.1055E 04 DY= 0.1000E 02*
* Z DOMAIN: 0.2729E 03 TO  0.4314E 04 DZ= 0.2021E 03*
* Z DOMAINS FOR THE CONTOURS :MAX VALUES FOR EACH    *
* NO. 1 0.2749E 03 NO. 8 0.1689E 04 NO.15 0.3104E 04*
* NO. 2 0.4770E 03 NO. 9 0.1892E 04 NO.16 0.3306E 04*
* NO. 3 0.6791E 03 NO.10 0.2094E 04 NO.17 0.3508E 04*
* NO. 4 0.8812E 03 NO.11 0.2296E 04 NO.18 0.3710E 04*
* NO. 5 0.1083E 04 NO.12 0.2498E 04 NO.19 0.3912E 04*
* NO. 6 0.1285E 04 NO.13 0.2700E 04 NO.20 0.4114E 04*
* NO. 7 0.1487E 04 NO.14 0.2902E 04 NO.21 0.4314E 04*
**********************************************************************
```

Figure P3.8 NUMERICAL VALUES OF CONTOURS IN FIGURE P3.9

is acceptable, then the closeness of the model to the vehicle is $C(\underline{p}^*) = 273.$ for the noisy case whereas it was $C(\underline{p}^*) = 0.$ for the noiseless case, compared to a maximum value of over 4000 in both cases. Thus, although the trough has remained essentially the same shape, the bottom has been raised significantly enough to somewhat obscure the true values of the parameters.

The contour in Figure P3.9 shows that v and w noises greatly affect the identifiability of the parameters; the question then becomes "For a step function input, which type of noise has the most significant effect on identifiability?" It must be kept in mind throughout this analysis that the primary effect of the v and w noises is to make the optimum $\underline{p}$ and the minimum cost function values $C(\underline{p}^*)$ into random variables of which these contours are samples. Contours were also generated for the cases (v=0, w=10%) and (v=10%, w=0), and the essential results of those contours along with the two

CONTOUR 4

```
                    *oooo* INCREMENT IS      0.50000000E 02
         0.55500000E 03            0.80500000E 03            0.10550000E 04
   0.56E 03    0.66E 03    0.76E 03    0.86E 03    0.96E 03    0.11E 04
                    *oooo*oooo*oooo*oooo*oooo*oooo*oooo*oooo*oooo*oooo*
-0.222E 02*3887776666555544444333333322222222222222222222222*
-0.219E 02*8887776665555444443333333222222222222222222222222*
-0.217E 02*8877766655555444444333333222222222222222222222222*
-0.214E 02*8877766655555444443333333222222222222222222222222*
-0.212E 02*7777666655554444433333332222222222222222222222222*
-0.209E 02*7776666555544444333333322222222222222222222222222*
-0.207E 02*7776665555544444333333322222222222222222222222222*
-0.204E 02*7766665555544444333333222222222222222222222222222*
-0.202E 02*7766655555444443333332222222222222222222222222222*
-0.199E 02*6666555554444433333322222222222222222222222222222*
-0.197E 02*5666555544444333333222222222222222222222222222223*
-0.194E 02*6665555444433333322222222222222222222222222222233*
-0.192E 02*6655554444433333222222222222222222222222222222333*
-0.189E 02*6655544444333332222222222222222222222222222223333*
-0.187E 02*5555444433333222222222222222222222222222222333333*
-0.184E 02*5555444433332222222222222222222222222222223333333*
-0.182E 02*5554444333333222222222222222222222222222233333333*
-0.179E 02*5544444333333222222222222222222222222222233333334*
-0.177E 02*5544443333332222222222222222222222222222233333344*
-0.174E 02*4444433333322222222222222222222222222223333334444*
-0.172E 02*4444333332222222222222222222222222222233333334444*
-0.169E 02*4444333332222222222222222222222222222233333344445*
-0.167E 02*4443333322222222222222222222222222223333333444455*
-0.164E 02*4433333222222222222222222222222222233333344444555*
-0.162E 02*3333332222222222222222222222222223333334444455555*
-0.159E 02*3333322222222222222222222222222233333334444455556*
-0.157E 02*3333222222222222221222222222222233333334444555566*
-0.154E 02*3332222222222222112222222222222333333344445555666*
-0.152E 02*3332222222222222112222222222223333333444445556667*
-0.149E 02*3332222222222221122222222222233333334444455556667777*
-0.147E 02*3322222222222211222222222222233333344444555666677778*
-0.144E 02*2222222222222112222222222222333334444445555666677788*
-0.142E 02*222222222221122222222222223333334444455556667778889*
-0.139E 02*22222222222122222222222223333334444555566677788899*
-0.137E 02*222222222222222222222222333333444455556667778899AA*
-0.134E 02*22222222222222222222222333334444455566667779889AAAB*
-0.132E 02*2222222222222222222222333333444445555666777889999AABB*
-0.129E 02*222222222222222222223333334444555666677788899AAABBCC*
-0.127E 02*22222222222222222223333344445555666777888999AABBCCCD*
-0.124E 02*2222222222222222223333334444555666677788899AABBBCCDDE*
-0.122E 02*22222222222222223333344444555666777888999AAABBCCDDEEF*
-0.119E 02*2222222222222223333344444555566677888899AAABBCCDDEEFFG*
-0.117E 02*2222222222222233333444455566677788999AABBCCDDEEFFGGH*
-0.114E 02*222222222223333344444555666777888999AABBBCCDDEEFGGHHJ*
-0.112E 02*2222222222333334444455566677788899AAABBCCDDEEFFGHHJJK*
-0.109E 02*2222222233333444455566677788999AABBCCDDEEFFGHHJJKLL*
-0.107E 02*222222233333444455566677388899AABBCCDDEEFFGHHJJKLLMM*
                    *oooo*oooo*oooo*oooo*oooo*oooo*oooo*oooo*oooo*oooo*
   0.56E 03    0.66E 03    0.76E 03    0.86E 03    0.96E 03    0.11E 04
```

Figure P3.9 SURGE, 10% w, 10%v, STEP FUNCTION CONTOURS

previously discussed are presented in Table P3.1. This table shows that both process and measurement noises have significant effects upon the identifiability characteristics of the parameters, but that v noise has more significance for $C(\underline{p}^*)$ than w noise.

Step Input	$p_1^*(-16.7)$	$p_2^*(755)$	$C\ (\underline{p}^*)$	C max	Comments
v=0, w=0	-16.7	755	0.0	4021.	Trough shape; Fig. N4.4 #9
v=0, w=10%	-17.9	825	11.6	3902.	Same
v=10%, w=0	-17.9	805	305.0	4390.	Same
v=10%, w=10%	-14.9	700	273.0	4314.	Same

Table P3.1 SURGE EQUATION IDENTIFIABILITY CHARACTERISTICS FOR A STEP INPUT

The detailed plots and contours for the step function input have been presented here to show the reader how the previous identifiability considerations and the numerical values in Table P3.1 were arrived at. For the remainder of the studies in this chapter, only the basic noiseless plots and contours will be presented. The noisy plots and contours will be described in tables of the same form as Table P3.1.

P3.2 MODEL REFERENCE SURGE CONTOURS USING A STAIRCASE FUNCTION INPUT

The step function input previously discussed only excited the dynamics of the vehicle for 90 seconds of the 470 second sea trial. The staircase function input in Figure P3.10 excites vehicle dynamics over most of the sea trial duration and proves to be a much better identification input than the step function. The DSRV propellor takes about 3 seconds to come up to speed (B-9), and so the step and

staircase function inputs are realistic from the standpoint of the 40 second vehicle time constant and the 470 second sea trial.

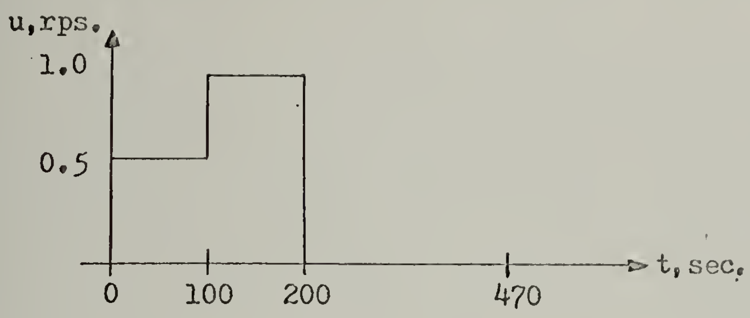


Figure P3.10 PROPELLOR STAIRCASE FUNCTION INPUT

The resulting noiseless sea trial data is given by Figure P3.11 and the noiseless contours are presented in Figures P3.12 and P3.13.

```
*****************************************************************
*                CONTOUR    1  PARAMETERS                       *
* X RANGE :-0.2220E 02 TO -0.1070E 02 DX= 0.2500E 00*
* Y RANGE : 0.5550E 03 TO  0.1055E 04 DY= 0.1000E 02*
* Z DOMAIN: 0.3228E-09 TO  0.1105E 04 DZ= 0.5525E 02*
* Z DOMAINS FOR THE CONTOURS :MAX VALUES FOR EACH      *
* NO. 1 0.5525E 00 NO. 8 0.3873E 03 NO.15 0.7741E 03*
* NO. 2 0.5580E 02 NO. 9 0.4426E 03 NO.16 0.8293E 03*
* NO. 3 0.1111E 03 NO.10 0.4978E 03 NO.17 0.8846E 03*
* NO. 4 0.1663E 03 NO.11 0.5531E 03 NO.18 0.9398E 03*
* NO. 5 0.2216E 03 NO.12 0.6083E 03 NO.19 0.9951E 03*
* NO. 6 0.2768E 03 NO.13 0.6636E 03 NO.20 0.1050E 04*
* NO. 7 0.3321E 03 NO.14 0.7188E 03 NO.21 0.1105E 04*
*****************************************************************
```

Figure P3.12 NUMERICAL VALUES OF CONTOURS IN FIGURE P3.13

The contours in Figure P3.13 show a much more clearly defined optimum than in the case of the step function input of Figure P3.6. The staircase input is designed to specifically excite both p_1 and p_2. The first two steps in the propellor excite the equation through p_2

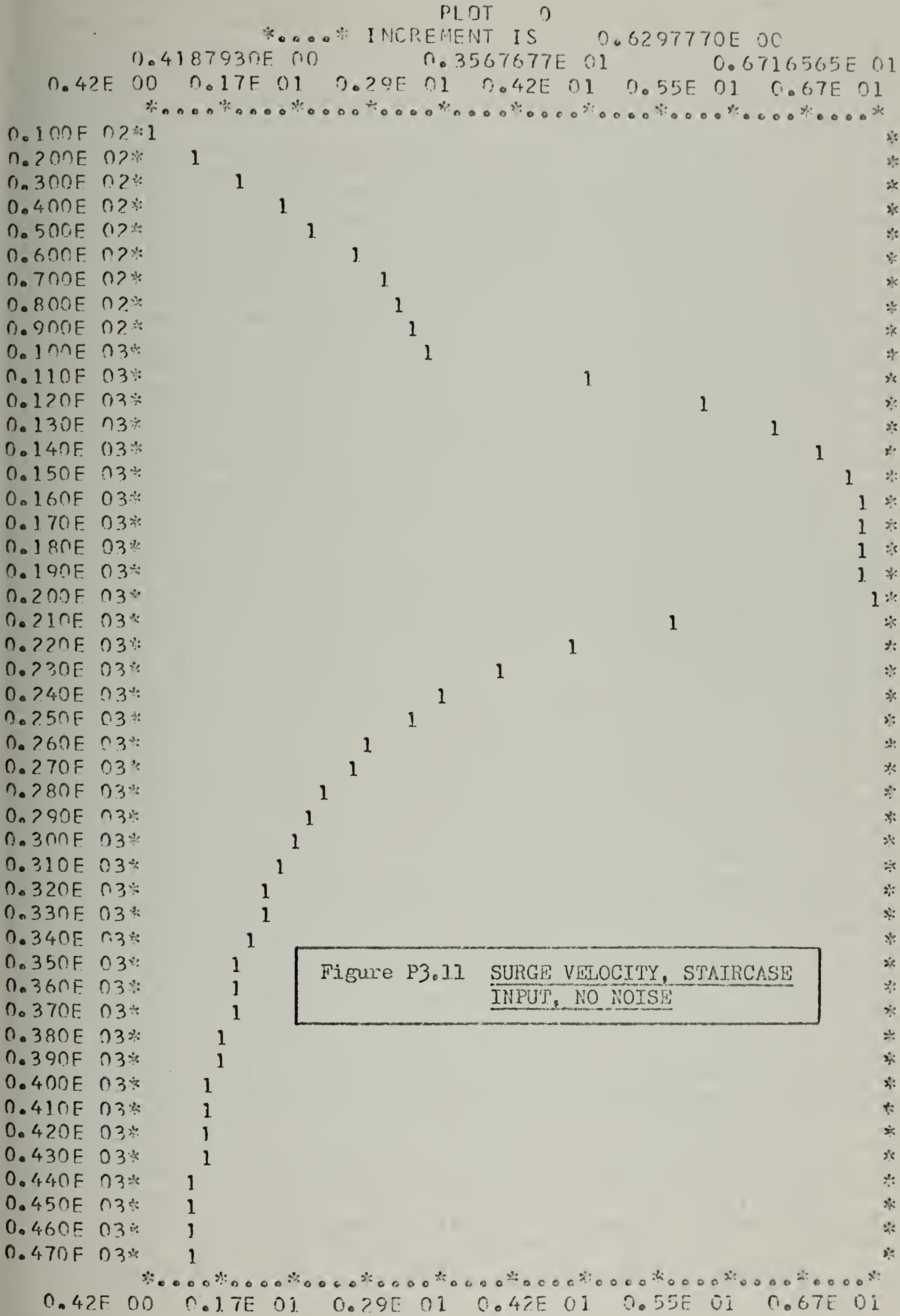


Figure P3.11 SURGE VELOCITY, STAIRCASE INPUT, NO NOISE

```
                              CONTOUR    1
                     *....* INCREMENT IS     0.5000000E 02
          0.5550000E 03            0.8050000E 03            0.1055000E 04
     0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
               *....*....*....*....*....*....*....*....*....*....*
-0.222E 02*888777666655554444444333333333332222222222222333333*
-0.219E 02*888777666655554444444333333333322222222222222333333*
-0.217E 02*887776666555554444433333333333222222222222222333333*
-0.214E 02*887776665555544444333333333322222222222222222333333*
-0.212E 02*777766665555444443333333333222222222222222222333333*
-0.209E 02*777666555544444433333333322222222222222222222333333*
-0.207E 02*777666555544444443333333322222222222222222222333333*
-0.204E 02*776666555544444333333333222222222222222222222333333*
-0.202E 02*776665555444443333333222222222222222222222222333333*
-0.199E 02*666655554444433333322222222222222222222222222333333*
-0.197E 02*666655544444333333322222222222222222222222222333333*
-0.194E 02*666555544443333333222222222222222222222222223333333*
-0.192E 02*666555444443333332222222222222222222222222223333333*
-0.189E 02*665554444443333322222222222222222222222222223333333*
-0.187E 02*555544443333332222222222222222222222222222223333334*
-0.184E 02*555444433333322222222222222222222222222222333333334*
-0.182E 02*555444433333222222222222222222222222222222333333344*
-0.179E 02*555444433333222222222222222222222222222223333333444*
-0.177E 02*554444333332222222222222222222222222222233333334444*
-0.174E 02*554444333322222222222222222222222222222333333344444*
-0.172E 02*444443333222222222222222222222222222223333333444444*
-0.169E 02*444443333222222222222222222222222222222333333444445*
-0.167E 02*444333332222222221122222222222222222233333344444555*
-0.164E 02*443333322222222221112222222222222222233333344444555*
-0.162E 02*443333322222222222112222222222222223333334444455556*
-0.159E 02*443333322222222222222222222222222233333344444455556*
-0.157E 02*443333322222222222222222222222222233333344444555566*
-0.154E 02*333332222222222222222222222222223333334444455555667*
-0.152E 02*333332222222222222222222222222223333334444555556677*
-0.149E 02*333322222222222222222222222222233333344444555566677*
-0.147E 02*333322222222222222222222223333334444455556667777888*
-0.144E 02*333322222222222222222222233333344445555666677788888*
-0.142E 02*333322222222222222222222333334444455556666777888899*
-0.139E 02*333322222222222222222233333344444555566677788899999*
-0.137E 02*333222222222222222222333333444445555666677788899AA*
-0.134E 02*3332222222222222222233333334444455566677788899 9AAB*
-0.132E 02*33332222222222222223333334444455566677778899 9AAABB*
-0.129E 02*333322222222222233333333444555566667778889 9AAABBCC*
-0.127E 02*33333322222223333333344444555666777888999AABBBCCD*
-0.124E 02*3333333333333333333444445555666777788999AABBBCCDDE*
-0.122E 02*333333333333333344444555566667778899AAABBCCDDEEE*
-0.119E 02*333333333333333344444555566667778899AAABBCCDDDEEFF*
-0.117E 02*3333333333334444444555566667778889 9AAABBCCDDDEEFFGH*
-0.114E 02*33333333334444444455556666777888999AABBCCDDDEEFFGHHJ*
-0.112E 02*444344444444444455556666777888999AABBCCDDDEEFGGHHJJK*
-0.109E 02*4444444444445555566667778889 99AABBCCDDDEFFGGHHJJKLL*
-0.107E 02*44444444445555566667778889 99AABBCCDDEEFFGGHHJJKLLMM*
               *....*....*....*....*....*....*....*....*....*....*
     0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
```

Figure P3.13 SURGE, NO NOISE, STAIRCASE FUNCTION CONTOURS

up to 200 seconds into the sea trial; and the propellor shutoff at 200 seconds causes only p_1, the drag coefficient to be in the dynamic equation of the vehicle until the end of the sea trial at 470 seconds.

A large number of identifiability studies for different noise combinations were conducted with the staircase input, and the essential results are shown in Table P3.2. There are several conclusions which can be drawn from this table relative to the identifiability characteristics of p_1 and p_2. In the first place, the parameters are essentially unidentifiable (to $\pm$ 50%) for v or w greater than 20%. This says that if the structure or measurements are 20% erroneous, then the parameters cannot be identified to within $\pm$ 50%

No.	Staircase Input		$p_1^*(-16.7)$	$p_2^*(755)$	C ($\underline{p}$*)	C max	Comments; Same as No.
	%v	%w					
1	0	0	-16.7	755	0	1105	Oval Shaped Bowl; Fig.N4.4 #9; 2%
2	0	1	-16.7	755	.41	1091	Same, Slightly Larger Min. 5%
3	0	10	-16.5	730	27	1213	Same, Larger Minimum 10%
4	0	100	-22.2	1000	4048R	5943	Bowl, Shifted Minimum
5	0	200	-22.2	550	7781R	12750	Fig N4.4 #7 Shifted Min.
6	1	0	-16.7	750	1.5	1072	Same as #2
7	1	1	-16.5	750	2.18	1093	" #2
8	1	10	-14.2	700	23.8	969	" #2
9	1	100	-22.2	1050	4740R	6627	" #4
10	1	200	-22.2	660	7163R	10460	" #5
11	10	0	-15.9	720	282	1421	" #2

Table P3.2 SURGE EQUATION IDENTIFIABILITY CHARACTERISTICS FOR A STAIRCASE INPUT

No.	Staircase Input		$p_1^*(-16.7)$	$p_2^*(755)$	C (p^*)	C max	Comments: Same as No.
	%v	%w					
12	10	1	-14.7	690	242	1305	Same as #2
13	10	10	-18.7	850	354	1457	" #2
14	10	100	-22.2	550	4079R	9080	" #5
15	10	200	-22.2	550	12940R	18380	" #5
16	100	0	-17.4	550	23470R	25600	" #5
17	100	1	-10.7	900	22500R	24790	Reversed Slope of No. 5 above
18	100	10	-22.2	730	29780R	33320	Same as #4
19	100	100	-22.2	1050	32410R	35190	" #4
20	100	200	-22.2	600	28390R	33490	" #5
21	200	0	-22.2	950	101600R	104600	" #4
22	200	1	-22.2	550	77510R	83280	" #5
23	200	10	-22.2	550	76700R	81800	" #5
24	200	100	-22.2	550	103600R	113600	" #5
25	200	200	-10.7	550	66830R	67960	" #4
26	2	2	-16.2	750	3.3	1112	" #1
27	2	5	-16.7	800	8,2	1044	" #1
28	2	20	-16.7	700	105.2	1400	" #3
29	2	30	-22.2	970	430R	1765	" #4
30	2	50	-20.7	660	821	2890	" #4
31	5	2	-16.2	750	3.0	1089	" #3
32	5	5	-15.7	700	11.1	1096	" #2
33	5	20	-12.7	680	84.8	950	" #2

Table P3.2 (Contd.) SURGE EQUATION IDENTIFIABILITY CHARACTERISTICS FOR A STAIRCASE INPUT

No.	Staircase Input		$p_1^*(-16.7)$	$p_2^*(755)$	C ($\underline{p}^*$)	C max	Comments: Same as No.
	%v	%w					
34	5	30	-22.2	970	466R	1751	Same as #4
35	5	50	-22.2	900	958	2456	" #4
36	20	2	-15.7	700	282	1397	" #2
37	20	5	-13.9	680	252	1301	" #2
38	20	20	-22.2	960	542	1941	" #4
39	20	30	-16.7	550	554R	2510	" #4
40	20	50	-22.2	930	2925R	5623	" #4
41	100	2	-17.7	550	23410R	25600	" #4
42	100	5	-10.7	880	22230R	24450	" #17
43	100	20	-22.2	700	29410R	33170	" #4
44	100	30	-18.4	1050	26390	28290	" #4
45	100	50	-22.2	900	24090	26700	" #4
46	200	2	-22.2	950	101800	104900	" #4
47	200	5	-22.2	555	77350	83120	" #5
48	200	20	-22.2	555	77010	82130	" #5
49	200	30	-22.2	555	98490	106300	" #5
50	200	50	-18.9	1050	63620	64470	" #4

Table P3.2 (Contd.) SURGE EQUATION IDENTIFIABILITY CHARACTERISTICS FOR A STAIRCASE INPUT

R = Relative Minimum

accuracy using model reference contouring. This gives some indication as to the importance of proper model structure and measurements in system identification. Secondly, the larger number of results in Table P3.2 than in Table P3.1 gives some indication of the randomness of $\underline{p}^*$ and C ($\underline{p}^*$) as the noise levels are increased. Thirdly, it is

obvious from Table P3.2 that the model reference contouring method is not well suited to extremely noisy (> 30%) systems because the contours, actually random variables, are only seen at one instant of time and cannot be averaged to get some "mean" of behavior Finally, this table shows that measurement noise v has a much more significant effect upon the value C ($\underline{p}^*$) and the identifiability of the parameters than does the process noise w in this case.

P3.3 MODEL REFERENCE SURGE CONTOURS USING A CRASHBACK FULL ASTERN INPUT

Whereas the staircase input was designed to somewhat evenly excite both the p_1 and p_2 dynamics, the crashback full astern input is designed to excite the entire equation with a large transient at the center of the sea trial. This input is shown in Figure P3.14 and the noiseless sea trial data resulting from it is shown in Figure P3.15.

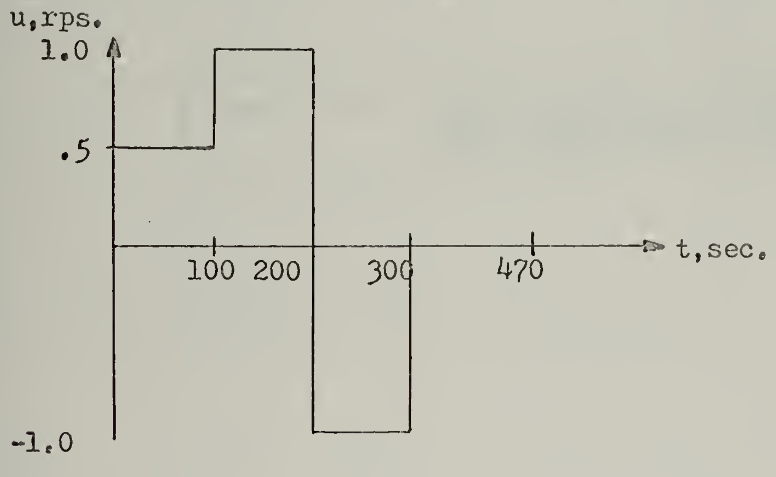


Figure P3.14 CRASHBACK FULL ASTERN INPUT FUNCTION

Since more vehicle dynamic motions are excited by this input than either the staircase or the step functions, the contours can be expected

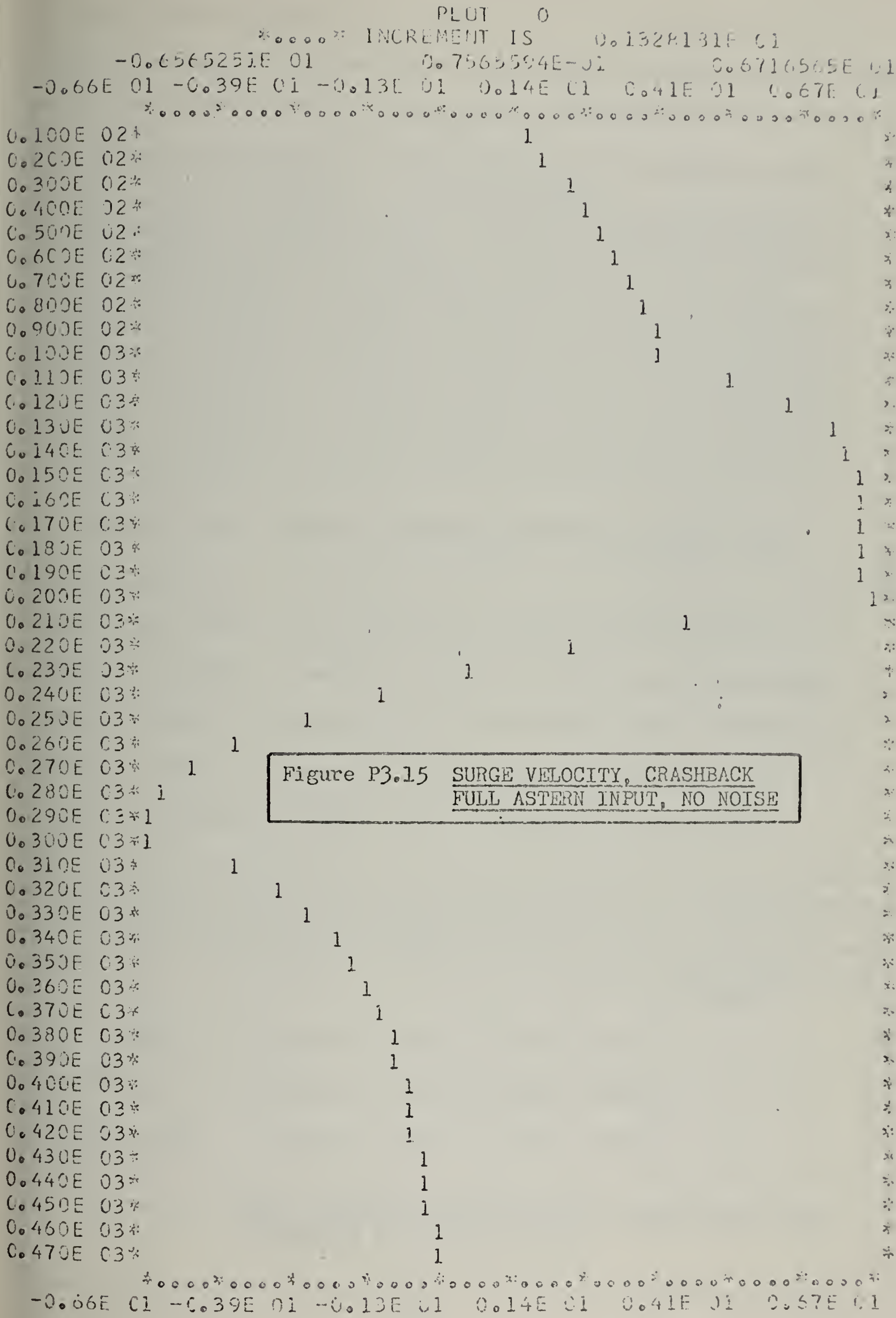


Figure P3.15 SURGE VELOCITY, CRASHBACK FULL ASTERN INPUT, NO NOISE

to have a more clearly defined optimum. This is precisely the case as seen in Figures P3.16 and P3.17. The contours in this case are "bowl"

```
*************************************************************
*                CONTOUR    1   PARAMETERS                  *
* X RANGE :-0.2220E 02 TO -0.1070E 02 DX= 0.2500E 00*
* Y RANGE : 0.5550E 03 TO  0.1055E 04 DY= 0.1000E 02*
* Z DOMAIN: 0.6229E-09 TO  0.1417E 04 DZ= 0.7083E 02*
* Z DOMAINS FOR THE CONTOURS :MAX VALUES FOR EACH    *
* NO. 1 0.7083E 00 NO. 8 0.4965E 03 NO.15 0.9923E 03*
* NO. 2 0.7154E 02 NO. 9 0.5673E 03 NO.16 0.1063E 04*
* NO. 3 0.1424E 03 NO.10 0.6382E 03 NO.17 0.1134E 04*
* NO. 4 0.2132E 03 NO.11 0.7090E 03 NO.18 0.1205E 04*
* NO. 5 0.2840E 03 NO.12 0.7798E 03 NO.19 0.1276E 04*
* NO. 6 0.3548E 03 NO.13 0.8506E 03 NO.20 0.1346E 04*
* NO. 7 0.4257E 03 NO.14 0.9215E 03 NO.21 0.1417E 04*
*************************************************************
```

Figure P3.16 NUMERICAL VALUES OF CONTOURS IN FIGURE P3.17

shaped with a fairly flat bottom but a clearly defined minimum. There is no longer any evidence of the partially linear dependence which was found in the case of the step function input. Several of the noise behavior characteristics for the crashback full astern input are presented in Table P3.3. The numbers in this table

Crashback Input	$p_1^*(-16.7)$	$p_2^*(755)$	$C(\underline{p}^*)$	C max	Comments
v=0, w=0	-16.7	755	0.	1417	Bowl Shaped, flat bottom
v=0, w=10%	-17.4	800	20.	1387	" "
v=10%, w=0	-15.7	700	290.	1875	" "
v=10%, w=10%	-15.4	850	246.	1190	" "

Table P3.3 SURGE EQUATION IDENTIFIABILITY CHARACTERISTICS FOR A CRASHBACK FULL ASTERN INPUT

```
                         CONTOUR    1
              *.....* INCREMENT IS     0.5000000E 02
     0.5550000E 03          0.8050000E 03          0.1055000E 04
 0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
          *....*....*....*....*....*....*....*....*....*....*
-0.222E 02*8888776666555554444443333333333333333334444444455555666*
-0.219E 02*8888776666555554444443333333333333333334444444455555666*
-0.217E 02*8887776665555544444433333333333333333334444444455555566*
-0.214E 02*8887776665555444444333333333333333333334444444455555566*
-0.212E 02*8887766655555444443333333333333333333333444444455555566*
-0.209E 02*8877766655554444443333333333333333333333444444455555566*
-0.207E 02*8877666555554444443333333333333333333333444444455555566*
-0.204E 02*8877666555544444333333333222222233333333344444455555666*
-0.202E 02*7776665555444443333333322222222222333333344444455555666*
-0.199E 02*7776665555444443333333222222222222233333334444445555566*
-0.197E 02*7776655554444333333322222222222222223333333444444555566*
-0.194E 02*7766655554444333333222222222222222223333333444444555566*
-0.192E 02*7766655444443333332222222222222222222333333344444555566*
-0.189E 02*7766555444443333322222222222222222222233333344444555566*
-0.187E 02*7766555444433333222222222222222222222233333344444555566*
-0.184E 02*6666554444333333222222222222222222222223333334444455555666*
-0.182E 02*6665554443333322222222222222222222222233333344445555666 7*
-0.179E 02*6665554443333222222222222222222222222233333344445555666 7*
-0.177E 02*6665544443333222222222222222222222222233333444445555666 7*
-0.174E 02*6665544433333222222222222222222222222233333444445556665 77*
-0.172E 02*6655544433332222222222222222222222222333334444555566677*
-0.169E 02*6655544433332222222222222222222222222333334444555666777*
-0.167E 02*6655444333322222222222211222222222222333334444555666777*
-0.164E 02*6655444333322222222222111222222222222333344445556667778*
-0.162E 02*6655444333322222222222211222222222222333344445556667778*
-0.159E 02*6655444333322222222222222222222222223333444455566677788*
-0.157E 02*5555444333322222222222222222222222233333444455555666777888*
-0.154E 02*5555444333322222222222222222222223333344445556667778889*
-0.152E 02*5555444333322222222222222222222223333344445555666777888999*
-0.149E 02*5555444333322222222222222222222233333344445555666777888999*
-0.147E 02*5555444333322222222222222222223333334444555566677788899A*
-0.144E 02*5555444333322222222222222222233333344445555666777888899AA*
-0.142E 02*5555444333322222222222222223333344445555666777888899AAB*
-0.139E 02*5555444333322222222222222233333344445555666777888999AABB*
-0.137E 02*5555444333333222222222223333333444455556667778899AABBC*
-0.134E 02*6655444433333322222222333333344444555566677788899AABBCC*
-0.132E 02*6655544433333333333333333333444444555666777888999AABBCCC*
-0.129E 02*6655544444333333333333333334444445555566677788899AABBCCDD*
-0.127E 02*6655544444333333333333334444455555666777888999AABBCCDDE*
-0.124E 02*6665554444433333333333444444555556667778889999AABBCCDDEEF*
-0.122E 02*6665555444444444344444444455555666777888999AABBCCDDEEFG*
-0.119E 02*6666555544444444444444444555566667778889999AABBCCDDEEFFGG*
-0.117E 02*7766555555444444444444555555666667778888999AABBCCDDEEFFGHH*
-0.114E 02*7766655555555444445555555566666777888999AABBCCDDEEFFGGHJJ*
-0.112E 02*7776666555555555555555556666677778889999AABBCCDDEEFFGHHJKK*
-0.109E 02*7777666666555555555556666677777888999AABBCCDDEEFFGGHHJJKLL*
-0.107E 02*8877766666666666666666777778889999AAABBCCDDEEFFGHHJKKLMN*
          *....*....*....*....*....*....*....*....*....*....*
 0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
```

Figure P3.17 SURGE, NO NOISE, CRASHBACK FULL ASTERN CONTOURS

indicate approximately that 10% noises still permit parameter identifications to within about 10% accuracy. The fact that the contours remain bowl shaped even when noises are added indicates that this input function is a good one for identifying these parameters using model reference contouring.

P3.4 <u>MODEL REFERENCE SURGE CONTOURS USING A SIMULATED IMPULSE FUNCTION INPUT</u>

An impulse function is an unrealistic input for the DSRV propellor but such a function may be simulated by the input function in Figure P3.18. This input function excites the dynamics of p_2 very

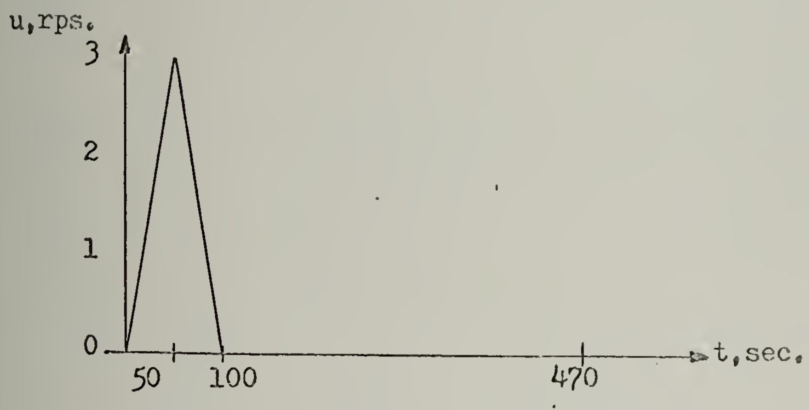


Figure P3.18 SIMULATED IMPULSE INPUT FUNCTION

strongly, but only for the first 100 seconds of the sea trial. The dynamics associated with p_1 are in effect for the entire sea trial.

The sea trial which results from the simulated impulse function input is shown in Figure P3.19, and the resulting model reference contours are shown in Figures P3.20 and P3.21. Several of the noise behavior characteristics for this input function are shown in Table P3.4.

```
                                    PLOT    0
                      *....* INCREMENT IS      0.2170053E 01
          0.0                        0.1085026E 02          0.2170053E 02
  0.0          0.43E 01  0.87E 01  0.13E 02  0.17E 02  0.22E 02
         *....*....*....*....*....*....*....*....*....*....*
0.100E 02*1                                                *
0.200E 02* 1                                               *
0.300E 02*     1                                           *
0.400E 02*                 1                               *
0.500E 02*                                  1              *
0.600E 02*                                                1*
0.700E 02*                               1                 *
0.800E 02*                           1                     *
0.900E 02*                   1                             *
0.100E 03*              1                                  *
0.110E 03*          1                                      *
0.120E 03*        1                                        *
0.130E 03*      1                                          *
0.140E 03*     1                                           *
0.150E 03*     1                                           *
0.160E 03*    1                                            *
0.170E 03*    1                                            *
0.180E 03*   1                                             *
0.190E 03*   1                                             *
0.200E 03*   1                                             *
0.210E 03*  1                                              *
0.220E 03*  1                                              *
0.230E 03*  1                                              *
0.240E 03*  1                                              *
0.250E 03* 1                                               *
0.260E 03* 1                                               *
0.270E 03* 1                                               *
0.280E 03* 1                                               *
0.290E 03* 1                                               *
0.300E 03* 1                                               *
0.310E 03* 1                                               *
0.320E 03* 1                                               *
0.330E 03* 1                                               *
0.340E 03* 1                                               *
0.350E 03*1                                                *
0.360E 03*1                                                *
0.370E 03*1                                                *
0.380E 03*1                                                *
0.390E 03*1                                                *
0.400E 03*1                                                *
0.410E 03*1                                                *
0.420E 03*1                                                *
0.430E 03*1                                                *
0.440E 03*1                                                *
0.450E 03*1                                                *
0.460E 03*1                                                *
0.470E 03*1                                                *
         *....*....*....*....*....*....*....*....*....*....*
  0.0          0.43E 01  0.87E 01  0.13E 02  0.17E 02  0.22E 02
```

Figure P3.19 SURGE VELOCITY, SIMULATED IMPULSE INPUT, NO NOISE

```
*****************************************************************
*              CONTOUR    1   PARAMETERS                        *
* X RANGE :-0.2220E 02  TO  -0.1070E 02  DX= 0.2500E 00*
* Y RANGE : 0.5550E 03  TO   0.1055E 04  DY= 0.1000E 02*
* Z DOMAIN: 0.1299E-08  TO   0.2808E 04  DZ= 0.1404E 03*
* 7 DOMAINS FOR THE CONTOURS :MAX VALUES FOR EACH       *
* NO. 1 0.1404E 01 NO. 8 0.9841E 03 NO.15 0.1967E 04*
* NO. 2 0.1418E 03 NO. 9 0.1124E 04 NO.16 0.2107E 04*
* NO. 3 0.2822E 03 NO.10 0.1265E 04 NO.17 0.2248E 04*
* NO. 4 0.4226E 03 NO.11 0.1405E 04 NO.18 0.2388E 04*
* NO. 5 0.5629E 03 NO.12 0.1546E 04 NO.19 0.2528E 04*
* NO. 6 0.7033E 03 NO.13 0.1686E 04 NO.20 0.2669E 04*
* NO. 7 0.8437E 03 NO.14 0.1826E 04 NO.21 0.2808E 04*
*****************************************************************
```

Figure P3.20 NUMERICAL VALUES FOR FIGURE P3.21

Simulated Impulse	$p_1^*(-16.7)$	$p_2^*(755)$	C ($\underline{p}^*$)	C max	Comments
v=0, w=0	-16.7	755	0.	2808.	Bowl shaped, flat bottom
v=0, w=10%	-16.7	760	28.	2697.	" "
v=10%, w=0	-17.2	755	301.	3338.	" "
v=10%, w=10%	-15.9	755	329.	2942.	" "

Table P3.4 SURGE EQUATION IDENTIFIABILITY CHARACTERISTICS FOR A SIMULATED IMPULSE INPUT

The contour shapes in Figure P3.21 are quite similar to those for the crashback full astern input in Figure P3.17. This indicates that the amount of dynamic behavior excited in each case is about the same and that the impulse function is an input which greatly facilitates the identification of p_1 and p_2. The noise behavior of this system for an impulse function is seen to be much better than that for a

```
                             CONTOUR    1
                  *....* INCREMENT IS    0.5000000E 02
        0.5550000E 03          0.8050000E 03          0.1055000E 04
   0.56E 03  0.66E 03  0.76E 03  0.86E 03  0.96E 03  0.11E 04
          *....*....*....*....*....*....*....*....*....*....*
-0.222E 02*888777666655555444444444333333333333333333333444444*
-0.219E 02*888777666655555444444444333333333333333333333444444*
-0.217E 02*887776666555554444444443333333333333333333334444444*
-0.214E 02*77777666555554444444333333333333333333333333344444*
-0.212E 02*777766665555444444433333333333333333333333333334444*
-0.209E 02*777666665555444444333333333333333333333333333334444*
-0.207E 02*777666555554444443333333333333333333333333333334444*
-0.204E 02*776665555544444333333333322222222333333333333334444*
-0.202E 02*776665555444443333333333222222222223333333333334444*
-0.199E 02*666655554444433333333322222222222222233333333333444*
-0.197E 02*666655544444333333332222222222222222222333333334444*
-0.194E 02*666555544443333333222222222222222222222333333334444*
-0.192E 02*665555444433333322222222222222222222222233333334444*
-0.189E 02*665544444333333222222222222222222222222333333334444*
-0.187E 02*555554443333332222222222222222222222223333333344444*
-0.184E 02*555444433333322222222222222222222222223333333444444*
-0.182E 02*555444443333222222222222222222222222233333333444444*
-0.179E 02*554444433333222222222222222222222222233333334444444*
-0.177E 02*554444333332222222222222222222222222223333333444445*
-0.174E 02*554444433332222222222222222222222222223333334444445*
-0.172E 02*554443333322222222222222222222222222233333334444455*
-0.169E 02*444443333222222222222222222222222222333333444444555*
-0.167E 02*444333332222222222221122222222222223333334444445555*
-0.164E 02*444333332222222222111222222222222223333333444455555*
-0.162E 02*444333332222222222211222222222222223333334444455556*
-0.159E 02*444333322222222222222222222222222333333444444555566*
-0.157E 02*444333332222222222222222222222223333334444445555666*
-0.154E 02*443333322222222222222222222222223333334444455556667*
-0.152E 02*443333322222222222222222222223333334444455556666677*
-0.149E 02*443333322222222222222222222223333334444555566666777*
-0.147E 02*443333322222222222222222222233333344444555566667778*
-0.144E 02*443333222222222222222222222333333444445555666777888*
-0.142E 02*443333222222222222222222223333334444455556666777889*
-0.139E 02*443333322222222222222223333333444445555666777788899*
-0.137E 02*443333333222222222223333333344444555566667778889999A*
-0.134E 02*443333333222222223333333344444455556666777888999AAA*
-0.132E 02*4433333333333333333333333344444455556666777888899AAABB*
-0.129E 02*444333333333333333333344444455566667778889999AABBBC*
-0.127E 02*444433333333333333334444444555566667778889999AAABBCCD*
-0.124E 02*444443333333333334444444555556666777888999AAABBCCDDD*
-0.122E 02*444444444444444444444455555666677788899AAABBCCCDDEE*
-0.119E 02*444444444444444444455555666677788899AAABBCCDDDEEFF*
-0.117E 02*554444444444444555555666677778889 9AAABBCCDDDEEFFGG*
-0.114E 02*55555555555555555556666667777888999AABBBCCDDEFEFFGGHH*
-0.112E 02*55555555555555566666677778889 99AAABBBCCDDEEFFGGHHJJK*
-0.109E 02*6655555555666666666677777888899 9AAABBCCCDDEEFFGGHHJJKKL*
-0.107E 02*66666666666666777778888999AAABBBCCDDEEEFFGGHJJKKLLMM*
          *....*....*....*....*....*....*....*....*....*....*
   0.56E 03  0.66E 03  0.76E 03  0.86E 03  0.96E 03  0.11E 04
```

Figure P3.21 <u>SURGE, NO NOISE, SIMULATED IMPULSE FUNCTION CONTOURS</u>

crashback full astern input. This is due partly to the fact that it is a better type of input function and partly to the fact that the large amplitude of x in Figure P3.19 makes the 10% step noise factors behave effectively as much lower percentage noises.

P3.5 MODEL REFERENCE SURGE CONTOURS USING SINUSOIDAL INPUT FUNCTIONS

The previous inputs examined using model reference contouring for the DSRV surge equation have all had discontinuous time derivatives and have been of a "persistently exciting" (A-13) nature. A single frequency sinusoidal input function has neither of these characteristics, but it is shown to be a valid input for the identification of parameters p_1 and p_2 when it has the proper period of oscillation. A too-long period of oscillation does not sufficiently excite the dynamic behavior of the equation to permit identification, and a too-short period of oscillation does not permit the surge velocity x to reach enough amplitude to override the 10% step noises or to permit identification. The sinusoidal inputs used in these tests have amplitudes of 1.414 rps. and periods of 100, 150, 300, and 500 seconds.

A sinusoidal input function enters the surge equations in Figure P3.1 as a constant value minus a cosine function of twice the frequency because of the trigonometric relationship in equation P3.4. With a step (P3.28 lower part) function input the system takes about

$$\sin^2 \theta t = (1 - \cos 2\theta t)/2 \qquad \text{P3.4}$$

$$\theta = 2\pi/TP \quad ; \quad TP = \text{Period} \qquad \text{P3.5}$$

80 seconds to approach steady state in Figure P3.3, therefore, it

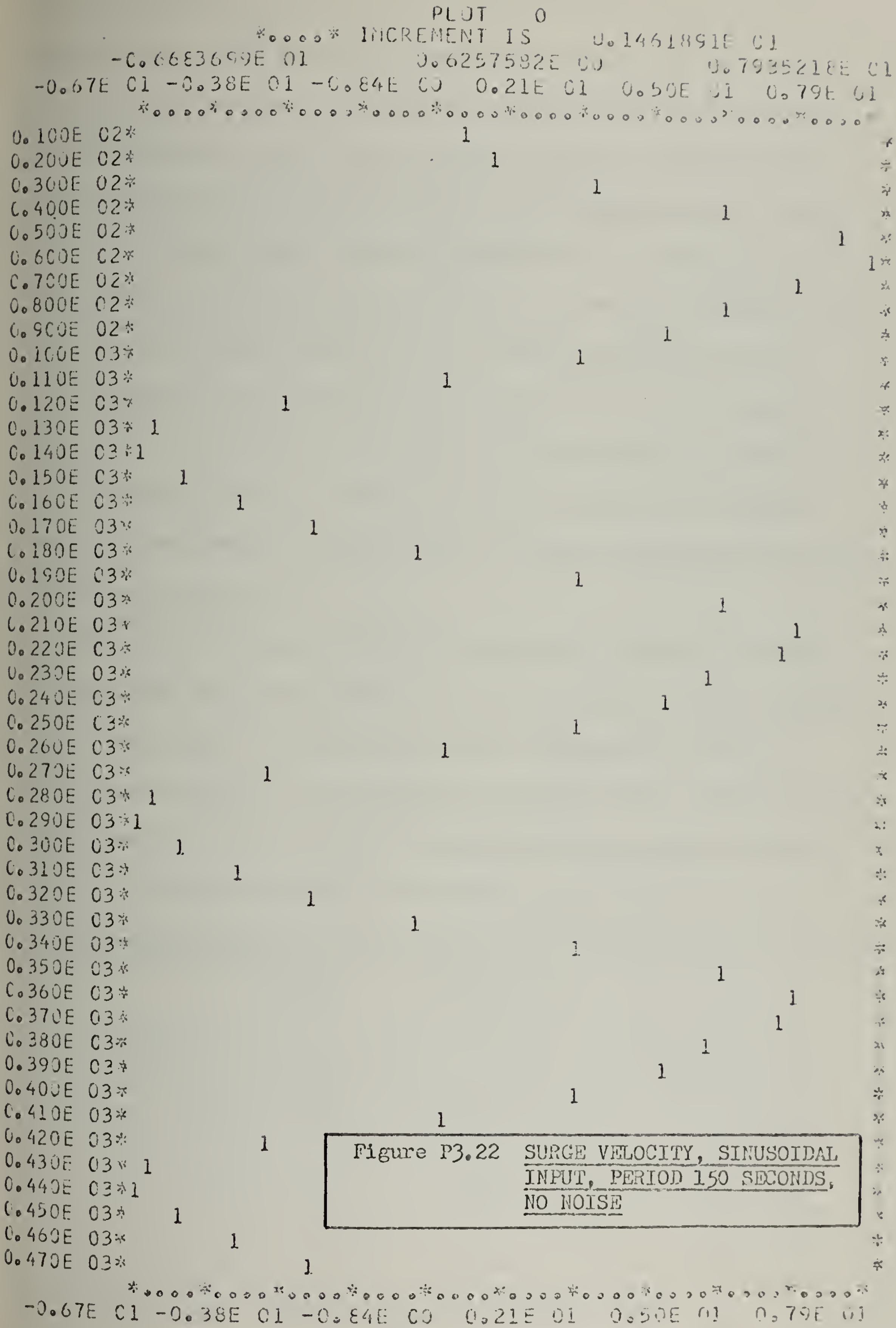


Figure P3.22 SURGE VELOCITY, SINUSOIDAL INPUT, PERIOD 150 SECONDS, NO NOISE

might be intuitively expected that the period of the sinusoidal function which would continually excite the dynamics of the surge equation would be about 160 seconds. Indeed, of the four different period sinusoidals used in these studies, the 150 second sinusoidal input yields the best identifiability for both parameters p_1 and p_2.

The sea trial data resulting from the use of a 150 second sinusoidal input is plotted in Figure P3.22. A close look at this figure shows that the output function is not exactly sinusoidal in shape but is similar to a combination of two $\sin^2$ terms as in equation P3.4, one for $x \geq 0$ and one for $x < 0$. The sea trial data plots for 100, 300, and 500 second sinusoidal function inputs are of approximately the same shape as Figure P3.22 with their corresponding periods and are not shown here.

The model reference contours for sinusoidal inputs of periods 100, 150, 300, and 500 seconds are presented in Figures P3.24 through P3.27 respectively. The detailed contour parameters for the 150 second sinusoidal input are presented in Figure P3.23. The parameters for the contours using the other sinusoidal inputs are not shown in detail, but their essential characteristics appear in later noise characteristics tables.

```
***********************************************************************
*                  CONTOUR    1   PARAMETERS                          *
* X RANGE :-0.2220E 02 TO -0.1070E 02 DX= 0.2500E 00                  *
* Y RANGE : 0.5550E 03 TO  0.1055E 04 DY= 0.1000E 02                  *
* Z DOMAIN: 0.1355E-08 TO  0.1775E 04 DZ= 0.8876E 02                  *
* Z DOMAINS FOR THE CONTOURS :MAX VALUES FOR EACH                     *
* NO. 1 0.8876E 00 NO. 8 0.6222E 03 NO.15 0.1244E 04                  *
* NO. 2 0.8965E 02 NO. 9 0.7110E 03 NO.16 0.1332E 04                  *
* NO. 3 0.1784E 03 NO.10 0.7997E 03 NO.17 0.1421E 04                  *
* NO. 4 0.2672E 03 NO.11 0.8885E 03 NO.18 0.1510E 04                  *
* NO. 5 0.3559E 03 NO.12 0.9773E 03 NO.19 0.1599E 04                  *
* NO. 6 0.4447E 03 NO.13 0.1066E 04 NO.20 0.1687E 04                  *
* NO. 7 0.5335E 03 NO.14 0.1155E 04 NO.21 0.1775E 04                  *
***********************************************************************
```

Figure P3.23 NUMERICAL VALUES FOR FIGURE P3.25

```
                          CONTOUR     5
               *.....* INCREMENT IS      0.50000000E 02
      0.55500000E 03              0.80500000E 03              0.10550000E 04
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
               *....*....*....*....*....*....*....*....*....*....*
-0.222E 02*AA99877655544333333333333333444556678899AABCCDEFGHJKLM*
-0.219E 02*AA99877655544333333333333333444555667889AABCDEFGHJKLM*
-0.217E 02*AA998776555444333322222333333444556677789AABCDEFGHJKLM*
-0.214E 02*AA998776555444333322222333334445566677899ABCDEFGHJKLM*
-0.212E 02*AA998776555444333322222223333344555677899ABCDEFGHJKLM*
-0.209E 02*AA998766555444333322222223333444555667889ABCCDEFGJKLM*
-0.207E 02*AA998766555444333222222222333444555667889ABCCDEFGHJLM*
-0.204E 02*AA998766555443333222222222233334455566788 9ABBCDEFGHJLM*
-0.202E 02*AA998766555443333222222222223334455566778 9AABCDEFGHJKM*
-0.199E 02*AA998766555443333222222222223334444555667789AABCDEFGHJKL*
-0.197E 02*AA998766555443332222222222222333444556677899ABCDEFGHJKL*
-0.194E 02*AA998766555443332222222222223333445556678 99ABCDEFGHJKL*
-0.192E 02*AA998766555443332222222222222333445556678899ABCDEFGHJKL*
-0.189E 02*AA998766555443332222222222222333445566678899ABCDEFGHJKL*
-0.187E 02*AA998766555443332222222222222233344555667889ABCCDEFHJKL*
-0.184E 02*AA998766555443332222222222222333444556677789ABBCDEFGJKL*
-0.182E 02*AA998766555443332222222222222333444556677789ABBCDEFGHKL*
-0.179E 02*AA998766555443332222222222222233344555667789AABCDEFGHKL*
-0.177E 02*AA998766555443332222222222222233344555677789AABCDEFGHJL*
-0.174E 02*AA998766555443332222222222222233344556677899ABCDEFGHJL*
-0.172E 02*AA998766555443332222222222222333445556678899ABCDEFGHJL*
-0.169E 02*AAA98766555443332222222222222333445556678899ABCDEFGHJK*
-0.167E 02*AAA98766555443332222221222222333445566678899ABCDEFGHJK*
-0.164E 02*AAA98776555443332222221222222333445556678899ABCDEFGHJK*
-0.162E 02*AAA98776555443332222221222222333445556678899ABCDEFGHJK*
-0.159E 02*BBA98776555443332222222222222333445556678899ABCDEFGHJK*
-0.157E 02*BBA98776555443332222222222222333445556678899ABCDEFGHJK*
-0.154E 02*BBA98776555443332222222222222333445556678899ABCDEFGHJK*
-0.152E 02*BBA98776555443332222222222222333445556678899ABCDEFGHJK*
-0.149E 02*BBA98776655443332222222222222333445556678899ABCDEFGHJK*
-0.147E 02*BBA99887665544433322222222222223334455566788 9ABCDEFGHJK*
-0.144E 02*BBA99887665554433322222222222222333445556678899ABCDEFGHJK*
-0.142E 02*BBA99887665554433322222222222222333445556678899ABCDEFGHJK*
-0.139E 02*BBA99887665554433322222222222222333445556678899ABCDEFGHJK*
-0.137E 02*BBA99887665554433322222222222223333445556678899ABCDEFGHJK*
-0.134E 02*BBA99887765554433332222222222223333445556678899ABCDEFGHJK*
-0.132E 02*BBAA9887765554443332222222222223333444556677899ABCDEFGHJL*
-0.129E 02*BBBA9887765554433332222222222223333444556677899ABCDEFGHJL*
-0.127E 02*CCBA99887665544333322222222223333344555667789AABCDEFGHJL*
-0.124E 02*CCBA99887665554443333222222333333444556677789AABCDEFGHKL*
-0.122E 02*CCBA99887665544433333222223333334445566677889AABCDEFGJKL*
-0.119E 02*CCBA99877655544333333223333333444556667889ABCCDEFGJKL*
-0.117E 02*CCBA99877665554443333333333333344555677889ABCCDEFGHJKL*
-0.114E 02*CCBAA9877665554443333333333333444555667789 9ABCDEFGHJKL*
-0.112E 02*CCBBA99887665555444333333333334444555667789 9ABCDEFGHJKL*
-0.109E 02*CCCBA998877665554444333333333344445556677889AABCDEFGHJKM*
-0.107E 02*DDCBA9988776655554444333333344445556677889ABBCDEFGHJLM*
               *....*....*....*....*....*....*....*....*....*....*
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
```

Figure P3.24 SURGE, NO NOISE, 100 SECOND SINUSOIDAL INPUT FUNCTION CONTOURS

```
                    CONTOUR    1
            *.....* INCREMENT IS      0.5000000E 02
       0.5550000E 03          0.8050000E 03          0.1055000E 04
  0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
            *....*....*....*....*....*....*....*....*....*....*
-0.222E 02*AA9887766554444443333333444444555666777788899AABCCDD*
-0.219E 02*AA988776655544444433333334444435566677778899AABCCDD*
-0.217E 02*999887766555444443333333334444455556677778899AABBCDD*
-0.214E 02*999887666554444433333333333444445556667778899AABBCDD*
-0.212E 02*99987766555444433333333333334444555666778899AABBCCD*
-0.209E 02*99987766555444333333333333333444445556677778899ABBCCD*
-0.207E 02*99887766554444433333333333333344445556667778899ABBCCD*
-0.204E 02*998877665544433333333333333333444445566677788999AABCCD*
-0.202E 02*998877665544433333222223333334444555667778899AABCCD*
-0.199E 02*998876655544433333222222333333444555667788999AABCCD*
-0.197E 02*998876655544333332222222223333344455556677788999AABCCD*
-0.194E 02*99887665544433332222222222233333444445566677889AABCCD*
-0.192E 02*99877665544433332222222222233334445566677889AABBCD*
-0.189E 02*998776655444333222222222222233333444555667789AABBCD*
-0.187E 02*99877665544333322222222222222333334445556677889AABBCD*
-0.184E 02*9987766554433332222222222222223334445556677889AABBCD*
-0.182E 02*998776655443333222222222222223333344555667789AABCCD*
-0.179E 02*99877665544333222222222222222333334445556677889AABCCD*
-0.177E 02*9987765544433322222222222222223333444555677889AABCCD*
-0.174E 02*998776554444333222222222222222333334445556677889AABCCD*
-0.172E 02*9987765544433322222222222222223333344455667789AABCCD*
-0.169E 02*99877655444333222222222222222223333444555667789AABCCD*
-0.167E 02*99877655444333222222212222222233344455566778899AABCDD*
-0.164E 02*99877655444333222222212222222233344455566778899ABBCDD*
-0.162E 02*998776554443332222222122222222333334445556677899ABBCDE*
-0.159E 02*99877655444333222222222222222233333445556678899ABBCDE*
-0.157E 02*99877665544333222222222222222223333344555667789AABCCDE*
-0.154E 02*99877665544333222222222222222223334445566677889AABCDDE*
-0.152E 02*998776655443332222222222222222333444556667789AABCDDE*
-0.149E 02*998876655443332222222222222222333334445556677899ABBCDEE*
-0.147E 02*998876655443333222222222222223333344555667889AABCCDEF*
-0.144E 02*9988766554443332222222222222223334445566677889AABCCDEF*
-0.142E 02*998876655444333222222222222223333444555667789ABBCDEEF*
-0.139E 02*999877655544333322222222222233333444555667889ABCCDEFG*
-0.137E 02*9998776655443333322222222222333334445556677889AABCCDEFG*
-0.134E 02*AA98776655444333332222222333333444555667789ABBCCDEEFG*
-0.132E 02*AA9887766554443333322222233333334445556677889ABCCDEFGH*
-0.129E 02*AA9887766555444333333333333333444555667789AABCCDEFGH*
-0.127E 02*AA9887766554443333333333333344445556677889ABBCCEFFGH*
-0.124E 02*AA99877665554443333333333344444555667788899AABCDDEFGHJ*
-0.122E 02*AA99837665554444333333333444445555667788899ABBCCEFFGHJ*
-0.119E 02*AAA98877665554444433333444444555666778899AABCCDEFGHJK*
-0.117E 02*BBA9987766555544444444444444555666778899AABCCDEFFGHJK*
-0.114E 02*BBA99887766555544444444444555666778899ABBCDDEFGHJKL*
-0.112E 02*BBAA9887766655554444444455556667788899AABCCDEFGGHJKL*
-0.109E 02*BBAA99877766655555544555555666777889AABBCDEEFGHJKLM*
-0.107E 02*BBBA99887766665555555555555566677788899ABBCDDEFGHJKKLM*
            *....*....*....*....*....*....*....*....*....*....*
  0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
```

Figure P3.25 <u>SURGE, NO NOISE, 150 SECOND SINUSOIDAL INPUT FUNCTION CONTOURS</u>

```
                         CONTOUR   1
              *....* INCREMENT IS     0.5000000E 02
      0.5550000E 03          0.8050000E 03          0.1055000E 04
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
           *....*....*....*....*....*....*....*....*....*....*
-0.222E 02*888877666655554444443333333333333333333333444444445*
-0.219E 02*888877666655554444443333333333333333333333344444445*
-0.217E 02*888777666555544444433333333333333333333333344444445*
-0.214E 02*888776666555444444333333333333333333333333334444445*
-0.212E 02*887776665555444444333333333333333333333333334444445*
-0.209E 02*887766655554444443333333333322222333333333334444445*
-0.207E 02*777766655544444433333333322222222223333333333444445*
-0.204E 02*777666555544444333333332222222222222333333334444445*
-0.202E 02*777666555444444333333322222222222222333333333444445*
-0.199E 02*777665555444433333332222222222222222233333334444455*
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-0.194E 02*776655554444433333322222222222222222233333333444455*
-0.192E 02*666655544443333333222222222222222222222333333444455*
-0.189E 02*666655544433333332222222222222222222222333334444555*
-0.187E 02*666555444433333222222222222222222222223333334444555*
-0.184E 02*666555444333332222222222222222222222223333333444555*
-0.182E 02*666554444333332222222222222222222222223333334444555*
-0.179E 02*665554443333322222222222222222222222223333344455556*
-0.177E 02*665554443333322222222222222222222222233333444455556*
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-0.172E 02*555544433333222222222222222222222222333333444555566*
-0.169E 02*555444333322222222222222222222222233333444455555666*
-0.167E 02*554444333222222222211222222222222333334444455556667*
-0.164E 02*555444333222222222221112222222222233333444455556667*
-0.162E 02*555444333222222222221122222222222333334444555566677*
-0.159E 02*555444333322222222222222222222222333334444555566677*
-0.157E 02*554443333322222222222222222222233333444445556667778*
-0.154E 02*554443333322222222222222222223333344444555666677788*
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-0.144E 02*554443333222222222222222233333444445555666778889 9A*
-0.142E 02*5544433333222222222222222333334444455556667788899AA*
-0.139E 02*55444333332222222222222223333334444555666778889 9AAB*
-0.137E 02*5544433333222222222222233333344445556667778889 9AABB*
-0.134E 02*554443333332222222222333333344445556667778899 9AABBC*
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-0.129E 02*5544443333333333333333333344444555666777889 9AAABBCCD*
-0.127E 02*555444443333333333333333344444555666677888899AABBCCDDE*
-0.124E 02*5554444443333333333333344444555566677788999AABBCCDDEF*
-0.122E 02*555444444433333333334444444555566677888899AABBBCDDEEFF*
-0.119E 02*5555444444443344444444445555666777888899AABBCCDDEEFGG*
-0.117E 02*5555544444444444444444555566666778889 9AAABBCCDEEFFGGH*
-0.114E 02*66555544444444444444555556666777889 99AABBCCDDEEFGGHHJ*
-0.112E 02*666555555444445555555666667777888899AABBBCDDEEFFGHHJJK*
-0.109E 02*6666555555555555555556666777888999AABBCCDDEFFGGHJJKLL*
-0.107E 02*666666555555555566666677788899 9AABBCCDDEEFGGHHJKKLMN*
           *....*....*....*....*....*....*....*....*....*....*
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
```

Figure P3.26 SURGE, NO NOISE, 300 SECOND SINUSOIDAL INPUT FUNCTION CONTOURS

```
                    CONTOUR    1
           *....* INCREMENT IS      0.5000000E 02
      0.5550000E 03           0.8050000E 03           0.1055000E 04
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
              *....*....*....*....*....*....*....*....*....*....*
-0.222E 02*888777666655554444333333333222222222222222222223333*
-0.219E 02*888777666655555444443333333332222222222222222222333*
-0.217E 02*888777666555554444443333333332222222222222222222333*
-0.214E 02*887776666555554444443333333322222222222222222222333*
-0.212E 02*887776666555554444443333333322222222222222222223333*
-0.209E 02*777766655554444433333333322222222222222222222223333*
-0.207E 02*777666665555444443333333332222222222222222222223333*
-0.204E 02*777666555554444433333333222222222222222222222233333*
-0.202E 02*776665555544444333333322222222222222222222222233333*
-0.199E 02*776665555444444333333222222222222222222222222333333*
-0.197E 02*666655554444433333332222222222222222222222222233333*
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-0.192E 02*666555554444433333332222222222222222222222223333333*
-0.189E 02*666555444443333332222222222222222222222222233333334*
-0.187E 02*665554444433333332222222222222222222222222333333334*
-0.184E 02*555554444333332222222222222222222222222222333333344*
-0.182E 02*555544444333333222222222222222222222222223333333444*
-0.179E 02*555544444333332222222222222222222222222233333334444*
-0.177E 02*555444443333322222222222222222222222222233333344444*
-0.174E 02*555444333332222222222222222222222222222333333444445*
-0.172E 02*554444333332222222222222222222222222222333333444455*
-0.169E 02*554443333322222222222221122222222222222333333444445*
-0.167E 02*444443333222222222222211222222222222223333334444555*
-0.164E 02*444433333222222222222111222222222222233333444445555*
-0.162E 02*444433332222222222222112222222222222333333444445556*
-0.159E 02*444333332222222222222222222222222223333333444445556*
-0.157E 02*444333322222222222222222222222222233333344444555666*
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-0.152E 02*443333222222222222222222222222233333344444555666777*
-0.149E 02*333333222222222222222222222222333333444455556666777*
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-0.144E 02*333332222222222222222222222233333344445555666777889*
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-0.139E 02*333332222222222222222222223333334444555566677788999A*
-0.137E 02*33332222222222222222222223333334444555566677788999AA*
-0.134E 02*3333222222222222222222233333344444555666777889999AABB*
-0.132E 02*33332222222222222222223333334444555566677788899AABC*
-0.129E 02*333322222222222222222333333344445555666777889999AABBCC*
-0.127E 02*3333322222222222223333333444445555666777888999AAABBCCDD*
-0.124E 02*333332222222222223333333344444555566677788999AABBCCCDDE*
-0.122E 02*33333322222333333333344444555566677788899AAABBCCDDEEF*
-0.119E 02*333333333333333333334444455555666777888999AABBCCDDEEFFG*
-0.117E 02*33333333333333333334444455555666777888999AABBCCCDEEFFGH*
-0.114E 02*333333333333333444445555566677788899AAABBCCDDEEFGGHHJ*
-0.112E 02*33333333333334444444555566677788999AABBCCCDDEEFFGHHJJK*
-0.109E 02*3333333333444444444555566677788999AABBCCDDEEFFGGHJJKLL*
-0.107E 02*44444444444444445555566677788899AABBCCDDEEFFGCHJJKLLMM*
              *....*....*....*....*....*....*....*....*....*....*
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
```

Figure P3.27 SURGE, NO NOISE, 500 SECOND SINUSOIDAL INPUT FUNCTION CONTOURS

The sinusoidal input contours in Figures P3.24 through P3.27 show that as the period of the sinusoid is increased (frequency decreased), parameter p_1 becomes more identifiable and parameter p_2 becomes less identifiable. The reason for this is that at the shorter periods the values of x are smaller, and p_1 is not as dynamically excited as at the longer periods. However, for the 500 second period input neither parameter is excited as much dynamically as for the 150 second period input, because the system responds fast enough to "keep up" with the 500 second period input. The most identifiable input of these four periods considering <u>both</u> parameters p_1 and p_2 is the 150 second period sinusoidal input.

The noise behavior characteristics of the four different period sinusoidal input functions are shown in Tables P3.5 through P3.8. In each of these noisy cases, the contours remained essentially the same shape as the no-noise versions in Figures P3.24 through P3.27. The most significant fact indicated by the numbers in these tables is that 10% noises permit about 10% accuracy identifications. The parameter p_1 is the lesser identifiable, and parameter p_2 is the more identifiable parameter with regard to accuracy. Considering parameter identification accuracy, contour shape, and contour min. and max. numbers, the 150 second-period sinusoidal input allows the best identification of p_1 and p_2 for the four different period sinusoidal inputs.

In comparing the behavior of the 150 second-period sinusoidal input to all other inputs used thus far in this chapter, it is found that this sinusoidal input offers the best identifiability for the combinations of 10% noises used. This is due to the fact that this input excites the system at its "natural frequency" and causes the

greatest amount of dynamic behavior over the length of the sea trial. But, then if the time parameters had not been known, this "natural frequency" would not have been known; and the "best" input could not have been chosen ahead of time. It is for these reasons that "persistently exciting" (A-13) inputs must be used to "guarantee" identifiability, whereas the sinusoidal input which excites the system at its natural frequency provides the best means for identification using model reference contouring.

Sinusoidal 100 sec.	$p_1^*(-16.7)$	$p_2^*(755)$	C (p^*)	C max.
v=0, w=0	-16.7	755	0	930
v=0, w=10%	-17.9	755	30	951
v=10%, w=0	-17.4	710	159	1336
v=10%, w=10%	-18.4	770	404	1252

Table P3.5 NOISE BEHAVIOR CHARACTERISTICS FOR A 100 SECOND-PERIOD SINUSOIDAL INPUT

Sinusoidal 300 sec.	$p_1^*(-16.7)$	$p_2^*(755)$	C (p^*)	C max.
v=0, w=0	-16.7	755	0	3243
v=0, w=10%	-17.2	800	15	3140
v=10%, w=0	-17.2	760	303	3712
v=10%, w=10%	-14.7	720	251	3193

Table P3.7 NOISE BEHAVIOR CHARACTERISTICS FOR A 300 SECOND-PERIOD SINUSOIDAL INPUT

Sinusoidal 500 sec.	$p_1^*(-16.7)$	$p_2^*(755)$	C (p^*)	C max.
v=0, w=0	-16.7	755	0	3996
v=0, w=10%	-16.4	750	30	3974
v=10%, w=0	-17.2	770	303	4519
v=10%, w=10%	-17.2	810	254	3834

Table P3.8 NOISE BEHAVIOR CHARACTERISTICS FOR A 500 SECOND-PERIOD SINUSOIDAL INPUT

No.	%v	%w	$p_1(-16.7)$	$p_2(755)$	C (p^*)	C max.	Comments
1	0	0	-16.7	755	0	1775	Oblong Bowl, Fig. N4.4, #5
2	0	1	-16.7	755	0.2	1774	Figure N4.4, #5
3	0	10	-16.9	755	17.	1908	Same as No. 1 above
4	0	100	-19.7	660	2660	5936	Flat Bowl, Min. Shifted
5	0	200	-22.2	560	2922	12360	Figure N4.4, #7
6	1	0	-16.7	755	1.7	1758	Same as No. 1 above
7	1	1	-16.8	755	2.2	1780	" No. 1 "
8	1	10	-16.8	750	33	1846	" No. 1 "
9	1	100	-22.2	800	3407	6647	" No. 4 "
10	1	200	-10.7	800	4034	6397	" No. 4 "
11	10	0	-16.7	760	282	2027	" No. 1 "
12	10	1	-18.2	770	229	2073	" No. 1 "

Table P3.6 NOISE BEHAVIOR CHARACTERISTICS FOR A 150 SECOND-PERIOD SINUSOIDAL INPUT

No.	%v	%w	$p_1(-16.7)$	$p_2(755)$	C (p^*)	C max.	Comments
13	10	10	-16.9	730	310	2383	Same as No. 1 above
14	10	100	-22.2	700	2009	5585	" No. 4 "
15	10	200	-12.4	1050	4078	8009	" No. 4 "
16	100	0	-10.7	600	22330	27930	" No. 5 "
17	100	1	-10.7	560	21460	28510	" No. 5 "
18	100	10	-14.7	550	29680	33800	" No. 5 "
19	200	0	-12.9	1050	114200	118500	" No. 5 "
20	200	1	-10.7	620	97840	103000	" No. 4 "
21	200	10	-10.7	1055	119500	126200	" No. 4 "
22	200	100	-22.2	555	106000	116900	" No. 5 "
23	200	200	-22.2	555	102700	114200	" No. 5 "
24	100	0	-18.5	1055	16520	20350	" No. 4 "
25	100	1	-22.2	850	17690	19890	" No. 4 "
26	100	10	-22.2	830	30740	34280	" No. 4 "
27	100	100	-22.2	860	24200	29270	" No. 4 "
28	100	200	-10.7	650	25570	29490	" No. 4 "
29	20	0	-16.9	770	1128	2844	" No. 1 "
30	20	1	-19.7	800	904	2851	" No. 1 "
31	20	10	-17.2	710	1223	3475	" No. 1 "
32	20	100	-22.2	680	2501	6445	" No. 4 "
33	20	200	-12.4	1030	3955	7893	" No. 4 "
34	50	0	-11.7	660	5504	8576	" No. 4 "
35	50	1	-14.7	580	5235	9273	" No. 4 "
36	50	10	-15.4	660	7289	10100	" No. 4 "

Table P3.6 (Contd.) NOISE BEHAVIOR CHARACTERISTICS FOR A 150 SECOND-PERIOD SINUSOIDAL INPUT

P3.6 FITTING A NONLINEAR VEHICLE WITH A LINEAR MODEL

For a sinusoidal input of 150 second-period to equation P3.6, the best linear model which fits the noiseless sea trial data is given by equation P3.7. The model reference contours for this case are shown in Figure P3.28, and the noise behavior characteristics for the linear model are presented in Table P3.9. The noise behavior

Non-Linear Veh.,Lin. Model	p_1^*	p_2^*	C (p^*)	C max
v=0, w=0	-95	900	158	2568
v=0, w=10%	-93	905	157	2593
v=10%, w=0	-91	900	545	3250
v=10%, w=10%	-95	860	346	2420

Table P3.9 NOISE BEHAVIOR OF A LINEAR MODEL FOR THE NONLINEAR SURGE EQUATION

$$4507\,\dot{x} = -16.7\,x\,|x| + 755\,u\,|u| \qquad \text{P3.6}$$

$$4507\,\dot{x} = -95x + 900u \qquad \text{P3.7}$$

for this model is quite good (5%), although the optimum noiseless cost function value of 158 indicates that the fit of the linear model to the nonlinear system for the 150 second-period sinusoidal input is about equivalent to a nonlinear fit with 10% v and w noises.

Extensive model reference contouring studies were conducted using a form of the linear equation P3.7 as the vehicle and using a linear model. The details and results of these linear system studies are presented in Appendix A13. These studies essentially show that the linear system parameters are more accurately identified than the

```
                              CONTOUR    1
                   *....* INCREMENT IS    0.5000000E 02
         0.5550000E 03           0.8050000E 03           0.1055000E 04
   0.56E 03  0.66E 03  0.76E 03  0.86E 03  0.96E 03  0.11E 04
             *....*....*....*....*....*....*....*....*....*....*
-0.150E 03*MMMLKKJJHGGFFEEDDCCBBBAAA9998888877777777766667777*
-0.148E 03*MMMLKKJJHGGFFFFDDCCBBBAAA9998888877777777766667777*
-0.146E 03*MMLLKJJHHGFFFFDDCCBBBAAA999888777777666666666666667*
-0.144E 03*MMLKKJHHGGFEEDDCCBBBAA99988887777766666666666666666*
-0.142E 03*LLLKJJHGGFFEDDCCCBBAA999888877776666666666666666666*
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-0.124E 03*HHGGFEEDCCBBAA998877766655554444444333333333334444444*
-0.122E 03*HHGFFEDDCCBAA99888776665555444443333333333333334444*
-0.120E 03*GGGFFEDCCBBAA998877666555544443333333333333333334444*
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-0.112E 03*FFEEDCCBBA99887776655544444333332222222222223333344*
-0.110E 03*FFFDDCCBAA998877665554444433333222222222222333333444*
-0.108E 03*FFEDDCBBA9988776665554443333332222222222222233333344*
-0.106E 03*EEEDCCBAA9988776655544433333222222222222222223333344*
-0.104E 03*EEDDCBBAA988776665544443333222222222222222223333344*
-0.102E 03*EEDCCBBA99887766555444333322222222222222222223333344*
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-0.580E 02*DDCCBAA999887777666666555555555666666677788899AABBCDD*
             *....*....*....*....*....*....*....*....*....*....*
   0.56E 03  0.66E 03  0.76E 03  0.86E 03  0.96E 03  0.11E 04
```

Figure P3.28 NO NOISE, SINUSOIDAL INPUT, 150 SECOND-PERIOD, NONLINEAR EQUATION, LINEAR MODEL CONTOURS

nonlinear system parameter for the same noise percentages and for the best inputs.

This chapter has presented in-depth model reference contouring studies of the DSRV surge equation. These studies were conducted using a variety of different inputs such as: step, staircase, crashback full astern, impulse, and sinusoidals of different periods. Of these inputs, the sinusoidal with a period near the "natural frequency" of the system was found to be the best for identification. Detailed noise studies showed in essence, that 10% v and w noises allowed about 10% identification accuracy for the two surge equation parameters.

The next chapter presents several detailed studies and identifications of the two parameters in the DSRV surge equation. As was done in this chapter, the parameters are studied for their identifiability characteristics using a variety of different inputs. However, the identifiability of the parameters in the next chapter using the extended Kalman filtering technique is determined based upon the accuracy of the final parameter estimates rather than upon the shape and minimum values of contours, as in this chapter.

CHAPTER P4

EXTENDED KALMAN FILTERING STUDIES OF SINGLE DEGREE OF FREEDOM DSRV EQUATIONS

This chapter presents the same types of analyses for the extended Kalman filtering identification technique as were described for the model reference technique in Chapter P3. The equations to be used in this section are those in Steps N3.9 through N3.16 of Chapter N3, and one form of the actual computer program used is given in Appendix A14. The vehicle equations and parameters are the same as those in Figure P3.1 except that the parameters are augmented into the new state vector $\underline{x}$ as in equation P4.1, and the value of δ is arbitrarily set at $\delta \approx p_i^*/4$.

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x \\ p_1 \\ p_2 \end{bmatrix} \quad \text{in Figure P3.1} \qquad \text{P4.1}$$

The application of extended Kalman filtering to the DSRV surge equation is a much more critical task than was the application of the model reference contouring technique. There are many factors which affect the behavior of the extended Kalman filter and determine both its rate and accuracy of convergence. One of these factors is that for given amounts of process and measurement noises, expressed by Q_n and R_n, the values of the corresponding values Q and R which are used in the filter may need to be adjusted for the best filter convergence over the sea trial length. These Q and R values determine how the matrix E behaves and how large or small the Kalman filter gain matrix K becomes.

This chapter begins the presentation of the extended Kalman filtering studies by showing some trial runs made for the purpose of determining proper values for Q and R. Then, using these values the inputs are varied using the step, staircase, crashback full astern, simulated impulse, and sinusoidal functions as was done in Chapter P3. Next, several studies are made using a longer sea trial length and varying the values of Q_n and R_n sequentially to study their effects upon the filtering process.

This chapter then presents some very dramatic filtering results which were obtained by properly "tuning" the filter by adjusting Q and R and concludes by utilizing the extended Kalman filter (EKF) on the remaining 5 single degree of freedom DSRV models in Table P2.1.

P4.1 ADJUSTING THE VALUES OF Q AND R IN THE EXTENDED KALMAN FILTER

Several extended Kalman filter runs were made with the staircase input, 1%v and 1%w noises for the purpose of determining the effects of Q and R upon the convergence of the filter. The staircase input run using $Q = Q_n$ and $R = R_n$ will be described in complete detail here, and then later runs will have only the results tabulated and comments made about them.

The sea trial data to be used in the extended Kalman filter studies for the staircase input function, 1%v, and 1%w noises is shown in Figure P4.1. This plot shows both the staircase input function (curve 1) which is the same as that of Figure P3.2 and the resulting noisy DSRV surge velocity (curve 2) which is similar to Figure P3.11 for the no-noise case.

The surge curves calculated for use in the extended Kalman filter are slightly different from those used in the model reference

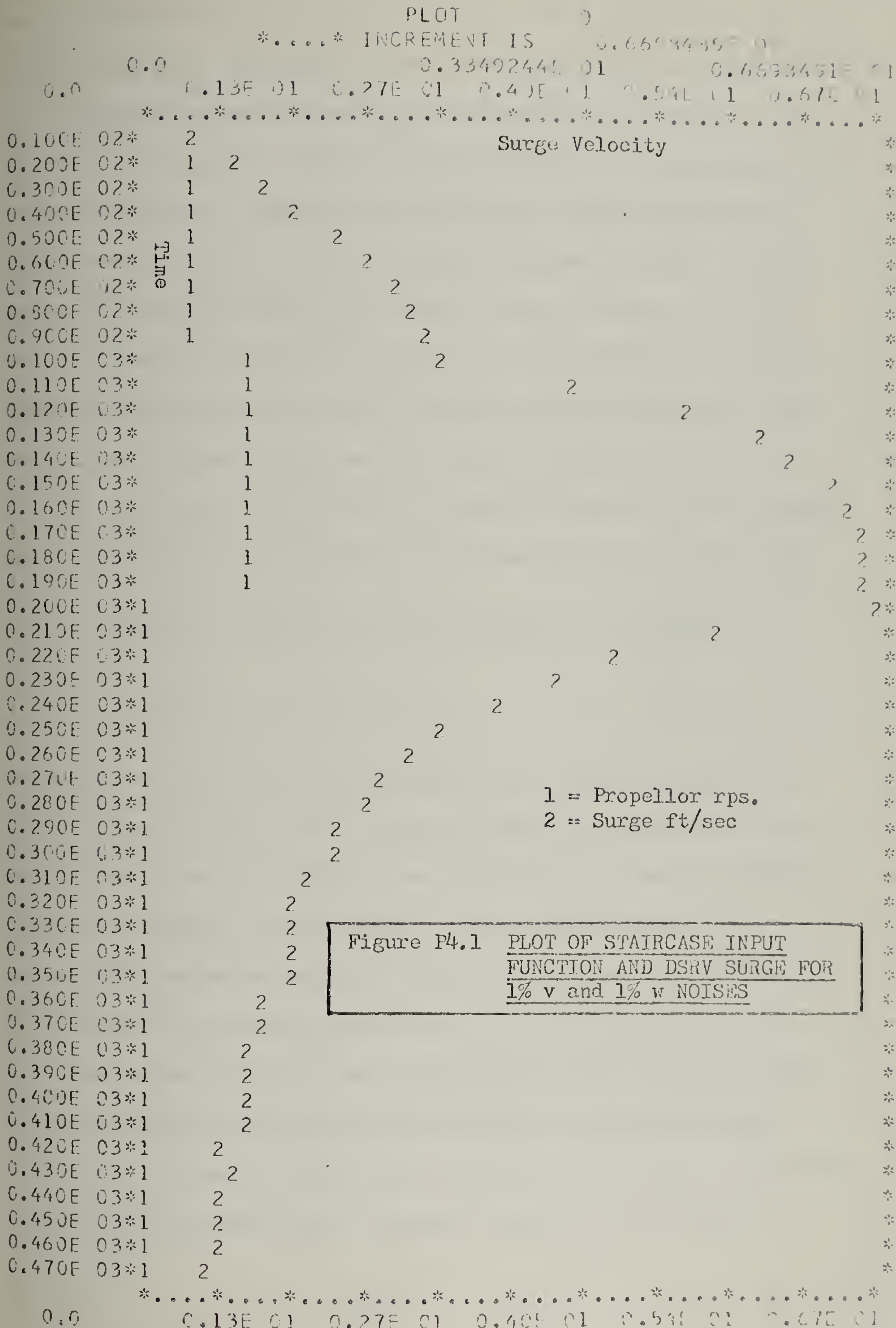


Figure P4.1 PLOT OF STAIRCASE INPUT FUNCTION AND DSRV SURGE FOR 1% v and 1% w NOISES

contours because the step size H for the filter is one-quarter of that used for the model reference studies. This means that four times as many points are generated, and Figure P4.1 merely represents a plot of every fourth point. This is necessary because the dynamics of the parameters and the E matrix are somewhat faster than those of the surge velocity, and more precise integration must be used. It must be kept in mind throughout this chapter that the greater number of sea trial data points and extended Kalman filter calculation points makes the noise percentages have a significantly greater effect than they did for the model reference contours of Chapter P3.

The equations used for the sea trial data generation are repeated here as equations P4.2 and P4.3. The maximum value of x during the

$$4597.\ \dot{x} = -16.7\ x\ |x| + 755\ u\ |u| + w \qquad \text{P4.2}$$

$$z = x + v \quad ; \quad x(t_0) = 0 \qquad \text{P4.3}$$

sea trial is about 7 feet/second and so the standard deviation for 1%v is chosen as 0.07, and R_n becomes 0.0049; the corresponding standard deviation for 1%w is chosen as $1.7*10^{-3}$ and Q_n becomes $2.89*10^{-6}$. Using these equations and numerical values subroutine RKNL in Appendix A14 generates the noisy sea trial data, which is plotted in Figure P4.1, by Euler integration.

Once the sea trial has been generated, then the extended Kalman filtering technique may be used to process the noisy data and to identify the "unknown" parameters p_1 (-16.7) and p_2 (755). The filter must be "told" initially what the best estimates of the states and

parameters are and be given their corresponding initial error covariances or standard deviations. The state equations to be used in the extended Kalman filter are given by equations P4.4 and P4.5 and the initial states and their standard deviations are given by equation P4.6. The initial values of the parameters are chosen at about 25%

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2\, x_1 |x_1| + x_3\, u|u| \\ 0 \\ 0 \end{bmatrix} \qquad \text{P4.4}$$

$$z_m = x_1 \qquad \text{P4.5}$$

$$\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} 1 \pm 0.7 \\ -11.2/M \pm 4.5/M \\ 943/M \pm 55/M \end{bmatrix} \quad \text{Where: } M = 4507 \text{ slugs} \qquad \text{P4.6}$$

away from the true parameters and approximately one standard deviation in confidence. For this run the filter is told exactly how much noise is in the data by equations P4.7 and P4.8.

$$Q = Q_n = 2.89*10^{-6} \qquad \text{P4.7}$$

$$R = R_n = 4.9*10^{-3} \qquad \text{P4.8}$$

With this information the extended Kalman filter may be run in passes over the sea trial data in Figure P4.1. The filter of Steps N3.9 through N3.16 is implemented in this case by using the MAIN program and subroutines PROP, GAIN, UPDAT, and STORE in Appendix A14. Each pass over the data generates six output plots ($x(t)$, $p_1(t)$, $p_2(t)$, $\sigma_x(t)$, $\sigma_{p_1}(t)$, and $\sigma_{p_2}(t)$) which represent the time values of

the filter estimates of the states and parameters over the length of the sea trial along with their corresponding estimated standard deviations. Successive passes are run using the final best estimates of the states and parameters in the previous pass as the starting values for the next pass.

The results of three extended Kalman filtering passes over the 1% noisy sea trial data in Figure P4.1 are shown in the 18 plots of Figures P4.2 through P4.19. These plots show how the noise is filtered out of the primary state x (Figure P4.2) and how the parameter values for p_1 (Figure P4.3) and p_2 (Figure P4.4) are arrived at by the filter. The corresponding time value of the standard deviations of the state σ_x (Figure P4.5) and the parameters σ_{p_1} and σ_{p_2} (Figures P4.6 and P4.7) are also shown. This sequence of six plots is repeated three times, once for each pass over the data, in Figure P4.2 through P4.19.

An enormous number of extended Kalman filter runs were made and the corresponding plots generated for the studies in this chapter. The results of these runs will be described here in tabular form by listing the noise characteristics, the initial values, and the final values of the parameters for each pass. The tabular form for Figure P4.1 through P4.19 is given in Table P4.1. The values at the end of each pass are taken from the state and parameter plots and their corresponding standard deviation plots.

The numbers in Table P4.1 show that the filtered state estimates are biased about 5% away from the true values and that the standard deviations are extremely low. This indicates that the filter has an extremely high degree of confidence in these values of the parameters.

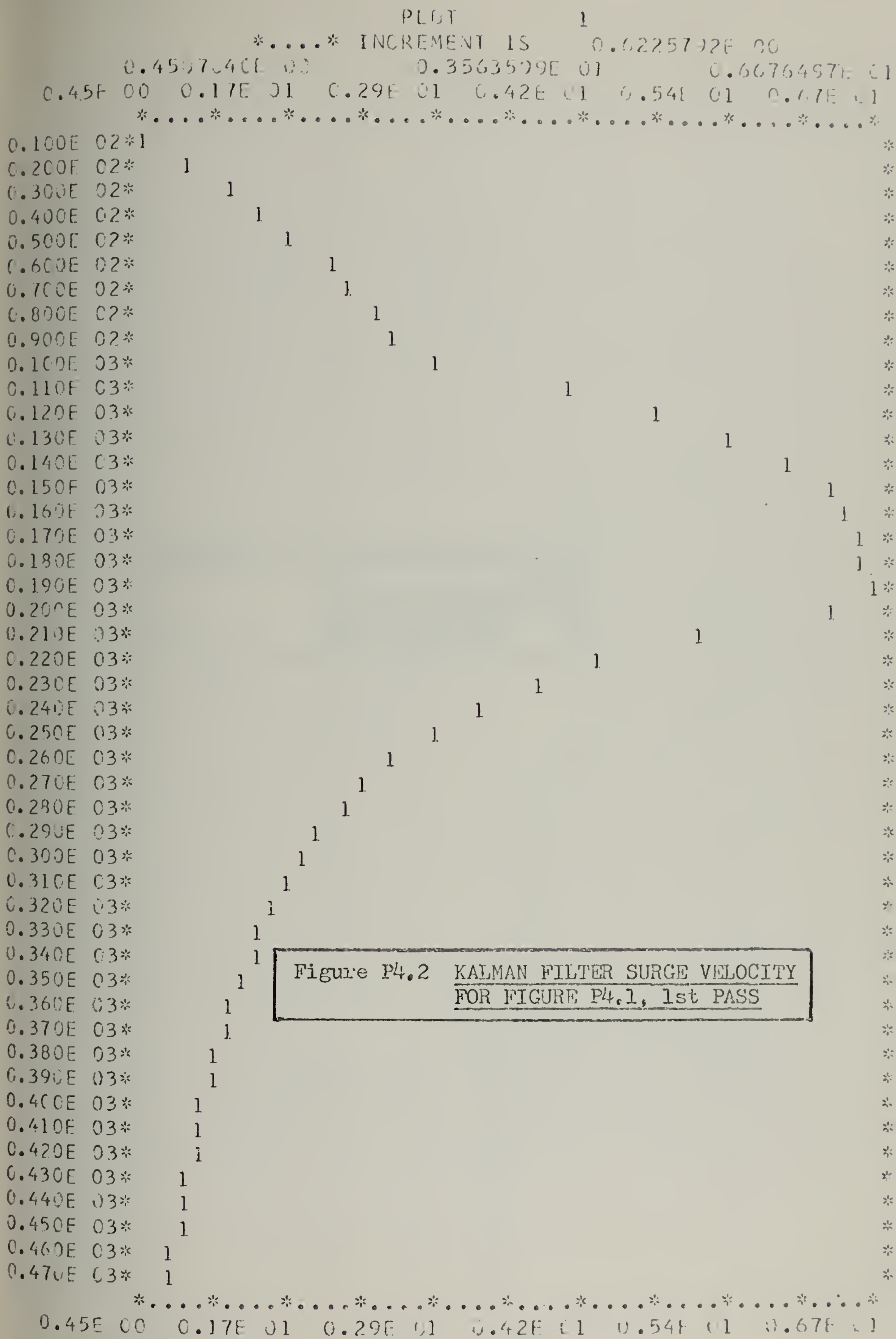


Figure P4.2 KALMAN FILTER SURGE VELOCITY FOR FIGURE P4.1, 1st PASS

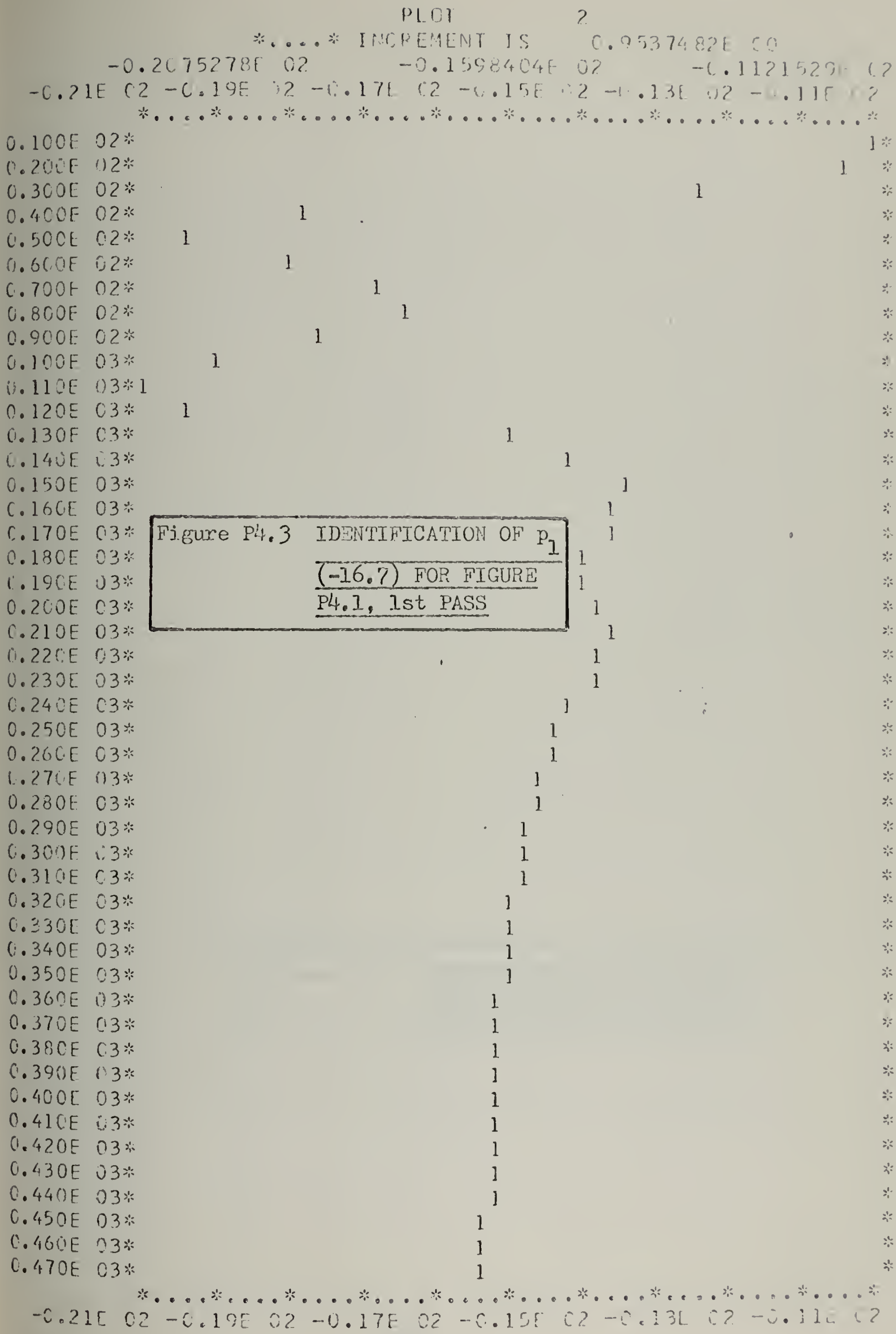


Figure P4.3 IDENTIFICATION OF p_1 (-16.7) FOR FIGURE P4.1, 1st PASS

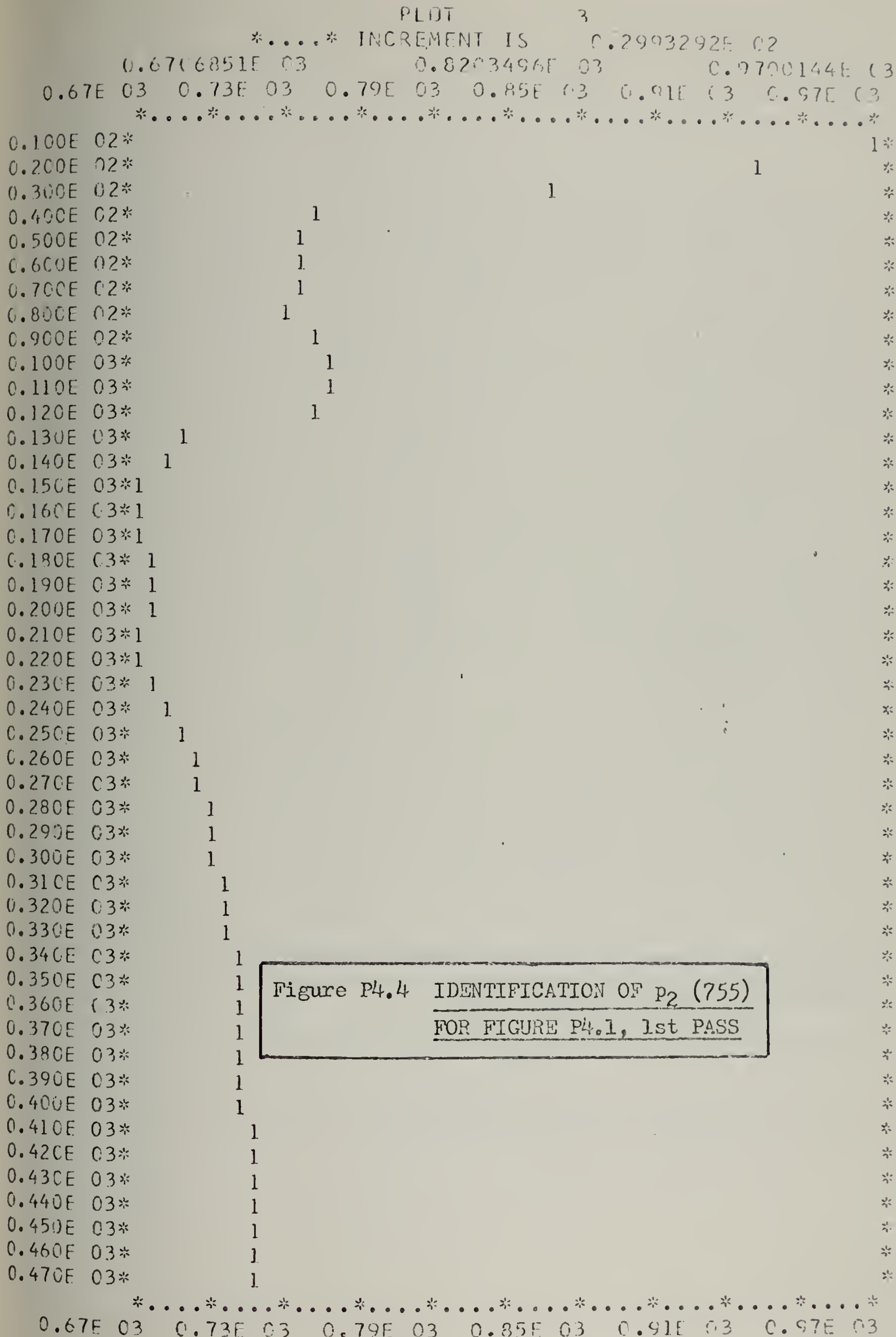


Figure P4.4 IDENTIFICATION OF p_2 (755) FOR FIGURE P4.1, 1st PASS

```
                              PLOT          4
                    *....* INCREMENT IS     0.2566176E-02
          0.1051692E-01          0.2334779E-01          0.3617867E-01
   0.11E-01   0.16E-01   0.21E-01   0.26E-01   0.31E-01   0.36E-01
          *....*....*....*....*....*....*....*....*....*....*
0.100E 02*                                                1*
0.200E 02*                                        1        *
0.300E 02*                                        1        *
0.400E 02*                                      1          *
0.500E 02*                                    1            *
0.600E 02*                                     1           *
0.700E 02*                                     1           *
0.800E 02*                                     1           *
0.900E 02*                                  1              *
0.100E 03*                        1                        *
0.110E 03*                      1                          *
0.120E 03*                          1                      *
0.130E 03*                                     1           *
0.140E 03*                                         1       *
0.150E 03*                                     1           *
0.160E 03*                                1                *
0.170E 03*                          1                      *
0.180E 03*                     1                           *
0.190E 03*                 1                               *
0.200E 03*                 1                               *
0.210E 03*                    1                            *
0.220E 03*                 1                               *
0.230E 03*             1                                   *
0.240E 03*          1                                      *
0.250E 03*       1                                         *
0.260E 03*     1                                           *
0.270E 03*   1                                             *
0.280E 03*  1                                              *
0.290E 03* 1                                               *
0.300E 03* 1                                               *
0.310E 03*1                                                *
0.320E 03*1                                                *
0.330E 03*1                                                *
0.340E 03*1                                                *
0.350E 03*1                                                *
0.360E 03*1                                                *
0.370E 03*1                                                *
0.380E 03*1                                                *
0.390E 03*1                                                *
0.400E 03*1                                                *
0.410E 03*1                                                *
0.420E 03*1                                                *
0.430E 03*1                                                *
0.440E 03*1                                                *
0.450E 03* 1                                               *
0.460E 03* 1                                               *
0.470E 03* 1                                               *
          *....*....*....*....*....*....*....*....*....*....*
   0.11E-01   0.16E-01   0.21E-01   0.26E-01   0.31E-01   0.36E-01
```

Figure P4.5 SURGE VELOCITY STANDARD DEVIATION FOR FIGURE P4.1, 1st PASS

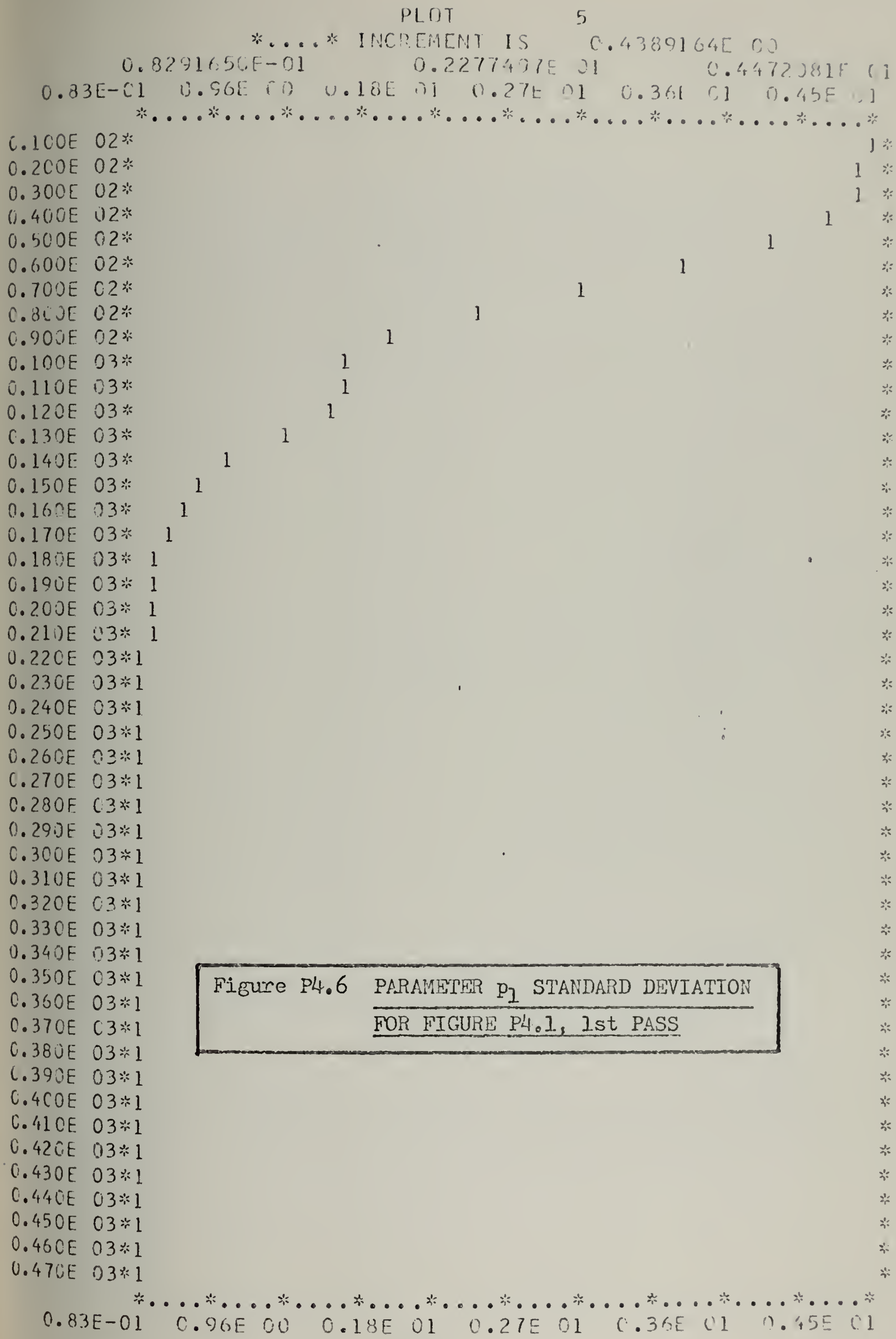


Figure P4.6 PARAMETER p_1 STANDARD DEVIATION FOR FIGURE P4.1, 1st PASS

```
                              PLOT         6
                    *....* INCREMENT IS    0.5381837E 01
       0.3347069E 01           0.3025624E 02           0.5716548E 02
  0.33E 01  0.14E 02  0.25E 02  0.36E 02  0.46E 02  0.57E 02
         *....*....*....*....*....*....*....*....*....*....*
0.100E 02*                                                  1*
0.200E 02*                                        1          *
0.300E 02*                            1                      *
0.400E 02*                     1                             *
0.500E 02*                   1                               *
0.600E 02*                   1                               *
0.700E 02*                   1                               *
0.800E 02*                 1                                 *
0.900E 02*               1                                   *
0.100E 03*              1                                    *
0.110E 03*              1                                    *
0.120E 03*              1                                    *
0.130E 03*           1                                       *
0.140E 03*        1                                          *
0.150E 03*      1                                            *
0.160E 03*    1                                              *
0.170E 03*   1                                               *
0.180E 03*   1                                               *
0.190E 03*  1                                                *
0.200E 03*  1                                                *
0.210E 03* 1                                                 *
0.220E 03* 1                                                 *
0.230E 03* 1                                                 *
0.240E 03*1                                                  *
0.250E 03*1                                                  *
0.260E 03*1                                                  *
0.270E 03*1                                                  *
0.280E 03*1                                                  *
0.290E 03*1                                                  *
0.300E 03*1                                                  *
0.310E 03*1                                                  *
0.320E 03*1                                                  *
0.330E 03*1                                                  *
0.340E 03*1                                                  *
0.350E 03*1                                                  *
0.360E 03*1                                                  *
0.370E 03*1                                                  *
0.380E 03*1                                                  *
0.390E 03*1                                                  *
0.400E 03*1                                                  *
0.410E 03*1                                                  *
0.420E 03*1                                                  *
0.430E 03*1                                                  *
0.440E 03*1                                                  *
0.450E 03*1                                                  *
0.460E 03*1                                                  *
0.470E 03*1                                                  *
         *....*....*....*....*....*....*....*....*....*....*
  0.33E 01  0.14E 02  0.25E 02  0.36E 02  0.46E 02  0.57E 02
```

Figure P4.7 PARAMETER p_2 STANDARD DEVIATION FOR FIGURE P4.1, 1st PASS

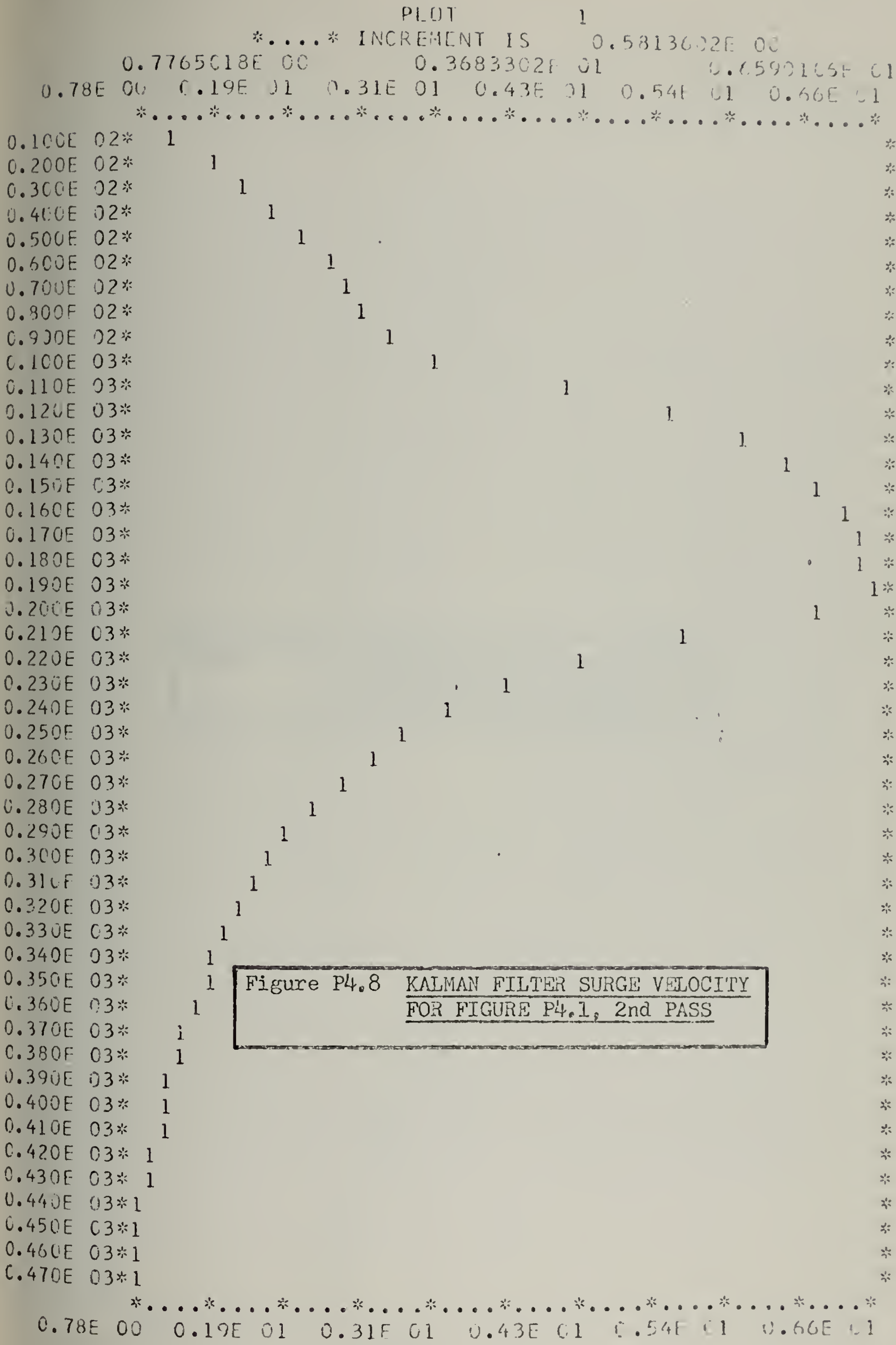


Figure P4.8 KALMAN FILTER SURGE VELOCITY FOR FIGURE P4.1, 2nd PASS

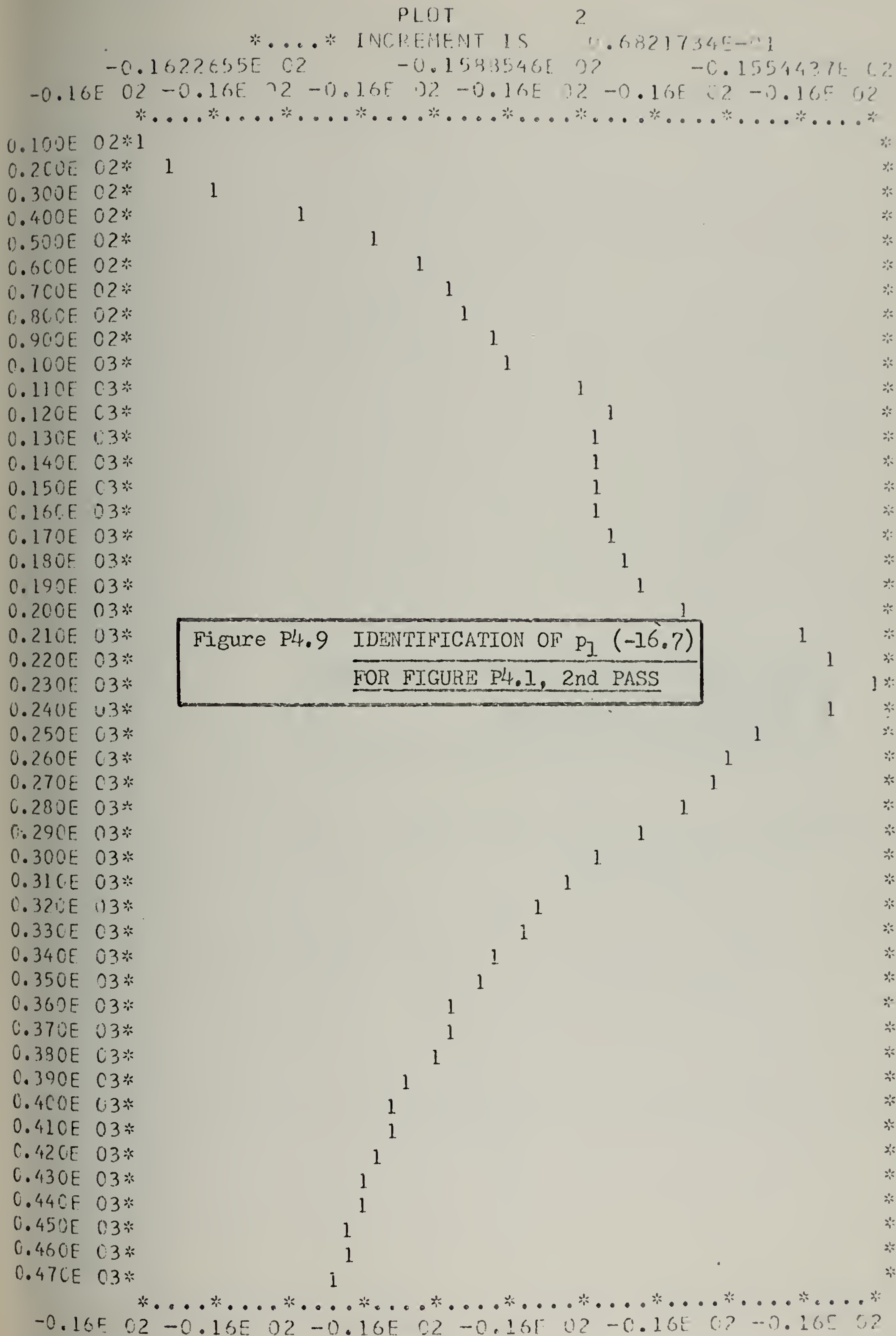


Figure P4.9 IDENTIFICATION OF p_1 (-16.7) FOR FIGURE P4.1, 2nd PASS

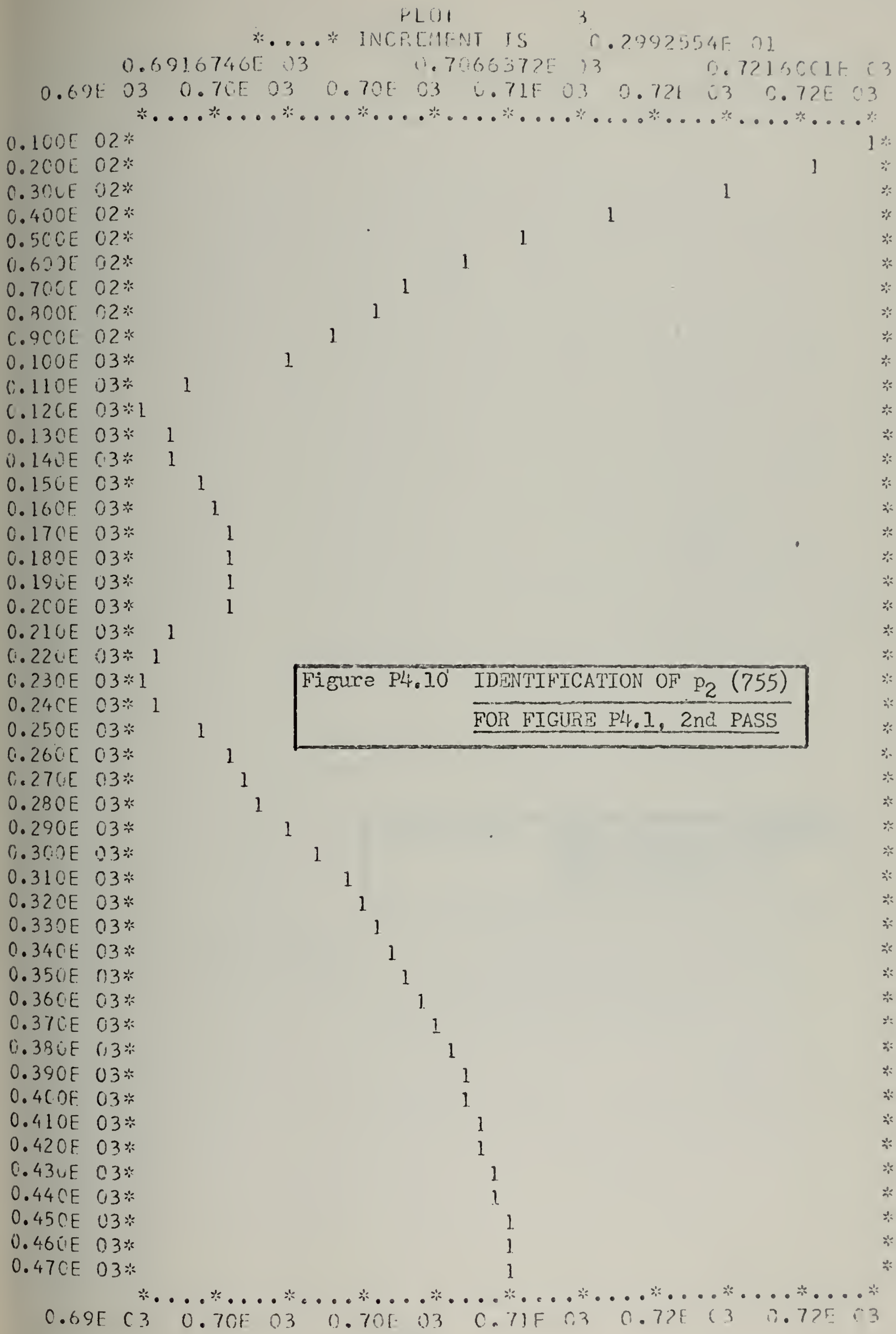


Figure P4.10 IDENTIFICATION OF p_2 (755) FOR FIGURE P4.1, 2nd PASS

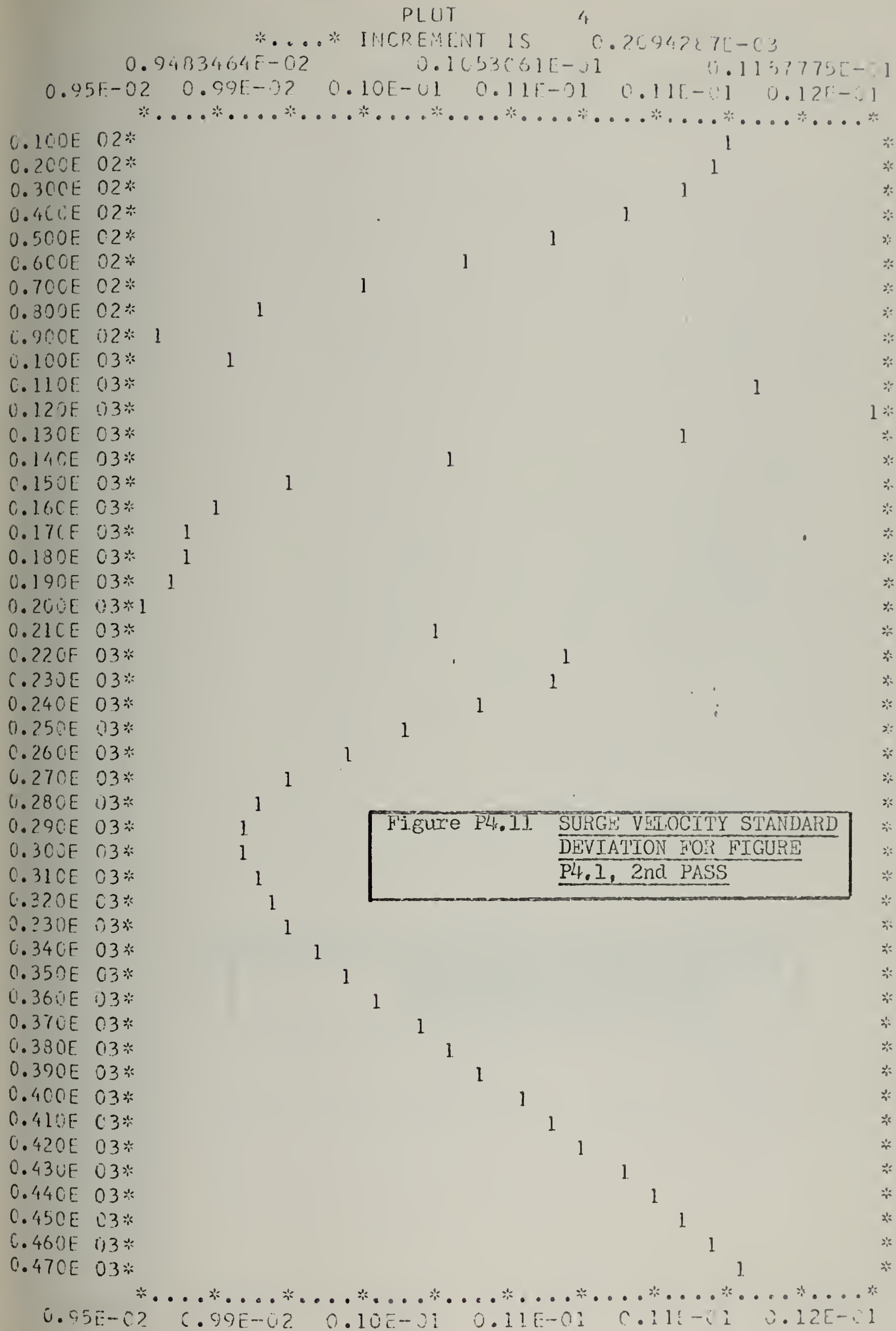


Figure P4.11 SURGE VELOCITY STANDARD DEVIATION FOR FIGURE P4.1, 2nd PASS

PLOT 5

.... INCREMENT IS 0.2155900E-02

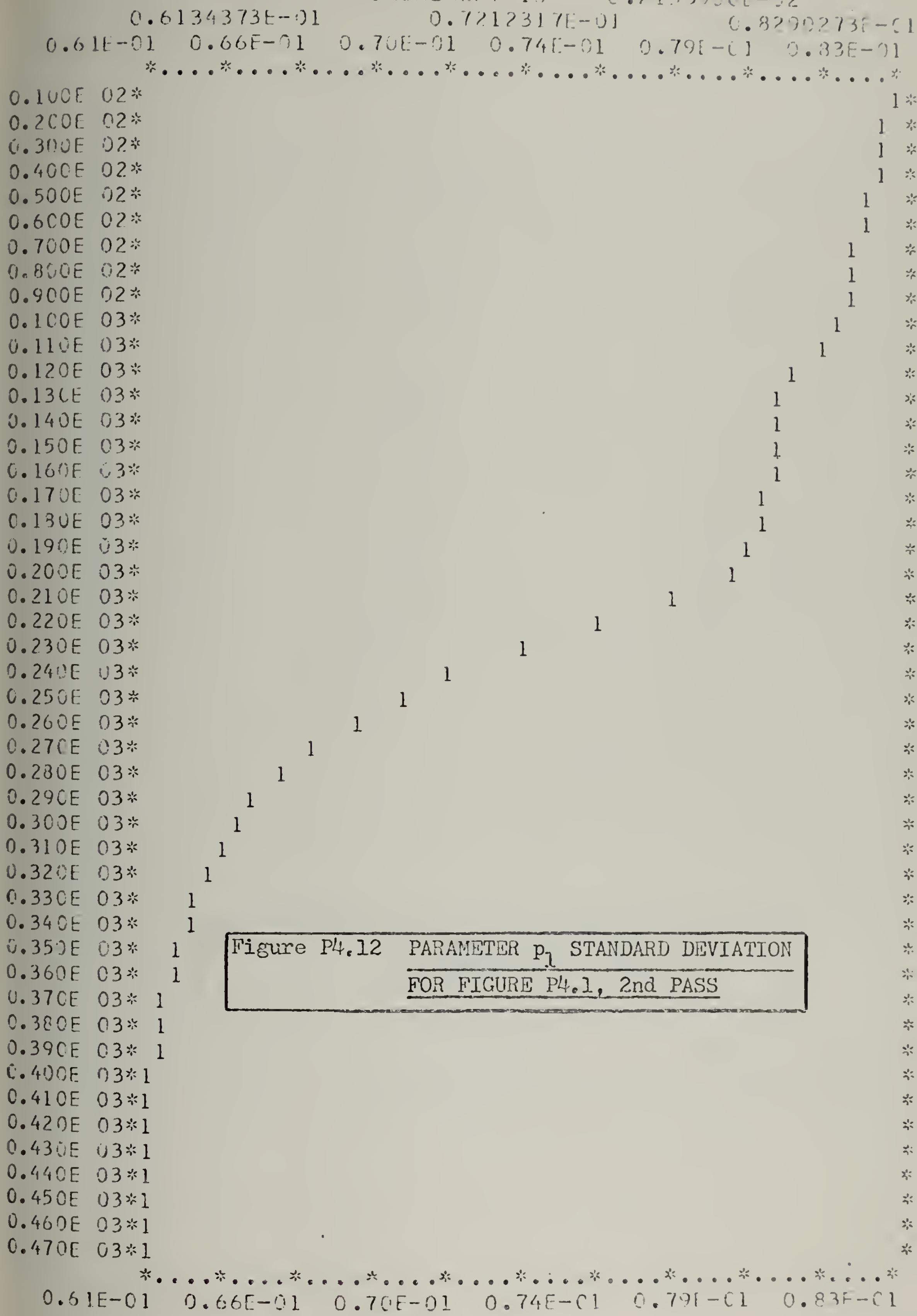


Figure P4.12 PARAMETER p_1 STANDARD DEVIATION FOR FIGURE P4.1, 2nd PASS

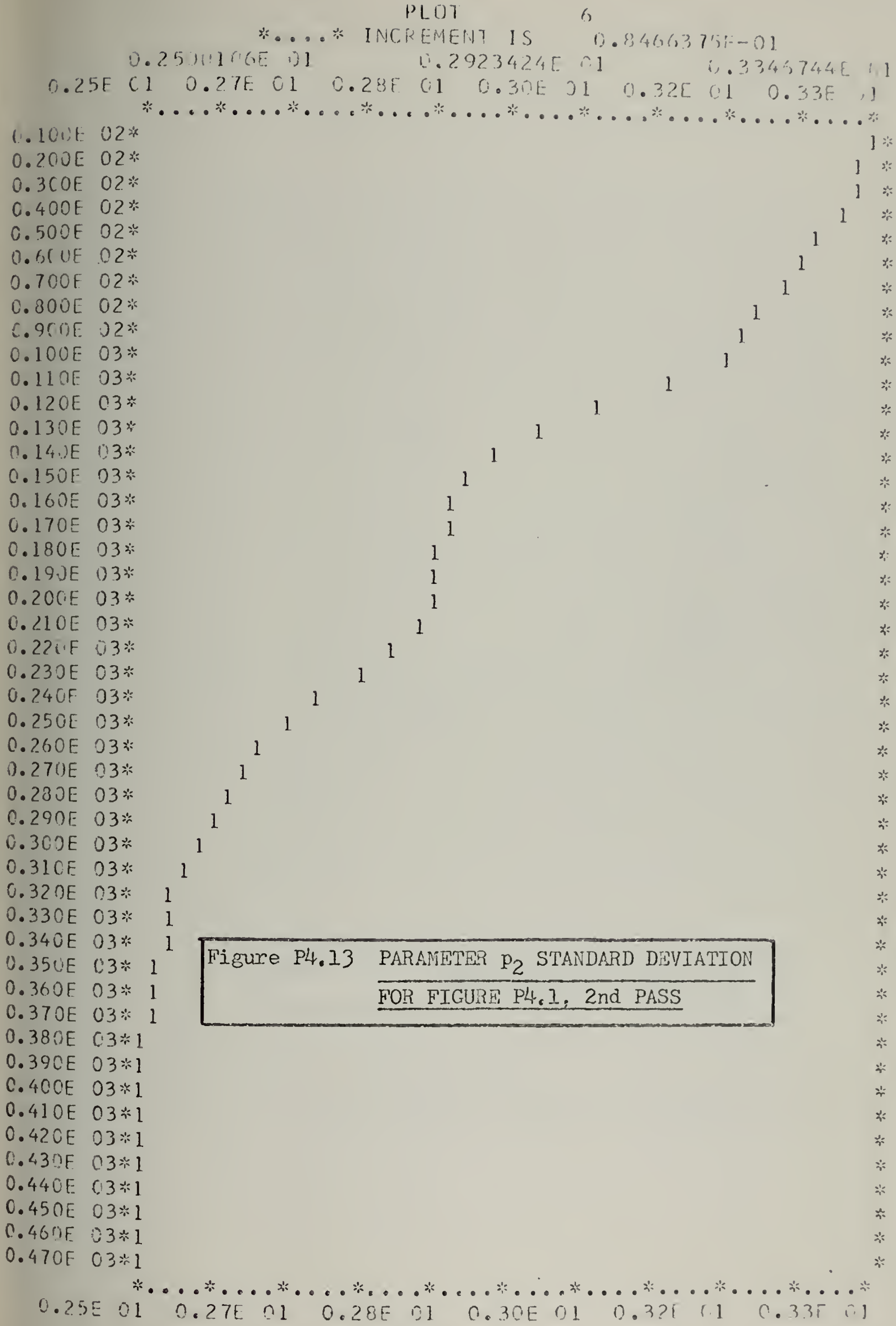


Figure P4.13 PARAMETER p_2 STANDARD DEVIATION FOR FIGURE P4.1, 2nd PASS

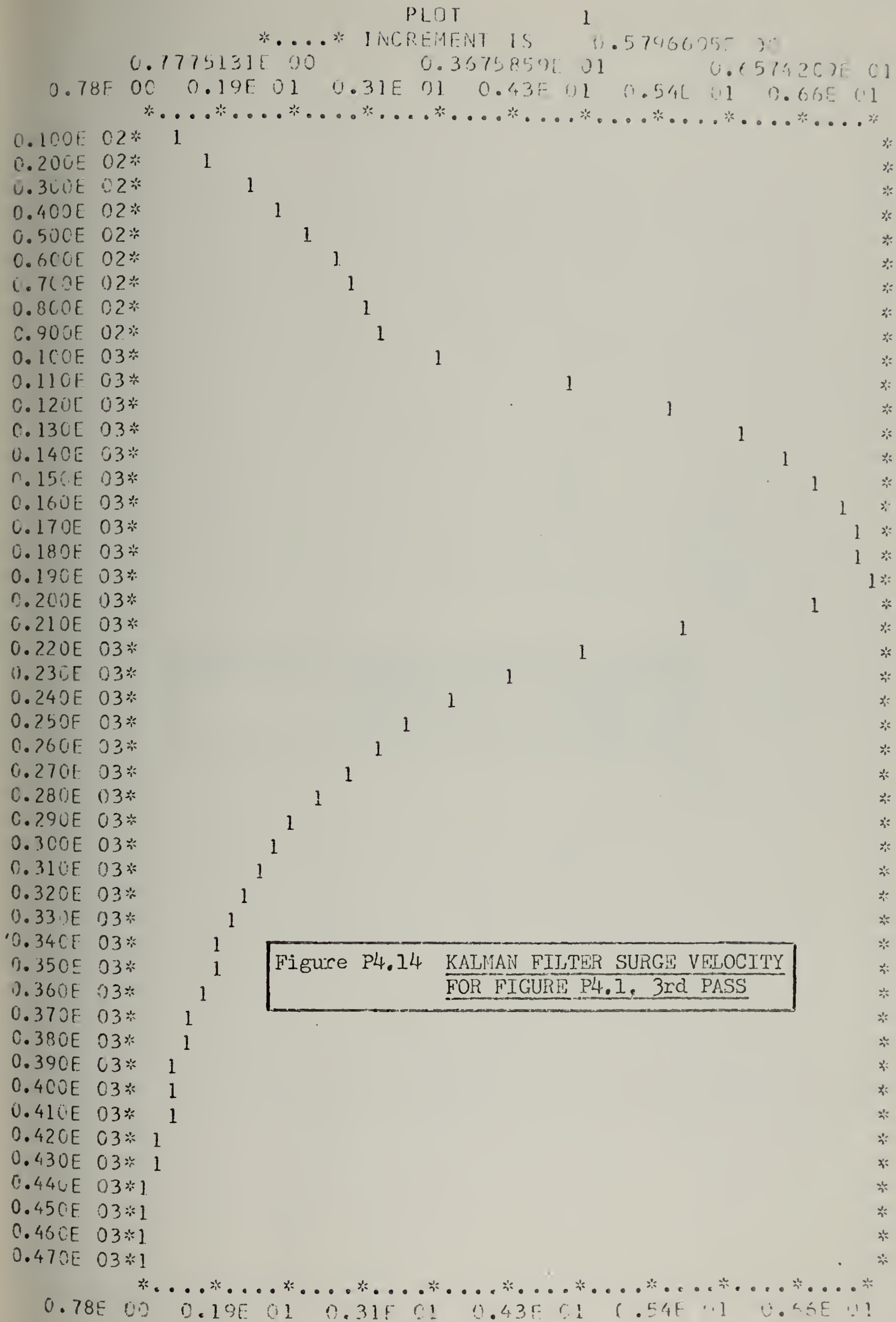


Figure P4.14 KALMAN FILTER SURGE VELOCITY FOR FIGURE P4.1, 3rd PASS

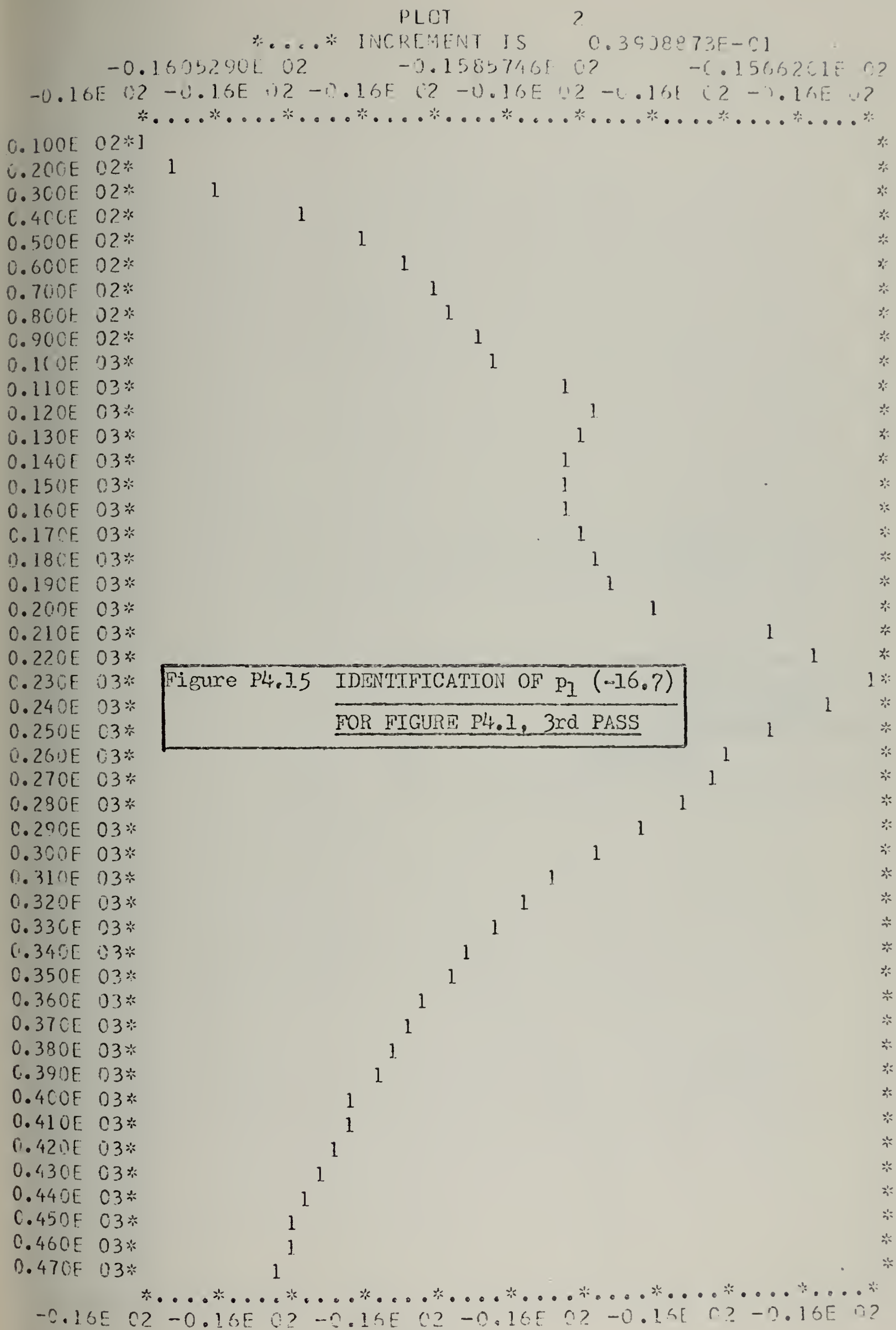


Figure P4.15 IDENTIFICATION OF p_1 (-16.7) FOR FIGURE P4.1, 3rd PASS

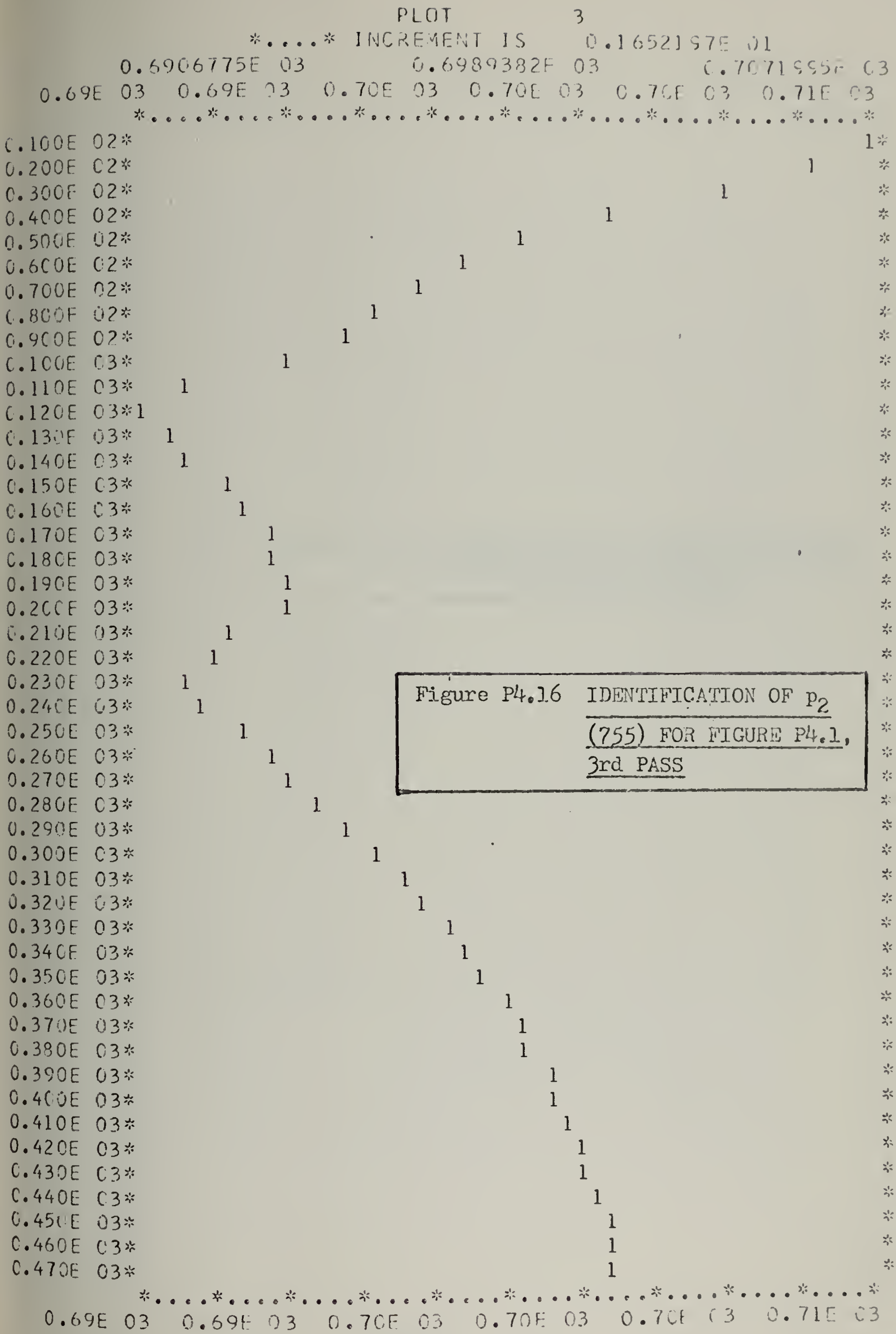


Figure P4.16 IDENTIFICATION OF p_2 (755) FOR FIGURE P4.1, 3rd PASS

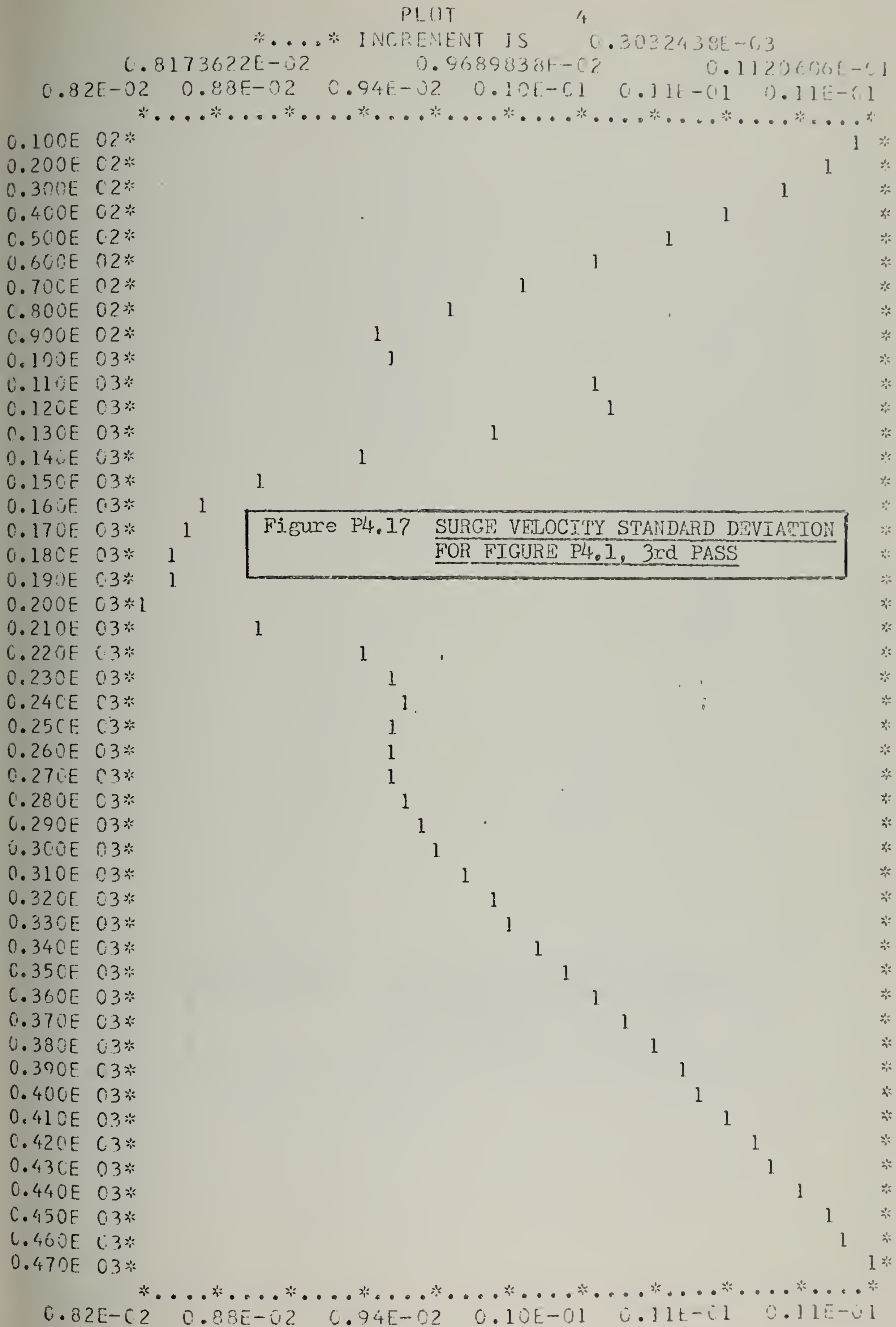


Figure P4.17 SURGE VELOCITY STANDARD DEVIATION FOR FIGURE P4.1, 3rd PASS

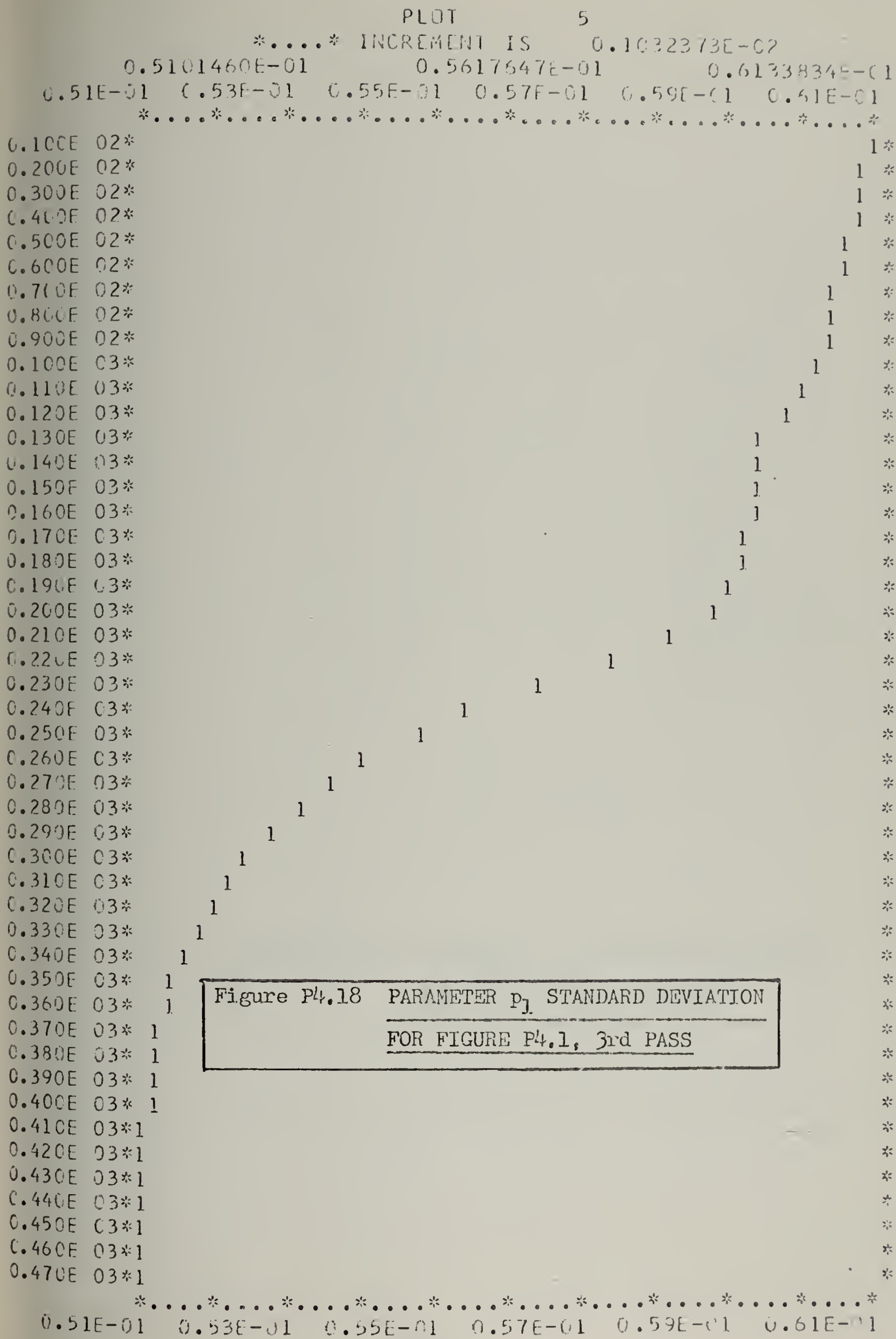


Figure P4.18 PARAMETER p_1 STANDARD DEVIATION FOR FIGURE P4.1, 3rd PASS

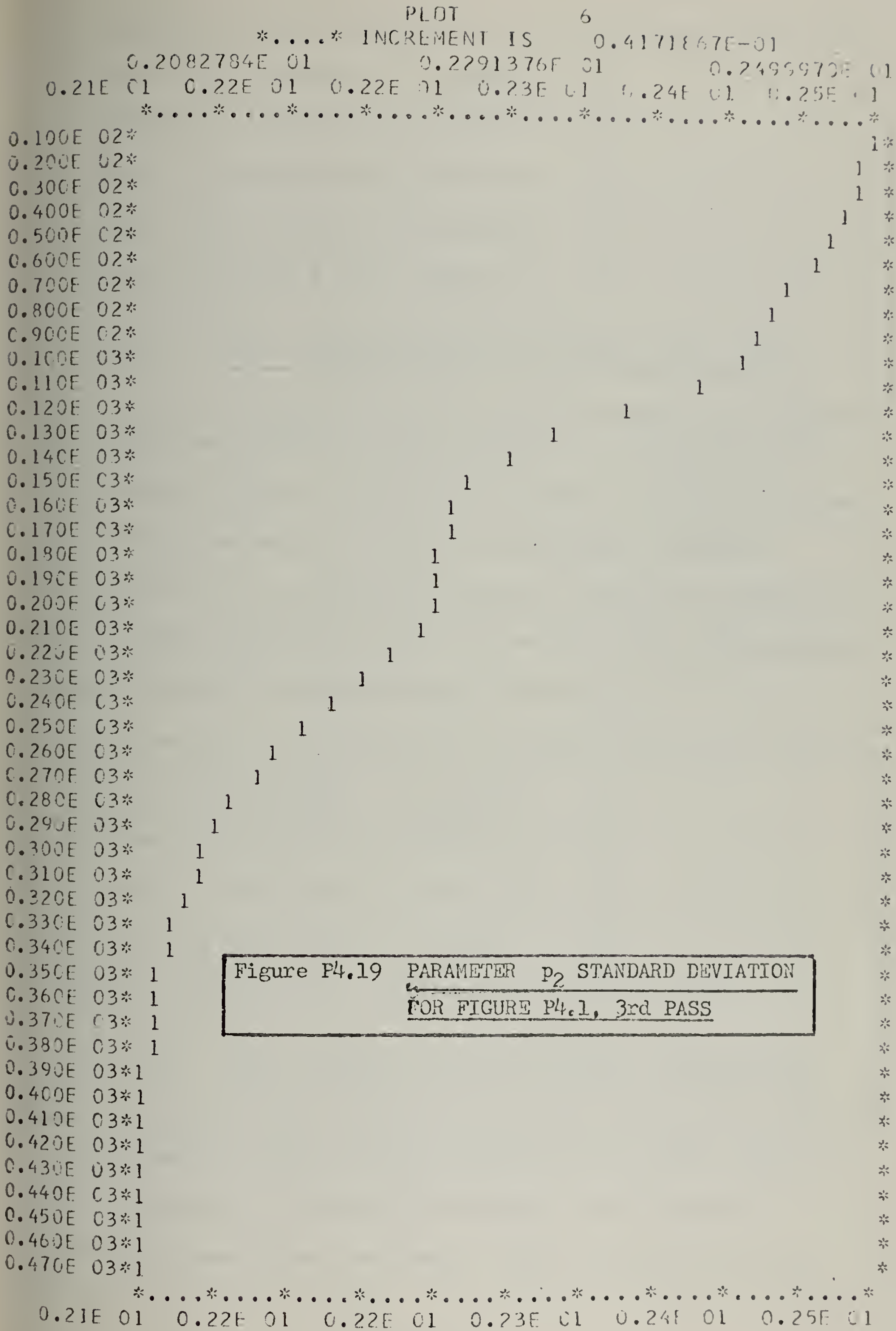


Figure P4.19 PARAMETER p_2 STANDARD DEVIATION FOR FIGURE P4.1, 3rd PASS

$Q/R = 1.$	$p_1(-16.7)$	$p_2(755)$
Initial	-11.2 ± 4.5	943 ± 55
Pass 1	-16.1 ± 0.08	721 ± 3
Pass 2	-16.1 ± 0.06	707 ± 2.5
Pass 3	-15.9 ± 0.05	702 ± 2

Table P4.1 EKF, STAIRCASE, 1%v, 1%w, $x_o=1\pm0.7$, $Q=Q_n$, $R=R_n$

This unwarranted confidence in the estimates of the parameters may be decreased by "telling" the filter that there is more noise in the data than there actually is. This is accomplished by setting $Q = 5Q_n$ and $R = 5R_n$, which has the effect of telling the filter that there is 2.2% v and 2.2% w noises. The results of the corresponding extended Kalman filter run are shown in Table P4.2.

$Q/R = 1.$	$p_1(-16.7)$	$p_2(755)$
Initial	-11.2 ± 4.5	943 ± 55
Pass 1	-16.3 ± 0.2	730 ± 8.3
Pass 2	-16.1 ± 0.1	709 ± 6
Pass 3	-16.0 ± 0.1	702 ± 5

Table P4.2 EKF, STAIRCASE, 1%v, 1%w, $x_o=1\pm0.7$, $Q=5Q_n$, $R=5R_n$

The results of Table P4.2 are essentially the same as those for Table P4.1 except that the filter confidence in the parameter estimates has been slightly decreased. What is happening in both

of these cases is that the extended Kalman filter E matrix for this nonlinear estimation problem is becoming too small, too rapidly over the length of the sea trial causing high confidences in biased values. It is beneficial to recall at this point that in the model reference studies, v noise had a significantly greater effect on the contours than w noise (Chapter P3.2). This suggests that the filter should be told that there is more v noise than there is w noise, or that $Q/R < 1$. The results of Table P4.2 suggest that both Q and R may need to be set significantly higher than the ~~time~~ TRUE noise amounts. As a result of these considerations the next run is made using $Q = 10Q_n$ and $R = 50R_n$. The results of this run are shown in Table P4.3.

$Q/R = 0.2$	$p_1(-16.7)$	$p_2(755$
Initial	-11.2 ± 4.5	943 ± 55
Pass 1	-17.1 ± 0.6	768 ± 25
Pass 2	-16.8 ± 0.4	729 ± 18
Pass 3	-16.6 ± 0.3	716 ± 15

Table P4.3 EKF, STAIRCASE, 1%v, 1%w, $x_o=1\pm0.7$, $Q=10Q_n$, $R=50R_n$

The filter has been "slowed down" considerably in Table P4.3 by the increased values of Q and R. The time value of p_1 has been found to within 1% and within 1 standard deviation whereas the time value of p_2 is about 5% away from the identified value. The error in p_2 and the high value of the filter confidence are due partly to the fact that the initial confidence of 55 represented a reasonably high confidence in an erroneous value. The values of the confidences in p_1 and

p_2 in this table represent about 3% of the true parameter values for a case in which the noises are 1%. This suggests that the values of Q and R may be slightly large for the 1% noise case. The next run is made using $Q = Q_n$ and $R = 10R_n$, and its results are shown in Table P4.4.

Q/R = 0.1	p_1(-16.7)	p_2(755)
Initial	-11.2 ± 4.5	943 ± 55
Pass 1	-16.7 ± 0.23	744 ± 11
Pass 2	-16.6 ± 0.17	718 ± 8
Pass 3	-16.5 ± 0.14	710 ± 6

Table P4.4 EKF, STAIRCASE, 1%v, 1%w, x_o=1±0.7, $Q=Q_n$, $R=10R_n$

In the run represented by Table P4.4 the filter was told that there were 3.2% v and 1% w noises when in actuality there was 1% of each. The identification passes show that p_1 is identified to within 1% mean with 1% standard deviation while p_2 is biased away about 5% with a 1% standard deviation. It is also evident from this table and from the previous three tables that the second and third extended Kalman filter passes really don't provide the significant part of the identification of p_1 and p_2. Most of the identification results for p_1 and p_2 are accomplished in one pass over the sea trial data for the DSRV surge equation.

For the DSRV surge equation with a staircase input, 1% noises, $x_o = 1 \pm 0.7$, and 25% erroneous initial values, the noise covariance values $Q = Q_n$ and $R = 10R_n$ are good values to use in estimating the

parameters. However, this does not necessarily mean that they are still good values to use when one of these major problem factors is changed. For a nonlinear state estimation problem such as this one, the filter must be "tuned" to each variation of the problem.

There is one computational problem which is sometimes encountered when using the extended Kalman filter with small amounts of noise. The E matrix rapidly approaches zero in the filter and may actually become negative due to computer or system noise. Negative E causes the E matrix propagation equation to become unstable, and so some provision should be made to prevent this from happening or to correct it after it happens. One possible and somewhat arbitrary correction procedure is given in Appendix A15.

The next group of studies to be made with the extended Kalman filtering technique and the DSRV surge equation represent the same input function variations as used in the model reference studies of Chapter P3 and the fixed values of $Q = Q_n$ and $R = 10R_n$. The input functions (step, staircase, crashback full astern, simulated impulse, and sinusoidal 100, 150, 300, 500) will be used with the 1% and 10% v and w noise combinations, and the results of 3 extended Kalman filter passes over each run will be tabulated.

P4.2 EXTENDED KALMAN FILTERING USING A STEP FUNCTION INPUT

The step function input is shown in Figure P3.2, and the results of the 1% and 10% v and w combinations of noises using the extended Kalman filter are presented in Table P4.5. These results essentially show that 1% noises permit 1% identifications and 10% noises permit 10% or better identifications using the step function input. A comparison of this table with the corresponding model reference results

in Table P3.1 shows essentially the same results but with the EKF giving slightly better identifications in the 10% noisy cases.

Initial Values					
Initial	p_1	-11.2 ± 4.5	x_o = 1 ± 0.7 Q/R = 0.1		
Values	p_2	943 ± 188	p_1^* = -16.7 p_2^* = 755.		
Noises		1%v 1%w	1%v 10%w	10%v 1%w	10%v 10%w
Pass 1	p_1	-16.9 ± .11	-16.5 ± .33	-15.9 ± 2.5	-14.5 ± 2.2
	p_2	756 ± .07	785 ± 13	730 ± 109	757 ± 108
Pass 2	p_1	-16.9 ± .08	-16.5 ± .28	-15.7 ± 2.1	-14.2 ± 1.9
	p_2	757 ± .01	785 ± 12	718 ± 91	739 ± 90
Pass 3	p_1	-16.9 ± .06	-16.5 ± .26	-15.7 ± 1.9	-14.1 ± 1.6
	p_2	757 ± .01	784 ± 11	712 ± 80	730 ± 78

Table P4.5 EKF, STEP, 1% AND 10% NOISES, $Q=Q_n$, $R=10R_n$

P4.3 EXTENDED KALMAN FILTERING USING A STAIRCASE FUNCTION INPUT

The staircase function input is shown in Figures P3.10 and P4.1, and the results of several noise combinations (1 and 10, 10 and 20, 1 and 100) are presented in Table P4.6. The initial values for Table P4.6 are the same as those in Table P4.5 and are not repeated. The results of Table P4.6 show that 1% noises permit 4% or better identifications, 10% noises permit 40% or better identifications, 20% noises permit 60% or better identifications, and 100% noises permit 80% or better identifications. The two 10% v 10% w cases are different because different noise starter integers were used in the noise generating subroutines.

The results of the staircase input function identifications using the EKF are surprisingly poor when compared with the step

function identifications and with the staircase model reference identifications in Table P3.2. The primary reason for this is that the v_n and w_n noises enter the EKF at 4 times as many points because of the reduced step size, and therefore these noises behave as the equivalent of about 4 times their actual percentages as determined from comparing Table P4.6 with Table P3.2. Thus the "modified" results are that 4% noises permit 4% identifications, 40% noises permit 40% identifications, 80% noises permit 60% identifications, and 400% noises permit 80% identifications.

These modified results show that the filter behaves about the same for the staircase input as it did for the step function (unmodified) and about the same as the model reference results in Table P3.2. The results in Table P3.2 for the larger noise cases are somewhat misleading in accuracy because the contours were constrained to $\pm$ 50% parameter variations. The 60% and 80% identifications for the 80% and 400% noises are possible because the filter is "climbing the wall" in Figure P3.13, which is caused by the fact that as p_1 approaches zero the surge equation approaches instability. The w_n noise for large percentages causes such large biases that the vehicle never slows below 5 ft/sec over the staircase input sea trial.

P4.4 EXTENDED KALMAN FILTERING USING A CRASHBACK FULL ASTERN INPUT

The crashback full astern input is shown in Figure P3.14, and the results of the 1% and 10% v and w noise combinations are presented in Table P4.7. These results show that for this input function 1% noises (4% modified) permit 10% or better identifications and 10% noises (40% modified) permit 20% or better identifications. These

Noises		1%v 1%w	1%v 10%w	10%v 1%w	10%v 10%w
Pass 1	p_1	-16.5 ± .22	-14.0 ± .44	-15.4 ± 1.8	-9.6 ± 1.6
	p_2	729 ± 9	700 ± 19	730 ± 87	630 ± 91
Pass 2	p_1	-16.5 ± .17	-14.0 ± .31	-15.5 ± 1.4	-9.4 ± 1.0
	p_2	732 ± 7	699 ± 14	719 ± 67	600 ± 61
Pass 3	p_1	-16.6 ± .14	-14.1 ± .25	-15.5 ± 1.2	-9.3 ± 0.8
	p_2	734 ± 6	698 ± 11	715 ± 57	590 ± 48

Noises		10%v 10%w	10%v 20%w	20%v 10%w	20%v 20%w
Pass 1	p_1	-22.6 ± 2.1	-12.1 ± 1.8	-13.3 ± 2.5	-8.4 ± 2.2
	p_2	786 ± 79	683 ± 96	781 ± 134	728 ± 135
Pass 2	p_1	-24.2 ± 1.8	-11.7 ± 1.3	-13.0 ± 2.1	-7.4 ± 1.5
	p_2	812 ± 68	652 ± 69	738 ± 112	630 ± 103
Pass 3	p_1	-25.1 ± 1.7	-11.5 ± 1.1	-12.8 ± 1.8	-6.9 ± 1.1
	p_2	826 ± 61	641 ± 56	714 ± 98	585 ± 83

Noises		1%v 1%w	1%v 100%w	100%v 1%w	100%v 100%w
Pass 1	p_1	Same as Above	-6.7 ± 1.4	-12.6 ± 4.0	-6.5 ± 3.7
	p_2	" "	608 ± 101	912 ± 183	969 ± 183
Pass 2	p_1	" "	-5.9 ± 1.1	-13.5 ± 3.7	-3.8 ± 2.5
	p_2	" "	539 ± 79	894 ± 178	935 ± 176
Pass 3	p_1	" "	-5.5 ± 0.9	-14.0 ± 3.5	-3.1 ± 1.6
	p_2	" "	508 ± 67	882 ± 174	883 ± 168

Initial Values Same as Table P4.5

Table P4.6 EKF, STAIRCASE, NOISE COMBINATIONS, $Q=Q_n$, $R=10R_n$

Noises		1%v 1%w	1%v 10%w	10%v 1%w	10%v 10%w
Pass 1	p_1	$-15.4 \pm .14$	$-14.5 \pm .24$	-15.3 ± 1.3	-14.5 ± 1.1
	p_2	720 ± 4	701 ± 8	732 ± 44	661 ± 39
Pass 2	p_1	$-15.4 \pm .10$	$-14.6 \pm .18$	-15.3 ± 0.9	-14.8 ± 0.8
	p_2	721 ± 3	704 ± 6	727 ± 32	664 ± 29
Pass 3	p_1	$-15.3 \pm .08$	$-14.7 \pm .15$	-15.3 ± 0.8	-15.0 ± 0.7
	p_2	722 ± 3	706 ± 5	725 ± 27	666 ± 24

Initial Values Same as Table P4.5

Table P4.7 EKF, CRASHBACK FULL ASTERN, 1% AND 10% NOISES, $Q=Q_n$, $R=10R_n$

values are not as good as those for the model reference contouring technique in Table P3.3, but the "modified" noises again cause misleading conclusions. The fact that effectively 40% noises permit 20% or better identifications indicates that the EKF is at least as accurate an identification technique as model reference contouring for this input function. These errors might also be reduced by adjusting Q and R for this input or by using a better integration technique.

P4.5 EXTENDED KALMAN FILTERING USING A SIMULATED IMPULSE FUNCTION INPUT

The simulated impulse function input is shown in Figure P3.18, and the results of the 1% and 19% v and w noise combinations are presented in Table P4.8. These results show that 1% (4% modified) noises permit 25% or better identifications and 10% (40% modified) noises permit 30% or better identifications using the simulated

Noises		1%v 1%w	1%v 10%w	10%v 1%w	10%v 10%w
Pass 1	p_1	-13.0 ± .06	-12.5 ± .08	-14.7 ± 1.0	-11.3 ± 0.9
	p_2	604 ± .01	604 ± .68	676 ± 44	583 ± 43
Pass 2	p_1	-13.4 ± .05	-12.5 ± .06	-14.7 ± 0.7	-11.3 ± 0.6
	p_2	608 ± .001	605 ± .64	673 ± 34	576 ± 30
Pass 3	p_1	-13.6 ± .04	-12.7 ± .05	-14.7 ± 0.6	-11.3 ± 0.5
	p_2	610 ± .001	605 ± .63	672 ± 28	573 ± 25

Initial Values Same as Table P4.5

Table P4.8 EKF, SIMULATED IMPULSE, 1% AND 10% NOISES, $Q=Q_n$, $R=10R_n$

impulse function input. The high degrees of confidence which the filter has in the biased values indicates either that this is not a good input for the EKF or that the values of Q and R need to be adjusted. The identifiability characteristics of the EKF using this input are far worse, even considering the modified noise behavior, than those of the model reference contouring technique in Table P3.4.

P4.6 EXTENDED KALMAN FILTERING USING SINUSOIDAL INPUT FUNCTIONS

The input functions used in these runs were sinusoidal functions of amplitude $\sqrt{2}$ and of periods 100, 150, 300, and 500 seconds. The results of multiple runs conducted with these four sinusoidal input functions are presented in Tables P4.9 through P4.12. The results in Table P4.9 for the sinusoidal-100 second input functions show some of the problems which may be encountered when using the EKF. For the low w noise cases in Tables P4.9 and P4.10 the filter dynamics were so fast that the Euler integrator, even with a step size of 2.5 seconds, could

not keep up. Also, it is evident that $Q = Q_n$ is much too low for the filter when using any of the four sinusoidal input functions of Tables P4.9 through P4.12.

The essential results of Tables P4.9 through P4.12 may be summarized in abbreviated form as follows:

100 second,	10%	noises	permit	25%	or better	identifications
150 second,	10%	"	"	30%	"	"
	20%	"	"	40%	"	"
	100%	"	"	50%	"	"
300 second,	1%	"	"	15%	"	"
	10%	"	"	10%	"	"
500 second,	1%	"	"	10%	"	"
	10%	"	"	20%	"	"

It may be expected that some of these figures could be bettered by readjusting the values of Q and R, by decreasing the integration step size and computing more points over the filtering process, by increasing the length of the sea trial, or by using a better integration method, such as Runge-Kutta. A comparison between these tables and the corresponding model reference results in Tables P3.5 through P3.8 shows that for sinusoidal inputs the EKF is not as accurate as model reference contouring unless the "modified" effects of the noises in the EKF are taken into consideration.

P4.7 EKF USING A LONGER SEA TRIAL, CLOSER INITIAL ESTIMATES, AND RUNGE-KUTTA INTEGRATION

The operation of the extended Kalman filter is much more critically dependent upon the input function, the initial state and parameter estimates, the values of Q and R, the length of the sea trial run, and

Noises		1%v 1%w	1%v 10%w	10%v 1%w	10%v 10%w
Pass 1	p_1	-17.7 ± 10^{-3}	-14.0 ± .16	-13.4 ± 1.3	-11.8 ± 1.2
	p_2	605 ± 0.1	652 ± .16	719 ± 29	702 ± 28
Pass 2	p_1	-26.0 ± 10^{-3}	-16.7 ± .02	-13.3 ± 1.0	-11.7 ± 0.9
	p_2	519 ± 0.1	610 ± .12	717 ± 21	701 ± 20
Pass 3	p_1	-38.6 ± 10^{-3}	-23.4 ± .003	-13.3 ± 0.8	-11.7 ± 0.7
	p_2	417 ± .05	525 ± 0.1	716 ± 17	701 ± 16

Noises		10%v 10%w	10%v 20%w	20%v 10%w	20%v 20%w
Pass 1	p_1	-14.5 ± 1.4	-12.4 ± 1.3	-12.9 ± 2.1	-11.1 ± 2.1
	p_2	714 ± 30	700 ± 29	724 ± 56	692 ± 54
Pass 2	p_1	-14.2 ± 1.1	-12.2 ± 0.9	-12.9 ± 1.7	-10.6 ± 1.6
	p_2	714 ± 21	698 ± 20	715 ± 40	684 ± 39
Pass 3	p_1	-14.1 ± 0.9	-12.1 ± 0.8	-12.8 ± 1.4	-10.3 ± 1.3
	p_2	714 ± 17	698 ± 17	712 ± 33	681 ± 32

Noises		1%v 1%w	1%v 100%w	100%v 1%w	100%v 100%w
Pass 1	p_1	Same as Above	-8.5 ± 1.0	-12.2 ± 4.0	-11.3 ± 4.0
	p_2	" "	679 ± 30	879 ± 161	786 ± 159
Pass 2	p_1	" "	-8.3 ± 0.7	-12.7 ± 3.8	-10.7 ± 3.8
	p_2	" "	674 ± 22	845 ± 142	692 ± 139
Pass 3	p_1	" "	-8.3 ± 0.6	-13.1 ± 3.6	-9.6 ± 3.6
	p_2	" "	672 ± 18	824 ± 129	623 ± 124

Initial Values Same as Table P4.5

Table P4.10 EKF, SINUSOIDAL, 150 SECOND PERIOD, $Q=Q_n$, $R=10R_n$

Noises		1%v 1%w	1%v 10%w	10%v 1%w	10%v 10%w
Pass 1	p_1	-13.6 ± .02	-11.7 ± .02	-12.7 ± 1.9	-12.6 ± 1.9
	p_2	529 ± 0.2	634 ± 0.4	744 ± 37	738 ± 37
Pass 2	p_1	-70.8 ± .02	-27 ± .02	-12.6 ± 1.5	-12.6 ± 1.5
	p_2	353 ± .07	479 ± 0.2	740 ± 26	734 ± 27
Pass 3	p_1	-119 ± .02	-52 ± .02	-12.6 ± 1.2	-12.6 ± 1.3
	p_2	250 ± .03	329 ± 0.1	739 ± 22	733 ± 22

Initial Values Same as Table P4.5

Table P4.9 EKF, SINUSOIDAL, 100 SECOND PERIOD 1% and 10% NOISES, Q-Q_n, $R=10R_n$

Noises		1%v 1%w	1%v 10%w	10%v 1%w	10%v 10%w
Pass 1	p_1	-14.3 ± 0.1	-14.7 ± 0.2	-14.6 ± 1.0	-14.0 ± 0.9
	p_2	653 ± .02	704 ± 5	700 ± 33	698 ± 31
Pass 2	p_1	-14.3 ± .07	-14.7 ± 0.1	-14.7 ± 0.7	-14.1 ± 0.6
	p_2	652 ± .01	704 ± 4	699 ± 24	699 ± 23
Pass 3	p_1	-14.4 ± .05	-14.7 ± 0.1	-14.7 ± 0.6	-14.2 ± 0.5
	p_2	652 ± .01	705 ± 3	698 ± 19	700 ± 19

Initial Values Same as Table P4.5

Table P4.11 EKF, SINUSOIDAL, 300 SECOND PERIOD, 1% AND 10% NOISES, $Q=Q_n$, $R=10R_n$

the speed and accuracy of the integration method than is the model reference contouring technique. The previous studies of this chapter have shown that with slight preliminary adjustments the EKF is nearly

Noises		1%v 1%w	1%v 10%w	10%v 1%w	10%v 10%w
Pass 1	p_1	-14.8 ± 0.1	$-14.9 \pm .21$	-15.4 ± 1.1	-13.3 ± 1.0
	p_2	$667 \pm .06$	688 ± 8	703 ± 45	616 ± 41
Pass 2	p_1	$-14.8 \pm .07$	$-14.8 \pm .15$	-15.2 ± 0.8	-13.2 ± 0.7
	p_2	$665 \pm .03$	684 ± 6	697 ± 32	612 ± 28
Pass 3	p_1	$-14.8 \pm .05$	$-14.8 \pm .12$	-15.2 ± 0.7	-13.1 ± 0.6
	p_2	$664 \pm .02$	682 ± 5	695 ± 26	611 ± 23

Initial Values Same as Table P4.5

Table P4.12 EKF, SINUSOIDAL, 500 SECOND PERIOD, 1% AND 10% NOISES, $Q=Q_n$, $R=10R_n$

as good as the model reference technique for low noise percentages and is a much better technique for high noise percentages. The following studies will show that for a better integration technique, a longer sea trial, and slightly closer initial estimates the EKF can be made to operate as well as model reference contouring in the low noise percentage cases.

A large number of EKF runs were made by the author using the staircase function input with Runge-Kutta integration in both the sea trial data generation subroutine and the EKF state and error covariance propagation subroutines. The sea trial time step was increased to H = 4 seconds, and the input function step times in Figure P3.10 were increased from 100 and 200 seconds, to 188 and 376 seconds respectively. The EKF was told that there was 2.2 times as much noise in the data as there was actually by setting $Q = 5Q_n$ and $R = 5R_n$. The initial values of p_1 were specified to within 25% and one standard deviation,

and the initial values of p_2 were specified to within 10% and one standard deviation. The noise percentages used were 5% v (20% effectively) and 2% w (8% effectively). The results of four of these runs are shown in Table P4.13 for four different combinations of initial estimates of the parameters.

The EKF runs described in Table P4.13 show how remarkably well this filter can identify the DSRV surge parameters when it is properly "tuned" and when it has a good integrator. These numbers show that 5% v (20% modified) and 2% w (8% modified) noises permit 5% or better identifications from any of four directions represented by the four different initial conditions which are 25% and 19% away from the true values initially. The Fortran IV statements for subroutines RKNL and PROP using Runge-Kutta integration are presented in Appendix A16.

Initial Values	$Q = 5Q_n$, $H = 4$ sec., Staircase steps at 188 and 376 sec. $R = 5R_n$, $\sigma_{p_1} = 6$, $\sigma_{p_2} = 50$, $x_o = 1 \pm$, Int. = Runge-Kutta			
Initial p's	$p_1=-13, p_2=800$	$p_1=-13, p_2=700$	$p_1=-21, p_2=800$	$p_1=-21, p_2=700$
Pass 1 p_1	-16.4 ± 0.9	-16.0 ± 0.8	-16.5 ± 0.9	-16.0 ± 0.8
p_2	760 ± 40	715 ± 39	781 ± 40	725 ± 39
Pass 2 p_1	-16.1 ± 0.7	-15.9 ± 0.7	-16.2 ± 0.7	-15.9 ± 0.7
p_2	750 ± 34	720 ± 33	753 ± 34	726 ± 33
Pass 3 p_1	-16.1 ± 0.6	-15.8 ± 0.6	-16.2 ± 0.6	-15.9 ± 0.6
p_2	738 ± 30	722 ± 29	742 ± 30	726 ± 29

Table P4.13 EKF, STAIRCASE, H=4, 5%v, 2%w, RUNGE-KUTTA, DIFFERENT INITIAL CONDITIONS

P4.8 SOME DRAMATIC EKF RESULTS WITH LARGE AMOUNTS OF NOISE

When the noise percentages are increased, and the "tuned" filter of Table P4.13 is used; then the identifications of Table P4.14 are generated by the filter. The noises used here appear so evidently

Initial p's	$p_1 = -13 \pm 4.5$	$p_2 = 800 \pm 64$
25%v 25%w	-16.5 ± 1.7	782 ± 61
75%v 75%w	-16.1 ± 2.9	791 ± 63

Initial Values same as Table P4.13 except as noted

Table P4.14 EKF, STAIRCASE, RUNGE-KUTTA, LARGE NOISES

in the sea trial data and are filtered out so dramatically by the EKF that the author decided to include the actual plots which were generated. The staircase input function and the sea trial data for the cases of 25% and 75% noises (v and w) are shown in Figures P4.20 and P4.24. The extended Kalman filter estimates of the vehicle surge velocity during the sea trial and of the parameters p_1 and p_2 are shown for these two noise percentages in Figures P4.21 through P4.23 and Figures P4.25 through P4.27 for the 25% and 75% noises respectively.

In the model reference contouring technique used in Chapter P3, the actual identifier was the "observer" of the contours. In Figure P4.24 for the 75% noisy sea trial, it is practically impossible to "observe" even the vehicle behavior over the sea trial, not to mention evaluating the mathematical model parameters. In fact, an observer might wonder whether or not there is any ocean vehicle behavior at all in the "hash" of Figure P4.24. A quick look at

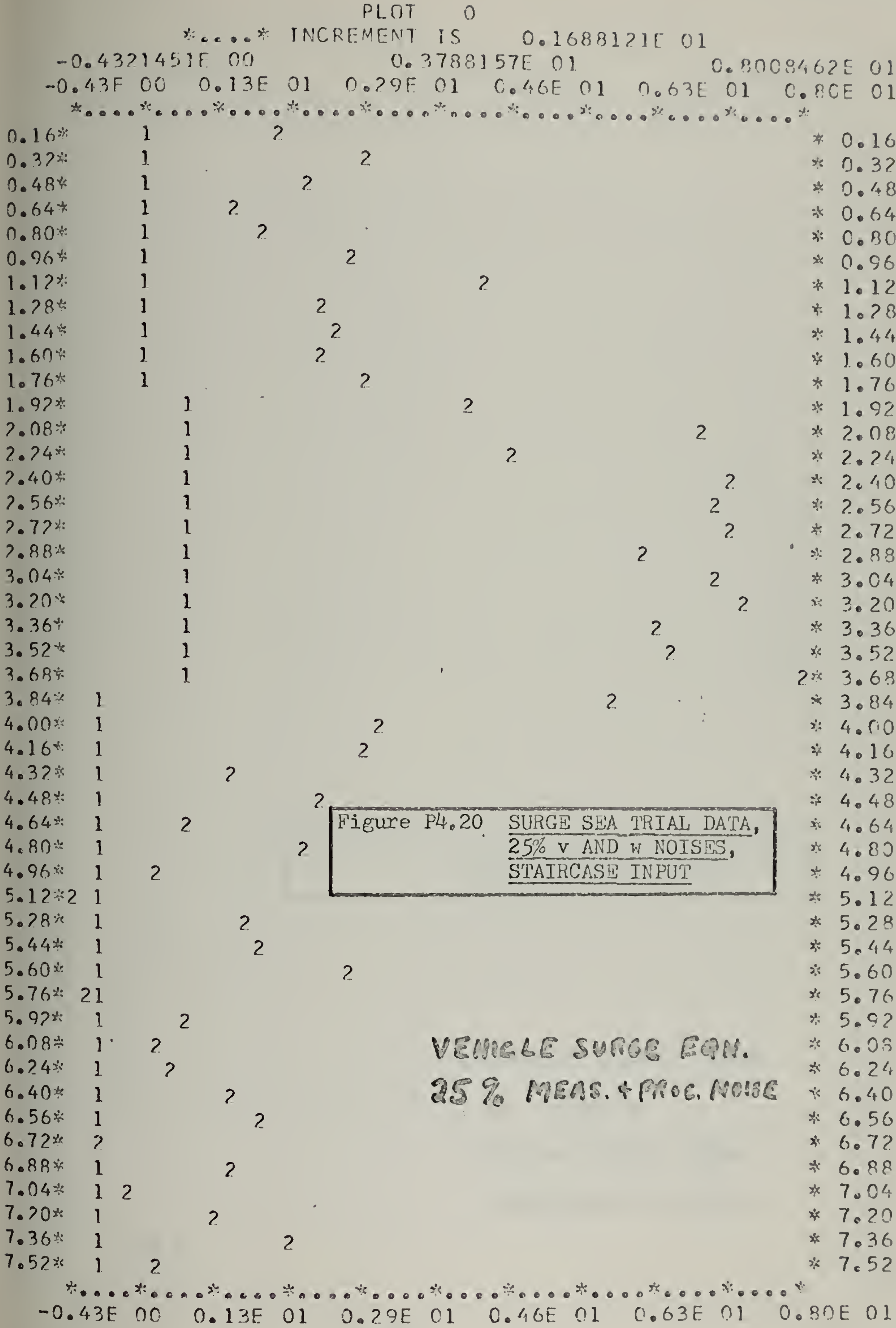


Figure P4.20 SURGE SEA TRIAL DATA, 25% v AND w NOISES, STAIRCASE INPUT

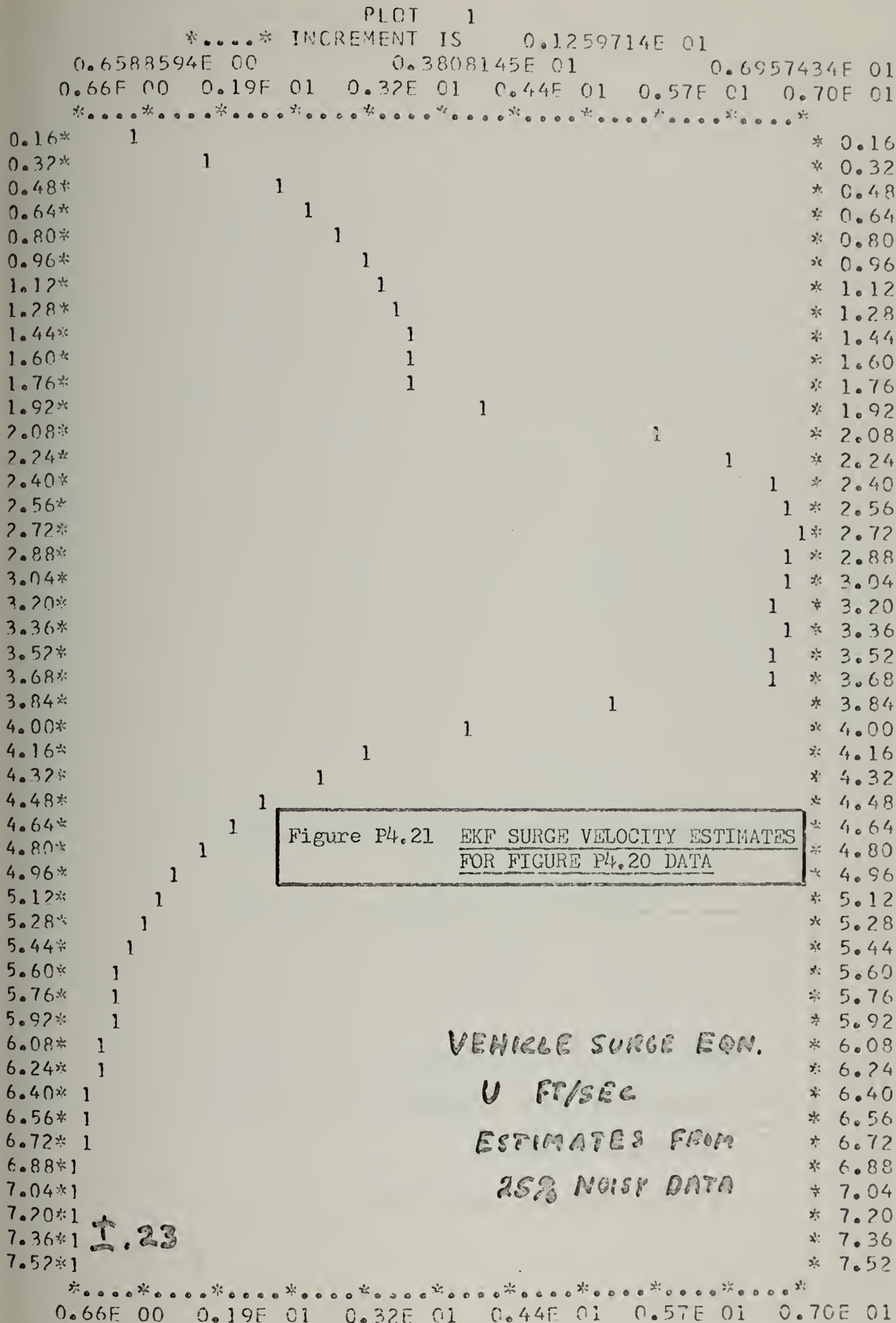


Figure P4.21 EKF SURGE VELOCITY ESTIMATES FOR FIGURE P4.20 DATA

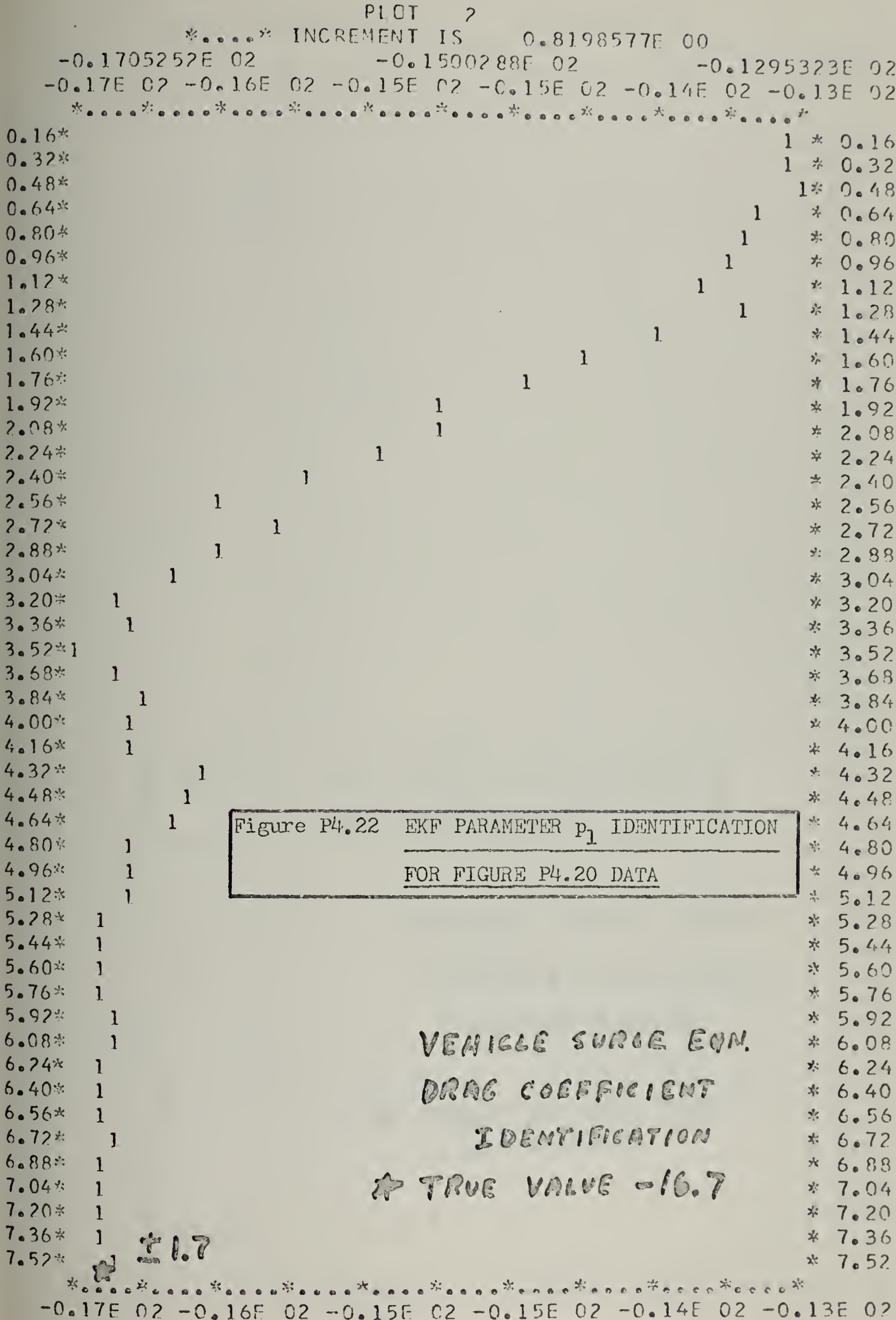


Figure P4.22 EKF PARAMETER p_1 IDENTIFICATION FOR FIGURE P4.20 DATA

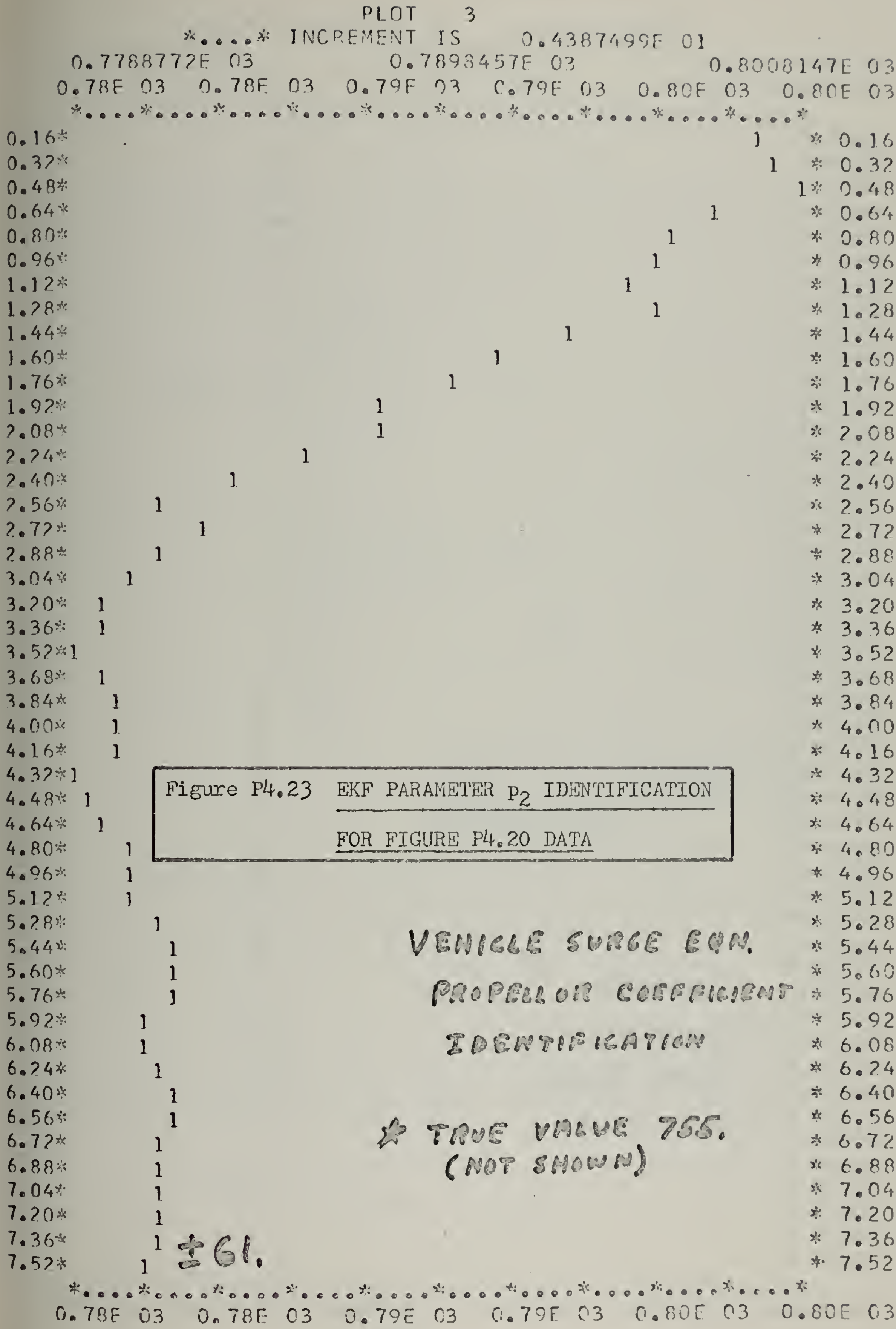


Figure P4.23 EKF PARAMETER p_2 IDENTIFICATION FOR FIGURE P4.20 DATA

VEHICLE SURGE EQN.
PROPELLOR COEFFICIENT
IDENTIFICATION

☆ TRUE VALUE 755.
(NOT SHOWN)

±61.

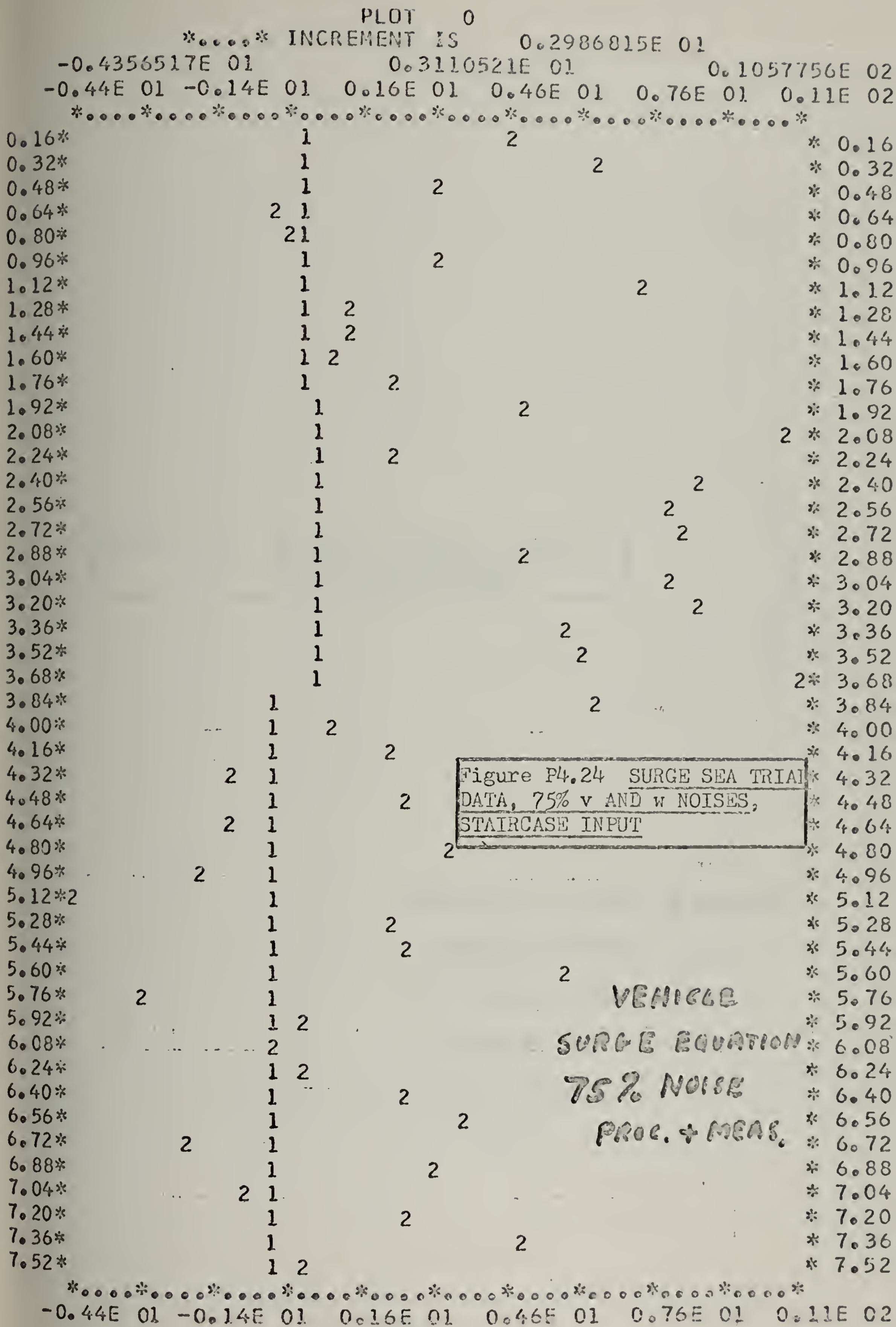


Figure P4.24 SURGE SEA TRIAL DATA, 75% v AND w NOISES, STAIRCASE INPUT

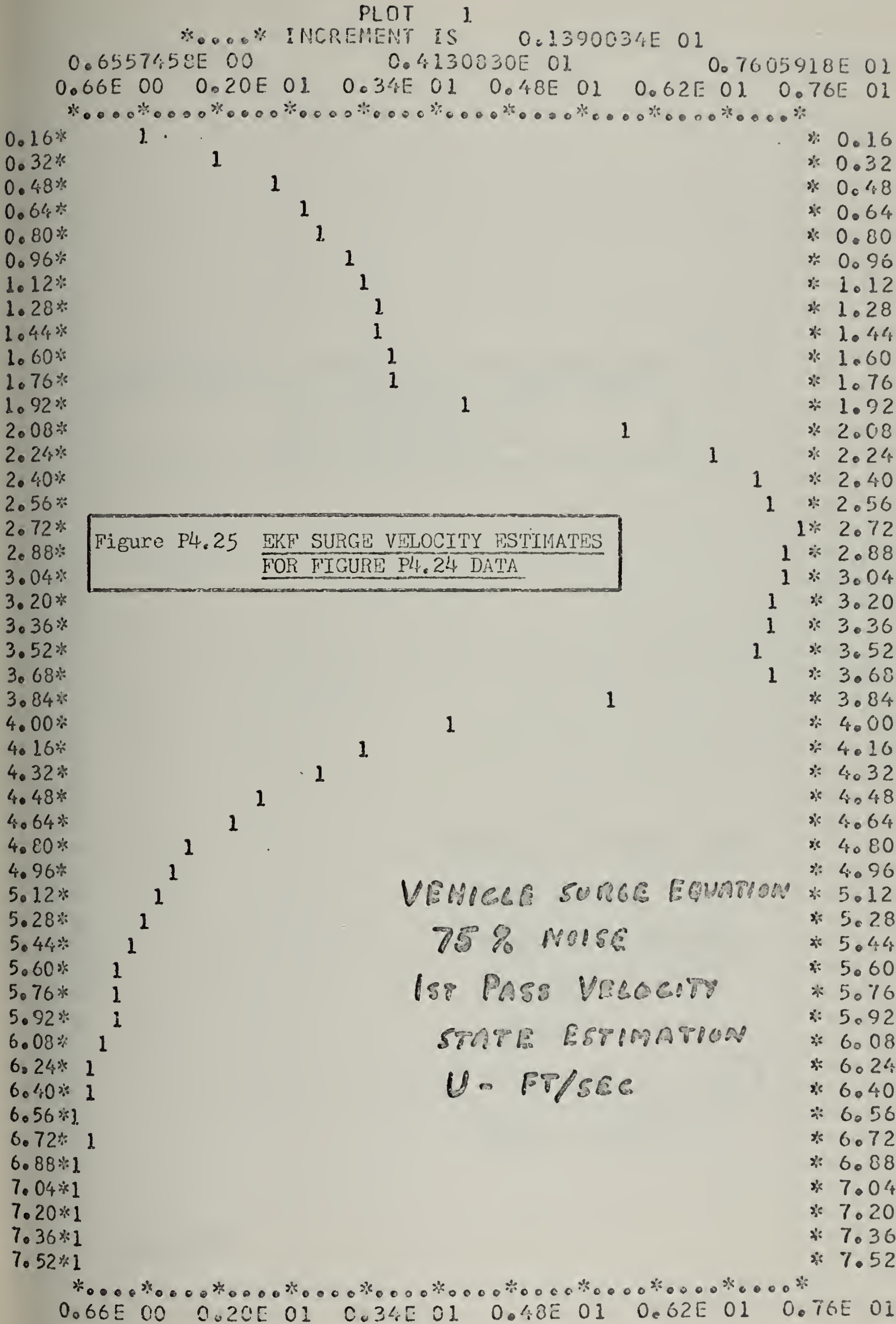


Figure P4.25 EKF SURGE VELOCITY ESTIMATES FOR FIGURE P4.24 DATA

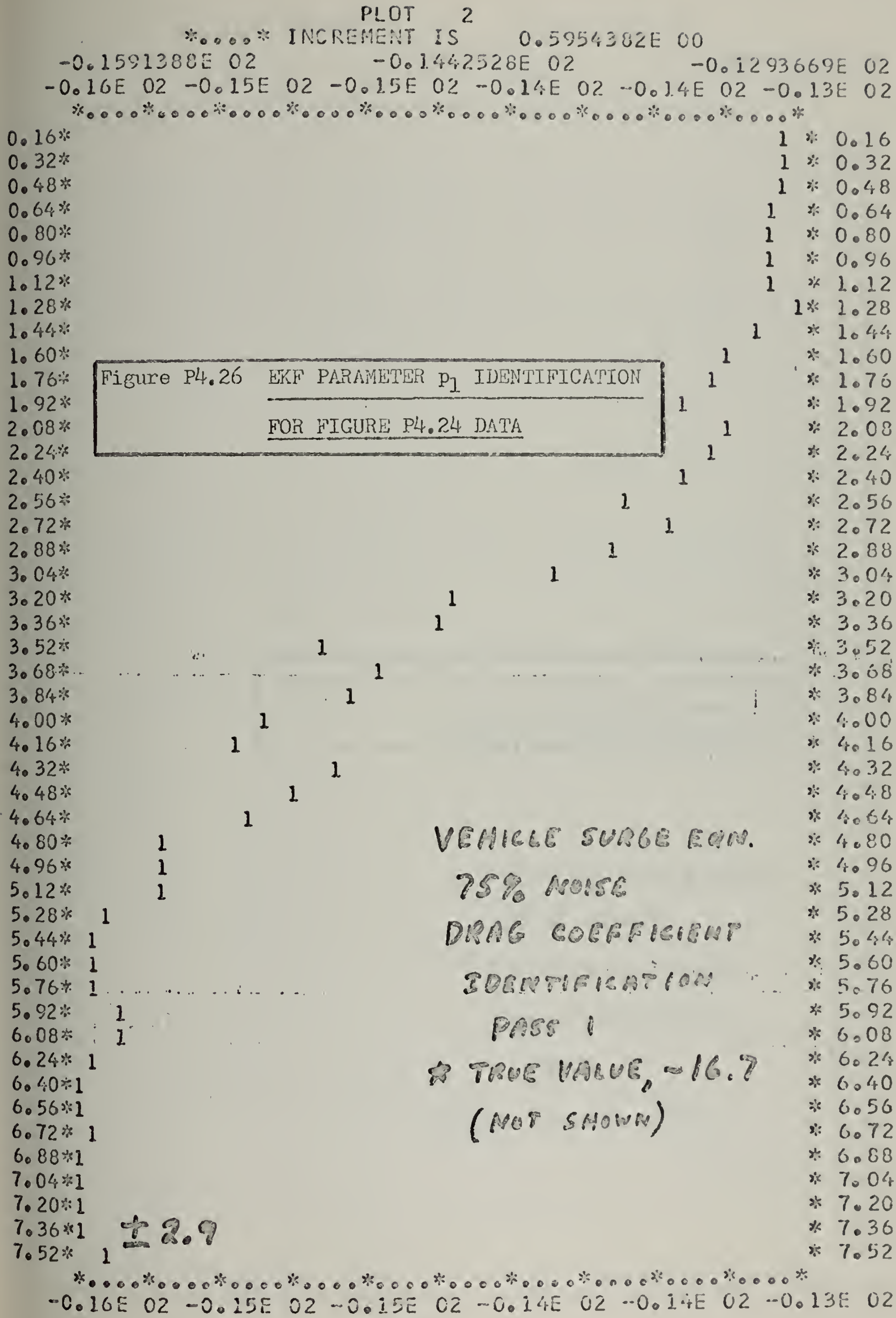


Figure P4.26 EKF PARAMETER p_1 IDENTIFICATION FOR FIGURE P4.24 DATA

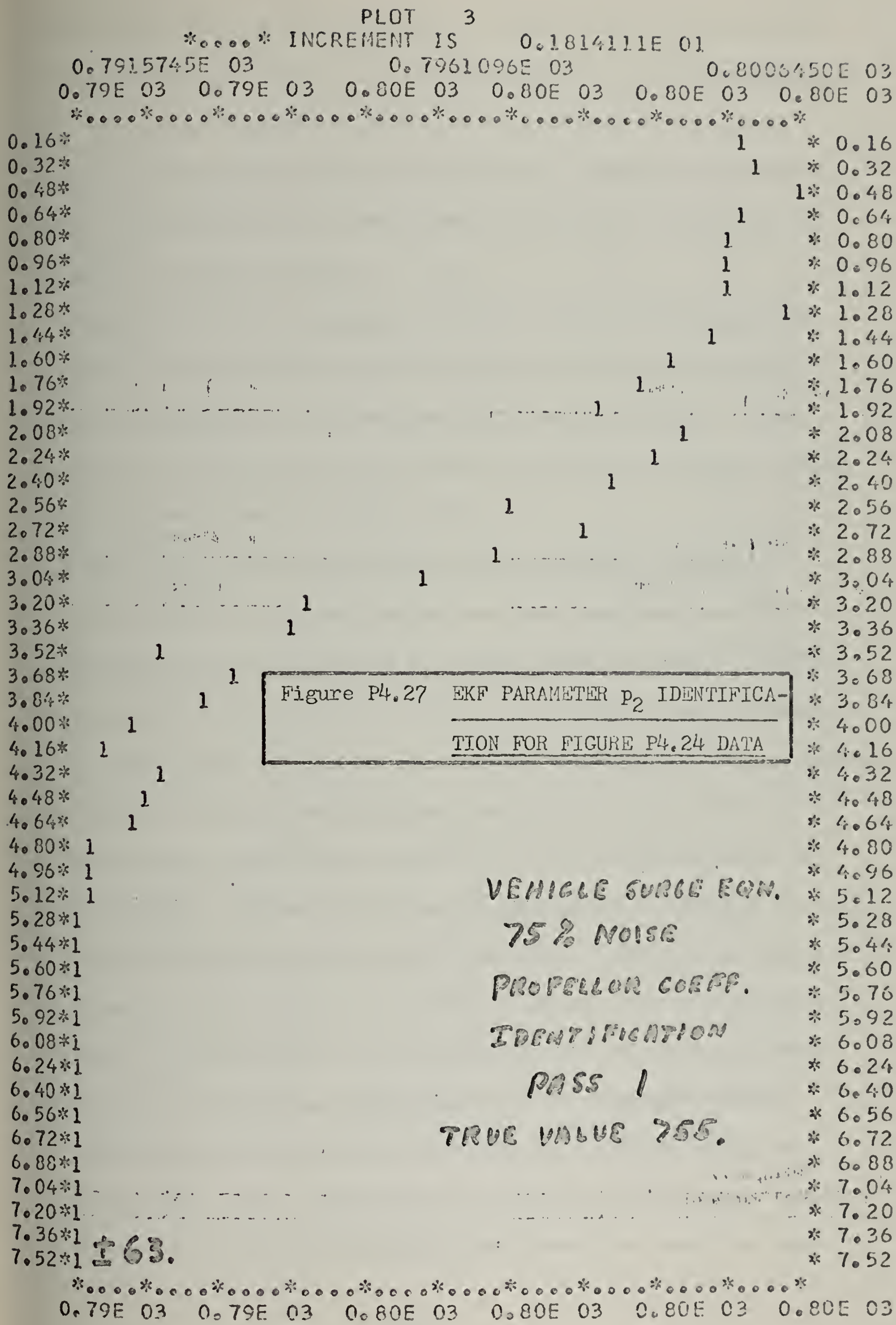


Figure P4.27 EKF PARAMETER p_2 IDENTIFICATION FOR FIGURE P4.24 DATA

Figures P4.25 through P4.27 reveals how dramatically the EKF picks out the DSRV surge velocity and the two vehicle parameters (especially p_1). The reason that the filter can do this is that it has been given the exact absolute square law structure (Equation P4.4) for the vehicle generating the data and it has been "tuned" to look for the parameters in this structure.

For the DSRV surge equation the EKF is much more difficult to implement and tune than the model reference contouring technique, but once operational it allows remarkable identifications. This version of the filter may now be applied to the six individual single degree of freedom models of Table P2.1.

P4.9 EKF RESULTS WITH DSRV SINGLE DEGREE OF FREEDOM EQUATIONS

Six single pass identification runs were made with the EKF using each of the single degree of freedom models in Table P2.1 for the purpose of proving that the working of the filter was independent of the model degree of freedom used and of the values of the parameters. The results of these identifications are shown in Table P4.15 for the DSRV surge, sway, heave, roll, pitch, and yaw single degree of freedom models. The sea trials and identifications conducted in each of these reduced models correspond to Blocks 1 through 6 in Figure P1.1. The primary results indicated by Table P4.15 are that the parameters are about 3% identifiable for 3% noises and that the EKF is a valid technique for each of the single degree of freedom DSRV models.

This chapter has presented the results of a large number of extended Kalman filter identification runs for the DSRV surge equation. It has been found that not only must the filter be "tuned" to this

Common Factors: $Q = 5Q_n$, $R = 5R_n$, Runge-Kutta Integration Method

Degree	%v	%w	x_0	Effector	Initial p_1	Initial p_2	H
Surge	5	2	$1 \pm .5$	Prop. 1rps	-13 ± 4.5	800 ± 64	4
Sway	3	3	$.1 \pm .1$	Thr. 8rps	-400 ± 47	8 ± 1.5	2
Heave	3	3	$.1 \pm .1$	Thr. 8rps	-300 ± 113	8 ± 1.4	2
Roll	3	0	$.01 \pm .03$	Prop. 1rps	$-2.5*10^4 \pm 4*10^3$	600 ± 73	.5
Pitch	3	0	$.01 \pm .03$	Thr. 8 rps	$-8*10^5 \pm 1.2*10^5$	100 ± 86	.5
Yaw	4	0	$.01 \pm .03$	Thr. 8rps	$-1.5*10^6 \pm 1.5*10^5$	300 ± 112	.5

Surge: $4507\,\dot{x} = (-16.7)\,x|x| + (755)\,u|u|$

Identified as $4507\,\dot{x} = (-16.6 \pm 10^6)\,x|x| + (746 \pm 10^{-3})\,u|u|$

Sway: $8683\,\dot{x} = (-346)\,x|x| + (9.5)\,u|u|$

Identified as $8683\,\dot{x} = (-323 \pm 8)\,x|x| + (9.1 \pm 0.3)\,u|u|$

Heave: $7963\,\dot{x} = (-207)\,x|x| + (9.5)\,u|u|$

Identified as $7963\,\dot{x} = (-198 \pm 4)\,x|x| + (9.2 \pm 0.2)\,u|u|$

Roll: $4.23*10^4\,\dot{x} = (-3.3*10^4)\,x|x| + (530)\,u|u|$

Identified as $4.23*10^4\,\dot{x} = (-3.2*10^4 \pm 570)\,x|x| + (525 \pm 8)\,u|u|$

Pitch: $8.62*10^5\,\dot{x} = (-6.69*10^5)\,x|x| + (193.6)\,u|u|$

Identified as $8.62*10^5\,\dot{x} = (-6.5*10^5 \pm 1.1*10^4)\,x|x| + (185 \pm 3.1)\,u|u|$

Yaw: $8.6*10^5\,\dot{x} = (-1.12*10^6)\,x|x| + (181.7)\,u|u|$

Identified as $8.6*10^5\,\dot{x} = (-1.11*10^6 \pm 3.1*10^4)\,x|x| + (184 \pm 5)\,u|u|$

Table P4.15 EKF IDENTIFICATIONS FOR THE SIX DSRV SINGLE DEGREE OF FREEDOM MODELS

nonlinear differential equation by adjusting the values of Q and R, but also it must have smaller time steps, more data points, and a better integration method than the model reference contouring technique. However, once the filter is adjusted and given reasonably good initial estimates of the parameters, it is a far more accurate identification technique in the presence of large noise percentages than is the model reference contouring technique. In addition, the extended Kalman filter in its general form may be used for the identification of many parameters at once, whereas the model reference contouring technique looks at the parameters two at a time.

This section has presented the detailed steps and results for the application of the techniques of model reference contouring and extended Kalman filtering to the ocean vehicle mathematical models of Section 2(M) by using, as examples, the single degree of freedom DSRV equation of motion. Now that these techniques have been validated for the single degree of freedom models, it may be reasonably expected that they will also work for more complex versions of the DSRV mathematical model. The next step is therefore to present the structure and parameters for a structure-selective DSRV mathematical model with any combination of six degrees of freedom or vehicle effectors and to develop the digital computer programs necessary to simulate the vehicle using this model. Section 5(D) accomplishes this step, and then Section 6(C) uses this programming to study several selected coefficients and parameters using multiple degree of freedom models.

SECTION 5

THE DSRV MATHEMATICAL MODEL (D)

A COMMENT REGARDING COMPUTERS:

"DAZZLED BY THEIR ABILITY TO DO ELEMENTARY THINGS AT TREMENDOUS SPEEDS AND TO PUT THESE TOGETHER IN STRUCTURES OF DAUNTING COMPLEXITY, SOME HAVE ALLOWED THE TERM 'GIANT BRAINS' TO GAIN CURRENCY AND, SEDUCED BY THE SIREN SONG OF SO SENSELESS A SOBRIQUET, HAVE SURRENDERED THEIR BIRTHRIGHT OF RATIONAL THOUGHT FOR A POTTAGE OF PUNCHED CARDS." - R. ARIS (A-7)

THE CENTRAL PURPOSE OF THIS THESIS IS THE DEVELOPMENT, PRESENTATION, DSRV UTILIZATION, AND ANALYSIS OF TECHNIQUES FOR THE IDENTIFICATION OF PARAMETERS IN AN OCEAN VEHICLE DYNAMIC MATHEMATICAL MODEL. THIS SECTION PRESENTS THE DETAILED EQUATIONS FOR THE STRUCTURE SELECTIVE MODEL AND GRADIENT PROGRAMMING TO SIMULATE A GENERAL OCEAN VEHICLE WITH SUBROUTINES SPECIFICALLY TAILORED TO THE DSRV. THIS SECTION IS NECESSARY FOR THE UTILIZATION OF IDENTIFICATION TECHNIQUES ON THE DSRV.

CHAPTER D1

GENERAL OCEAN VEHICLE AND DSRV FORMULATIONS

The purpose of this chapter is to provide a bridge between the general ocean vehicle mathematical models in Section 2(M) and the specific DSRV mathematical model of the Lockhead Missiles and Spacecraft Company (B-9). To do this, the LMSC model must be converted to state space form and broken up into additive blocks of coefficient and effector equations. The resulting equations can then be seen to fit into the corresponding terms in equation D1.1, which is discussed in Section 2(M). The nomenclature for the terms and coefficients is standard and is defined in references (S-9) and (A-1).

$$\boxed{A\dot{\underline{x}} = \underline{f}(\underline{x},\underline{p}) + \underline{f}_{eff}(\underline{x},\underline{u},\underline{p}) + G\underline{w} \qquad \text{D1.1}}$$

D1.1 THE DSRV DYNAMIC EQUATIONS

The complete mathematical model for the DSRV is presented, with some variations, in the three references: (B-9), (T-1), and (B-2). The equations to be used here are taken from (B-9), modified slightly, and presented here as equations D1.2 through D1.7.

Surge

$$m\left[\dot{u} + qw - rv - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})\right] =$$
$$X_{\dot{u}}\,\dot{u} + X_{qw}\,qw + X_{rv}\,rv + X_{u|u|}\,u|u| + X_{rr}\,r^2 + X_{qq}\,q^2 +$$
$$X_{rp}\,rp + X_{grav} + X_{shroud} + X_{prop} + X_{thr} + X_{dist} \qquad \text{D1.2}$$

Sway

$$m\left[\dot{v} + ru - pw - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})\right] =$$
$$Y_{\dot{v}}\,\dot{v} + Y_{\dot{r}}\,\dot{r} + Y_{\dot{p}}\,\dot{p} + Y_{r|u|}\,r|u| + Y_{pw}\,pw + Y_{qp}\,qp +$$
$$Y_{v|w|}\,v|w| + Y_{p|u|}\,p|u| + Y_{v|u|}\,v|u| + Y_{secondary} +$$
$$Y_{grav} + Y_{shroud} + Y_{thr} + Y_{dist} \qquad \text{D1.3}$$

Heave

$$m\left[\dot{w} + pv - qu - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})\right] =$$
$$Z_{\dot{w}}\,\dot{w} + Z_{\dot{q}}\,\dot{q} + Z_{pp}\,p^2 + Z_{pv}\,pv + Z_{pr}\,pr + Z_{vv}\,v^2 + Z_{rv}\,rv +$$
$$Z_{rr}\,r^2 + Z_{u|u|}\,u|u| + Z_{w|u|}\,w|u| + Z_{q|u|}\,q|u| + Z_{secondary} +$$
$$Z_{grav} + Z_{shroud} + Z_{prop} + Z_{thr} + Z_{dist} \qquad \text{D1.4}$$

Roll

$$m\left[y_G(\dot{w} + pv - qu) - z_G(\dot{v} + ru - pw)\right] + I_x\,\dot{p} + (I_z -$$
$$I_y)qr = K_{\dot{p}}\,\dot{p} + K_{\dot{v}}\,\dot{v} + K_{qr}\,qr + K_{v|w|}\,v|w| + K_{rw}\,rw +$$
$$K_{vq}\,vq + K_{pw}\,pw + K_{v|u|}\,v|u| + K_{v|v|}\,v|v| + K_{p|u|}\,p|u| +$$
$$K_{r|u|}\,r|u| + K_{p|p|}\,p|p| + K_{grav} + K_{prop} + K_{dist} \qquad \text{D1.5}$$

Pitch

$$m\left[z_G(\dot{u} + qw - rv) - x_G(\dot{w} + pv - qu)\right] + I_y\dot{q} + (I_x -$$
$$I_z)pr = M_{\dot{q}}\,\dot{q} + M_{\dot{w}}\,\dot{w} + M_{pr}\,pr + M_{|u|w}\,|u|w + M_{q|u|}\,q|u| +$$
$$M_{vp}\,vp + M_{u|u|}\,u|u| + M_{vv}\,v^2 + M_{rv}\,rv + M_{rr}\,r^2 +$$
$$M_{secondary} + M_{grav} + M_{shroud} + M_{prop} + M_{thr} + M_{dist}$$
$$\text{D1.6}$$

Yaw

$$m\left[x_G(\dot{v} + ru - pw) - y_G(\dot{u} + qw - rv)\right] + I_z\dot{r} + (I_y - I_x)pq =$$
$$N_{\dot{r}}\,\dot{r} + N_{\dot{v}}\,\dot{v} + N_{pq}\,pq + N_{|u|v}\,|u|v + N_{r|u|}\,r|u| + N_{wp}\,wp +$$
$$N_{vq}\,vq + N_{p|u|}\,p|u| + N_{|w|v}\,|w|v + N_{secondary} + N_{grav} +$$
$$N_{shroud} + N_{prop} + N_{thr} + N_{dist} \qquad \text{D1.7}$$

D1.2 THE DERIVATIVE COEFFICIENT MATRIX AND PRIMARY DERIVATIVES

Expanding equations D1.2 through D1.7 and collecting the derivative terms leads to the vector representation in equation D1.8. The derivative coefficient matrix A is then given by equation D1.9.

$$A\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\\ \dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}\dot{u}'\\ \dot{v}'\\ \dot{w}'\\ \dot{p}'\\ \dot{q}'\\ \dot{r}'\end{bmatrix} = A\dot{\underline{x}} = \dot{\underline{x}}' \qquad \text{D1.8}$$

$$\begin{bmatrix}(m - X_{\dot{u}}) & 0 & 0 & 0 & mz_G & -my_G\\ 0 & (m - Y_{\dot{v}}) & 0 & -(mz_G + Y_{\dot{p}}) & 0 & (mx_G - Y_{\dot{r}})\\ 0 & 0 & (m - Z_{\dot{w}}) & my_G & -(mx_G + Z_{\dot{q}}) & 0\\ 0 & -(mz_G + K_{\dot{v}}) & my_G & (I_x - K_{\dot{p}}) & 0 & 0\\ mz_G & 0 & -(mx_G + M_{\dot{w}}) & 0 & (I_y - M_{\dot{q}}) & 0\\ -my_G & (mx_G - N_{\dot{v}}) & 0 & 0 & 0 & (I_z - N_{\dot{r}})\end{bmatrix} = A$$

D1.9

The terms remaining in the expanded equations D1.2 through D1.7 are designated the primed derivatives and correspond directly to the forces and moments acting on the DSRV. These terms are presented here for completeness as equations D1.10 through D1.15.

Surge

$$\dot{u}' = X_{u|u|}\, u|u| + (X_{vr} + m)\, vr + (X_{wq} - m)\, wq + my_G\, pq + (X_{pr} - mz_G)\, pr + (X_{qq} + mx_G)\, q^2 + (X_{rr} + mx_G)\, r^2 + X_{grav} + X_{shroud} + X_{prop} + X_{thr} + X_{dist} \qquad \text{D1.10}$$

Sway

$$\dot{v}' = Y_{|u|v}\, |u|v + Y_{|u|p}\, |u|p + (Y_{|u|r}\, \text{sign}(u) - m) \cdot ur + Y_{v|w|}\, v|w| + (Y_{wp} + m)\, wp + my_G\, p^2 + (Y_{pq} - mx_G)\, pq - mz_G\, qr + my_G\, r^2 + Y_{secondary} + Y_{grav} + Y_{shroud} + Y_{prop} + Y_{thr} + Y_{dist} \qquad \text{D1.11}$$

Heave

$$\dot{w}' = Z_{u|u|}\, u|u| + Z_{|u|w}\, |u|w + (Z_{|u|q}\, \text{sign}(u) + m)\, uq + Z_{vv}\, v^2 + (Z_{vp} - m)\, vp + Z_{vr}\, vr + (Z_{pp} + mz_G)\, p^2 + (Z_{pr} - mx_G)\, pr + mz_G\, q^2 - my_G\, qr + Z_{secondary} + Z_{grav} + Z_{shroud} + Z_{prop} + Z_{thr} + Z_{dist} \qquad \text{D1.12}$$

Roll

$$\dot{p}' = K_{|u|v} |u| v + K_{|u|p} |u| p + my_G uq + (K_{|u|r} \operatorname{sign}(u) + mz_G) ur + K_{v|v|} v|v| + K_{v|w|} v|w| - my_G vp + K_{vq} vq + (K_{wp} - mz_G) wp + K_{wr} wr + K_{p|p|} p|p| + (I_y - I_z + K_{qr}) qr + K_{grav} + K_{prop} + K_{dist} \qquad \text{D1.13}$$

Pitch

$$\dot{q}' = M_{u|u|} u|u| + M_{|u|w} |u| w + (M_{|u|q} \operatorname{sign}(u) - mx_G) uq + M_{vv} v^2 + (M_{vp} + mx_G) vp + (M_{vr} + mz_G) vr - mz_G wq + (M_{pr} - I_x + I_z) pr + M_{rr} r^2 + M_{secondary} + M_{grav} + M_{shroud} + M_{prop} + M_{thr} + M_{dist} \qquad \text{D1.14}$$

Yaw

$$\dot{r}' = N_{|u|v} |u| v + N_{|u|p} |u| p + (N_{|u|r} \operatorname{sign}(u) - mx_G) ur + N_{v|w|} v|w| + N_{vq} vq - my_G vr + (N_{wp} + mx_G) wp + my_G wq + (N_{pq} + I_x - I_y) pq + N_{secondary} + N_{grav} + N_{shroud} + N_{prop} + N_{thr} + N_{dist} \qquad \text{D1.15}$$

D1.3 STATE SPACE FORM FOR THE DSRV EQUATIONS

The general techniques for converting ocean vehicle mathematical models to state space form have been presented in Section 2 (M). The conversion of the DSRV equations D1.10 through D1.15 and D1.8 to state space form is a straightforward application of those techniques. Thus, the terms u v w p q r become the first six states in the state vector $\underline{x}$. Since the DSRV model contains only second degree coefficients and since an augmented state space model is desired, the second degree coefficients (126 of them at most) become the next

126 states in $\underline{x}$. In addition, the fact that the reverse mode coefficients are in some cases different leads to the inclusion of the reverse mode coefficients as the next 126 states. This last step is not essential if other provisions are made within the model to include the reverse mode coefficients.

The vector $\underline{x}$ is now of dimension 258 by 1 and includes the vehicle linear and angular velocities and the vehicle forward and reverse mode second degree coefficients. Had linear, third degree, or other coefficients been present in the model equations, they would have been augmented into the general state vector.

For ease of computation the second degree coefficients were included in a special order running from X_{uu} through N_{rr}. A general second degree coefficient can be represented as X^{k}_{ij}, where k, i, and j run from 1 through 6 in correspondence with u v w p q r and X Y Z K M N. In addition, some of the coefficients are symmetric in that $X^{k}_{ij} = X^{k}_{ji}$. Thus, one order in which the X^{k}_{ij} can be placed in $\underline{x}$ is in correspondence with equation D1.16.

$$(k, i, j) = i + j(j-1)/2 + n(n+1)\,(k-1)/2 + n$$

$$\text{where: } n = 6 \qquad \text{D1.16}$$

The augmented state vector then appears as in equation D1.17. If in addition all of the vehicle parameters other than the states and coefficients are lumped into one parameter vector $\underline{p}$, the DSRV equations are seen to be in the form of equation D1.1.

$$\underline{x} = [u\ v\ w\ p\ q\ r\ X_{uu} \cdots N_{rr}\ X_{uu}\text{rev} \cdots N_{rr}\text{rev}]^{T} \qquad \text{D1.17}$$

With the equations specified in this form and with the assumed or determined values of the coefficients and parameters, it is now possible to simulate a general ocean vehicle with the DSRV as a special case. The computations and structure selection are greatly facilitated by the state space form of the equations and by the coefficient computation order.

This chapter has presented the DSRV dynamic equations in state space form. The detailed numerical values of the vehicle coefficients and parameters are presented in Appendix A1. The next chapter presents the programming details for calculating a general ocean vehicle state derivative $\dot{\underline{x}}$ in a structure selective manner.

OCEAN VEHICLE MODEL SUBROUTINES (OVMOD)

This chapter presents a programming description and a flow-chart for structure-selectively calculating $\dot{\underline{x}}$ in equation D1.1 with and without noise. This subroutine is very general in that by specifying inputs and by including proper input subroutines, a very large class of ocean vehicles may be simulated. The Fortran IV statements for this chapter are presented in Appendix A6.

D2.1 INPUTS AND OUTPUTS OF SUBROUTINE OVMOD

All of the inputs and outputs of subroutine OVMOD are either vectors or constants to permit variable dimensioning. Any numbers listed here are those used for the DSRV.

OVMOD (ME, AV, XF, P, IN, U, NE, X, XN, XD, XDN, KB, L)

Inputs:

ME - Vector of 20 integers describing the vehicle effectors and parameter locations in vector P

ME(1) - ME(9) - Model selectors of 0 or 1 depending upon
ME(19) - ME(20)
whether a particular effector is not or is to be used. If 0, effector subroutine is not called

1 - Tanks (TANKS)

2 - Secondary drag terms (SECAL)

3 - Shroud (SHCAL)

4 - Propellor (PRCAL)

5 - Thrusters (THCAL)

6 - Noise (XDIST)

7 - Constant terms (COCAL)

8 - Center of buoyancy $\neq$ 0 (GRAVT)

9 - Noiseless trajectory calculated in addition to noisy one

19 - Vehicle angles are integrated angular velocities

20 - Vehicle angles are plotted as output

ME(10) - ME(18) - Integer location of first effector parameter in vector P

10 -	Tanks	44
11 -	Secondary drag terms	72
12 -	Shroud	103
13 -	Propellor	229
14 -	Thrusters	266
15 -	Noise	560
16 -	Constant	572
17 -	Gravity terms	578
18 -	General constants	586

AV - Vector of 126 integers describing the absolute value characteristics of each of the second degree coefficient terms in the mathematical model in the order of equation D1.16; see Appendix A2

-1 - $X^k_{ij}\ |x_i|\ x_j$

0 - $X^k_{ij}\ x_i\ x_j$

1 - $X^k_{ij}\ x_i\ |x_j|$

XF - Vector of 126 integers designating second degree coefficient X_{ij}^{k} as a function to be evaluated instead of a constant and designating its functional branch in subroutine XFUNS which is used in subroutine XDCAL; see Appendix A2

P - Vector of all vehicle and environmental parameters with initial locations specified by ME, total of 587 in DSRV; Appendix A1

IN - Vector of integers to be used as starters for the noise generation subroutines (6)

U - Vector of inputs to the ocean vehicle (18)

U(1) - U(7) - Liquid loadings of the 7 tanks in lbf.

U(8) - U(9) - Shroud angles in radians (DP, DY)

U(10) - U(11)- Propellor RPS and RPS/SEC (n, $\dot{n}$)

U(12) - U(15)- Thrusters RPS (yf, ya, zf, za)

U(16) - U(18)- Vehicle Angles (PHI, THETA, PSI)

NE - Vector of integers describing the number of each type of input and its location in input vector U and describing some effector parameter group characteristics in P

NE(1) - NE(10) - Number of types of effectors and locations in input vector U

1 - Tanks (7)	6 - Tanks location (1)
2 - Shroud (2)	7 - Shroud location (8)
3 - Propellor (2)	8 - Prop. location (10)
4 - Thrusters (4)	9 - Thr. location (12)
5 - Angles (3)	10 - Angles location (16)

NE(11) - NE(27) - Number of straight line regions in the parametric descriptions of the shroud and thrusters

11 - Shroud lift regions (5)

12 - Shroud drag regions (10)

13 - 18 Thruster regions (8, 8, 3, 5, 7, 7)

19 - Noise parameters (6)

20 - Shroud lift derivative regions (9)

21 - Shroud drag derivative regions (15)

22 - 27 - Thruster derivative regions (11, 9, 5, 9, 10, 9)

X - Vector of all vehicle states and augmented foreward and reverse mode coefficients, noiseless (258), Appendix A1

XN - Vector of all vehicle states and augmented foreward and reverse mode coefficients, with added and integrated noise (258)

KB - Integer $1 \leq KB \leq KS$ describing the number of states in the model structure, KS = 6 for DSRV

L - Vector describing the individual states (total of KB) in the model structure

OUTPUTS:

XD - Vector of primary state derivatives selected by KB and L without noise (maximum of 6)

XDN - Vector of primary state derivatives selected by KB and L with added and integrated noise (maximum of 6)

D2.2 PROGRAMMING DESCRIPTION AND FLOWCHART OF SUBROUTINE OVMOD

The purpose of subroutine OVMOD is to calculate the noisy and noiseless state derivatives, $\dot{\underline{x}}$ and $\dot{\underline{x}}_n$, in a structure selective manner. Thus, subroutine OVMOD solves equation D2.1, which represents a large class of ocean vehicles. Equation D2.1 is a subcase of

$$A\dot{\underline{x}} = \sum_{i=1}^{6} \sum_{j=i}^{6} \underline{X}_{ij}\, x_i\, x_j + \underline{X}_{sec} + \underline{X}_{grav} + \underline{X}_{eff} + \underline{X}_{dist} \qquad \text{D2.1}$$

equation D1.1. The summations may also be written j = 1, 6 and i = 1, j. The structure selective aspect of solving equation D2.1 comes from allowing i, j, and k (where $\underline{x} = x_k$, k = 1, 6) to take on only the values of the desired model.

The programming necessary to solve D2.1 structure selectively begins with the establishment of the center of gravity, weight, buoyancy, and center of buoyancy. Normally, the center of buoyancy is taken to be the coordinate system origin. A new center of gravity and vehicle weight are calculated based upon the loadings in the tanks. Then the C. G. and W are used to calculate the gravity forces on the vehicle and the values of the A matrix.

At this point in subroutine OVMOD, provision is made for the calculation of two separate vehicle trajectories, a noisy and a noiseless one. The two paths are almost identical except that in the noisy path, moisy variables are used and disturbances are added to the forces. The basic solution in both paths involves calculation of the shroud forces, calculation of coefficient forces and

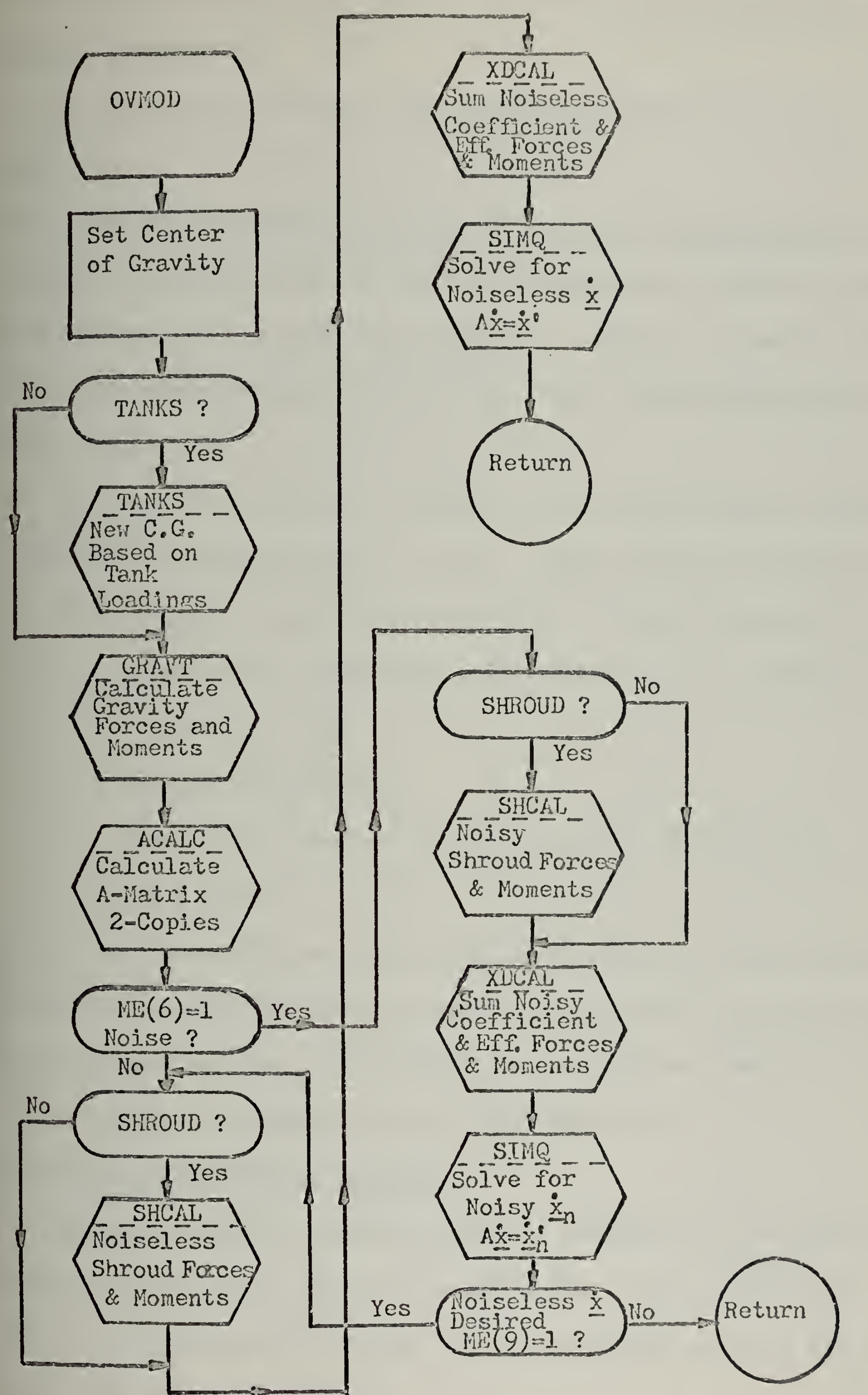


Figure D2.1 ABBREVIATED FLOWCHART FOR SUBROUTINE OVMOD

other effector forces using XDCAL, and solving the simultaneous equation D1.8 for $\underline{\dot{x}}$.

An abbreviated flowchart for subroutine OVMOD is shown in figure D2.1.

D2.3 SUBROUTINE XDCAL (ME, AV, XF, P, PG, U, N, X, XD, KB, L, K)

Subroutine XDCAL computes a selected, untransformed linear or angular acceleration for a general ocean vehicle. XDCAL is a general subroutine for second-degree coefficients, but it uses subroutines specific to the DSRV.

The inputs and outputs to XDCAL are the same as those to subroutine OVMOD except for PG, N, and K. XDCAL operates basically by

PG - Vector of parameters describing the vehicle weight (1), buoyancy (5), center of gravity (2 - 4), and center of buoyancy (6 - 8)

N - Same as NE in OVMOD

K - Index of the desired linear or angular acceleration, $1 \leq K \leq KS$

summing the second-degree coefficient terms using the coefficients calculated by XFUNS and then by adding on the forces and moments of the desired effectors. A listing of the coefficient functions and their selection constants is given in Appendix A2.

D2.4 SUBROUTINES ACALC AND SECAL

Subroutine ACALC is used by OVMOD to calculate the derivative coefficient matrix, A, given by equation D1.9. These values are calculated selectively using the vehicle parameter vector P and the gravity and buoyancy vector PG.

Subroutine SECAL is used by OVMOD to calculate the cross-flow drag or secondary-drag forces and moments encountered when the vehicle moves with v, w, q, or r. The secondary drag equations are highly nonlinear equations involving coefficients Y_{vv} (24) and Z_{ww} (48). These equations are presented in Appendix A3.

Both subroutines ACALC and SECAL are specifically written for the DSRV.

The Fortran IV statements for subroutines OVMOD, XDCAL, XFUNS, SECAL, and ACALC are included in Appendix A6. The next chapter describes the DSRV effectors and presents the input-output descriptions of the effector subroutines.

EFFECTOR SUBROUTINES

The subroutines which are called by OVMOD as effectors are: TANKS, GRAVT, SHCAL, PRCAL, THCAL, COCAL, and XDIST. Each of these effectors will be described and its equations presented. The Fortran IV subroutines for the simulation of each effector are presented in Appendix A7. The information presented in this chapter is not intended to be a complete description of each effector, but merely an aid in understanding and using the subroutines. For more information on the development of these equations and more details of the effectors, see (B-9) and (B-2).

D3.1 TANKS

Tanks equations for vehicle center of gravity

$$x_G = \frac{\Sigma W_{io} x_i + \Sigma \delta W_i x_i}{\Sigma W_{io} + \Sigma \delta W_i} \qquad \text{D3.1}$$

Same form for y_G and z_G except y_i and z_i are used

Where W_{io} = equilibrium tank loadings

$\delta W_i = W_i - W_{io}$ = incremental tank loadings

$W_o = \Sigma W_{io}$ = equilibrium vehicle weight

$W = W_o + \Sigma \delta W_i$ = vehicle weight

x_i, y_i, z_i = location of C.G. of i'th tank

TANKS (P, M, U, NT, PG)

P - Parameter vector containing the tank equilibrium loadings and the tank locations in the order (starting at P(M)) of W_{io}, x_i, y_i, z_i

M - Location of first tank parameter in P

U - Input tank loadings starting at U(1), W_i

NT - Number of Tanks

PG - Gravity and Buoyancy Vector, described in D2.3

Subroutine TANKS is valid for a general system of NT tanks.

D3.2 GRAVT

Gravity forces and moments equations D3.2

$$X_G = -(W - B) \sin \theta$$

$$Y_G = (W - B) \cos \theta \sin \phi$$

$$Z_G = (W - B) \cos \theta \cos \phi$$

$$K_G = (y_G W - y_B B) \cos \theta \cos \phi - (z_G W - z_B B) \cos \theta \sin \phi$$

$$M_G = -(x_G W - x_B B) \cos \theta \cos \phi - (z_G W - z_B B) \sin \theta$$

$$N_G = (x_G W - x_B B) \cos \theta \sin \phi + (y_G W - y_B B) \sin \theta$$

Where B = vehicle buoyancy

θ, ϕ = vehicle angles with respect to equilibrium

These equations simplify when x_B, y_B, z_B are zero.

GRAVT (M, A, N, P, X, K, L)

M - Center of Buoyancy = 0 indicator, M = 0 means C.B. = 0

A - Input Vehicle angles in radians

N - Location of first vehicle angle in vector A

P - Same as PG vector in D2.3

X - Output forces and moments calculated

K - Number of terms to be calculated, same as KB in OVMOD

L - Vector of indices of terms to be calculated

D3.3 SHCAL

The shroud is, from a simulation standpoint, the most complex effector on the DSRV. The shroud equations are developed in reference (B-9). The shroud angles are δp and δy. In the foreward mode ($u > o$) the angles are defined such that $+\delta p$ produces $-Z$ and $-M$ while $+\delta y$ produces $+Y$ and $-N$. The shroud is almost ineffective for $u < 2$ ft./sec.

Shroud Equations

$$\begin{bmatrix} u_{2s} \\ v_{2s} \\ w_{2s} \end{bmatrix} = \begin{bmatrix} \cos\delta y & \sin\delta y & 0 \\ -\sin\delta y & \cos\delta y & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\delta p & 0 & -\sin\delta p \\ 0 & 1 & 0 \\ \sin\delta p & 0 & \cos\delta p \end{bmatrix} *$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -x_s \\ 0 & 0 & 1 & 0 & x_s & 0 \end{bmatrix} * [u\ v\ w\ p\ q\ r]^T \qquad \text{D3.3}$$

$$\alpha_s = \tan^{-1} \frac{(v_{2s}^2 + w_{2s}^2)^{\frac{1}{2}}}{u_{2s}} = \tan^{-1} \frac{v_{ns}}{u_{2s}} \qquad \text{D3.4}$$

$$\text{LIFT} = \tfrac{1}{2}\rho\, V^2\, S_r\, C_L\, (\alpha_s) \qquad \text{D3.5}$$

$$\text{DRAG} = \tfrac{1}{2}\rho\, V^2\, S_r\, C_D\, (\alpha_s) \qquad \text{D3.6}$$

$$N_f = \text{LIFT} \cos\alpha_s + \text{Drag} \sin\alpha_s \qquad \text{D3.7}$$

$$F_{us} = \text{LIFT} \sin\alpha_s - \text{Drag} \cos\alpha_s \qquad \text{D3.8}$$

$$F_{vs} = -\frac{v_{2s}}{v_{ns}} N_f \qquad \text{D3.9}$$

$$F_{ws} = -\frac{w_{2s}}{v_{ns}} N_f \qquad \text{D3.10}$$

$$\begin{bmatrix} F_{xs} \\ F_{ys} \\ F_{zs} \end{bmatrix} = \begin{bmatrix} \cos\delta y\cdot\cos\delta p & \sin\delta y & -\cos\delta y\cdot\sin\delta p \\ -\sin\delta y\cdot\cos\delta p & \cos\delta y & \sin\delta y\cdot\sin\delta p \\ \sin\delta p & 0 & \cos\delta p \end{bmatrix}^{-1} * \begin{bmatrix} F_{us} \\ F_{vs} \\ F_{ws} \end{bmatrix} \qquad \text{D3.11}$$

$$\left.\begin{array}{l} X_{sh} = F_{xs} \\ Y_{sh} = F_{ys} \\ Z_{sh} = F_{zs} \end{array}\right\} = T^{-1}_{s22s} * \begin{bmatrix} F_{us} \\ F_{vs} \\ F_{ws} \end{bmatrix} \qquad \text{D3.12}$$

$$K_{sh} = 0$$

$$M_{sh} = x_s * Z_{sh}$$

$$N_{sh} = -x_s * Y_{sh}$$

Where:

u_{2s}, v_{2s}, w_{2s} = velocities of shroud in shroud coordinates

α_s = angle between incoming flow and shroud centerline

x_s = shroud location on the vehicle (-24.59 ft.)

ρ = fluid specific gravity (1.994)

S_r = surface area (14.64)

$C_L(\alpha_s)$ = shroud lift coefficient, calculated by RGCOM using a series of connected straight lines for different regions of α_s, there

are 16 parameters beginning at P(103), 6 ranges, 5 slopes, 5 intercepts which are used by RGCOM to calculate C_L, straight lines approximate curves in (B-9)

$C_D(\alpha_s)$ = shroud drag coefficient, calculated same as $C_L(\alpha_s)$, 31 parameters beginning at P(119), 11 ranges, 10 slopes, and 10 intercepts. These straight lines for C_L, C_D are approximations for the curves given in reference (B-9).

V^2 = $u_{2s}^2 + v_{2s}^2 + w_{2s}^2$

SHCAL (P, M, X, U, N, NS, NL, ND, XS, KB, L)

P - vehicle parameter vector, same as in OVMOD

M - location of first shroud parameter in P

X - vehicle augmented state vector

U - vehicle input vector

N - location of first shroud angle in U (8)

NS - number of shroud input angles (2)

NL - number of regions in lift coefficient C_L (5)

ND - number of regions in drag coefficient C_D (10)

XS - output vector of shroud forces and moments

KB,L - structure selection inputs as in OVMOD

The shroud equations are so structured that it is best to calculate all selected shroud forces at once rather than independently.

The subroutines used by SHCAL are ASCOM, RGCOM, UTCOM, and SIMQ. Subroutine ASCOM computes the shroud angle α_s using equation D3.4. Subroutine RGCOM is used to compute the straight-line approximate values of the lift and drag coefficients, C_L and C_D. Subroutine UTCOM computes the product of the three matrices in equation D3.3. Subroutine SIMQ is a simultaneous equation solver from reference (I-1), the IBM/SSP and is presented in Appendix A4.

D3.4 PRCAL

The propellor equations are of similar structure for each degree of freedom but have different parameters depending upon the region of operation.

Propellor Equations, propellor inputs are n, $\dot{n}$

$$v_p = v + x_p r$$

$$w_p = w - x_p q$$

Regions for X_p, K_p equations	X_p Parameters			
$1 = u \geq 0,\ n/u \geq -0.21$	0	1	2	32
$2 = u \geq 0,\ n/u \leq -0.21$	3	4	5	32
$3 = u < 0,\ n \geq 0$	6	7	8	32
$4 = u < 0,\ n < 0$	9	10	11	32

Parameters located at M + Index

$$X_p = P_0\, n\,|n| + P_1\, un + P_2\, u^2 + P_{32}\, n\, (v_p^2 + w_p^2)^{\frac{1}{2}} \tag{D3.13}$$

$$K_p = P_{16}\, n\,|n| + P_{17}\, un + P_{18}\, u^2 + P_{33}\, n\, (v_p^2 + w_p^2)^{\frac{1}{2}} + P_{34}\, \dot{n} \tag{D3.14}$$

Region	Parameter indexes				
1	16	17	18	33	34
2	19	20	21	33	34
3	22	23	24	33	34
4	25	26	27	33	34

Regions for Y_p, Z_p, M_p, N_p

1 $n \geq 0$

2 $n < 0$

$$Y_p, N_p = P_{12}\, n\, v_p \qquad \text{D3.15}$$

$$Z_p, M_p = P_{14}\, n\, w_p \qquad \text{D3.16}$$

Parameter indexes

Region	Y_p	N_p	X_p	M_p
1	12	30	14	28
2	13	31	15	29

Where:

x_p = propellor location on vehicle, P(M + 36), (-25.5)

n = propellor rotation, rps

$\dot{n}$ = propellor acceleration, rps^2

PRCAL (P, M, X, U, N, NP, XP, K)

P, M, X, U, N - same as in SHCAL, except N = (10)

NP - number of propellor inputs (2)

XP - output of propellor force or moment selected by K

K - index of desired force or moment

D3.5 THCAL

For the purposes of this thesis, the steady state (dn/dt = 0) thruster equations are used.

Thruster equations (B-9) D3.17

$X_{th} = \Sigma X_{th\ mn}$ $m = y, z$

$Y_{th} = Y_{th\ yf} + Y_{th\ ya}$ $n = f, a$ foreward, aft

$Z_{th} = Z_{th\ zf} + Z_{th\ za}$

$K_{th} = 0$

$M_{th} = \Sigma M_{th\ mn}$

$N_{th} = \Sigma N_{th\ mn}$

Individual equations D3.18

$X_{th\ mf} = n_{mf}^2\ T_3$

$X_{th\ ma} = n_{ma}^2\ T_4$

$Y_{th\ yf} = n_{yf}\ |n_{yf}|\ T_1$

$Y_{th\ ya} = n_{ya}\ |n_{ya}|\ T_2$

$Z_{th\ zf} = n_{zf}\ |n_{zf}|\ T_1$

$N_{th\ ya} = P_{124}\ n_{ya}\ |n_{ya}|\ M_2$

$Z_{th\ za} = n_{za}\ |n_{za}|\ T_2$

$M_{th\ zf} = P_{120}\ n_{zf}\ |n_{zf}|\ M_1$

$M_{th\ za} = P_{121}\ n_{za}\ |n_{za}|\ M_2$

$M_{th\ yn} = P_{122}\ n_{yn}\ |n_{yn}|$

$N_{th\ yf} = P_{123}\ n_{yf}\ |n_{yf}|\ M_1$

$N_{th\ zn} = P_{125}\ n_{zn}\ |n_{zn}|$

Where:

$m = y, z$ - thruster directions

$n = f, a$ - fore and aft thruster sets

T_i - thruster force coefficients approximated as straight lines and calculated using RGCOM, (B-9) curves

M_i - thruster moment coefficients, calculated same as T_i

All coefficients are functions of $A = u/|n|$

THCAL (P, M, U, N, X, XT, K)

P, M, U, X, N - same as in SHCAL, except N = (12)

XT - output thruster force or moment selected by K

K - index of desired force or moment

D3.6 COCAL

COCAL (P, M, X, XC, K) Constant forces and moments selected from P

P, M, X, K - same as previous effectors

XC - selected constant force or moment

D3.7 XDIST

Subroutine XDIST is a general disturbance generator which uses subroutines GAUSS and RANDU from the IBM/SSP (I-1).

XDIST (I, P, M, X, K, N)

I - vector of noise generation integers used by RANDU

P,M - same as previous effectors

X - output selected noise force or moment

K - selection index

N - number of possible forces and moments (6)

The Fortran IV statements for subroutines TANKS, GRAVT, SHCAL, PRCAL, THCAL, COCAL, XDIST, ASCOM, RGCOM, and UTCOM are included in Appendix A7. Subroutines SIMQ, GAUSS, and RANDU from reference (I-1) are included in Appendix A4.

These three chapters, D1, D2, and D3, have presented the equations and the input-output descriptions of the subroutines necessary to simulate the DSRV in any selected combination of six degrees of

freedom. The next three chapters, D4, D5, and D6, present the equations and the subroutine descriptions necessary to calculate the gradient of the DSRV forces and moments selectively with respect to the vehicle states, coefficients, and parameters. These gradient subroutines are used in Kalman Filtering.

GENERAL OCEAN VEHICLE AND DSRV GRADIENT FORMULATIONS

The state space gradient for an ocean vehicle is essential to almost all identification techniques. It is required for propogation of the error covariance matrix in a Kalman Filter and is required for best estimate calculations in many techniques which linearize the state equations about a certain trajectory. Given a general state space system as in equation D4.1, the desired gradient is given by equation D4.2.

$$\dot{\underline{x}} = \underline{f}\ (\underline{x},\ \underline{u}) \qquad \text{D4.1}$$

$$F = \frac{\partial \dot{\underline{x}}}{\partial \underline{x}} = \frac{\partial f\ (\underline{x},\ \underline{u})}{\partial \underline{x}} \qquad \text{D4.2}$$

The gradient can now be expressed in terms of a general ocean vehicle equation D1.1 repeated here as D4.3. The gradient of D4.3 is given by D4.4 for the case in which A is a constant matrix. For the case in which A(p) is dependent upon vehicle parameters, equation D4.5 must be used, and calculation of the A derivative term does appear to be straightforeward, as seen in equation D4.6.

$$A(\underline{p})\ \dot{\underline{x}} = \underline{f}(\underline{x},\ \underline{p}) + \underline{f}_{eff}\ (\underline{x},\ \underline{u},\ \underline{p}) + G\ \underline{w} \qquad \text{D4.3}$$

$$A \left[\frac{\partial \dot{\underline{x}}}{\partial \underline{x}} \;\vdots\; \frac{\partial \dot{\underline{x}}}{\partial \underline{p}} \right] = \left[\frac{\partial \underline{f}}{\partial \underline{x}} + \frac{\partial \underline{f}_{eff}}{\partial \underline{x}} \;\vdots\; \frac{\partial \underline{f}}{\partial \underline{p}} + \frac{\partial \underline{f}_{eff}}{\partial \underline{p}} \right] \qquad \text{D4.4}$$

$$\left[\frac{\partial \dot{\underline{x}}}{\partial \underline{x}} \;\vdots\; \frac{\partial \dot{\underline{x}}}{\partial \underline{p}} \right] = \left[A^{-1}\left(\frac{\partial \underline{f}}{\partial \underline{x}} + \frac{\partial \underline{f}_{eff}}{\partial \underline{x}}\right) \;\vdots\; \frac{\partial A^{-T}}{\partial \underline{p}} (\underline{f} + \underline{f}_{eff} + G\ \underline{w}) + A^{-1} \left(\frac{\partial \underline{f}}{\partial \underline{p}} + \frac{\partial \underline{f}_{eff}}{\partial \underline{p}}\right) \right] \qquad \text{D4.5}$$

$$\frac{\partial A^{-T}}{\partial \underline{p}} = -\left[A^{-1} \left(\frac{\partial A}{\partial \underline{p}}\right)^T A^{-1} \right] \qquad \text{D4.6}$$

The gradient subroutine in this thesis uses equation D4.4 and does not permit identification of those parameters in the A-matrix (added mass and moment of inertia terms) with Kalman filtering.

The calculation of the gradient matrix for a general ocean vehicle now proceeds by applying equation D4.4 to equation D2.1 and then developing the equations and programming for the calculation of the coefficient and effector derivatives. The details of the equations and programming for the gradient routines are presented in chapters D5 and D6.

CHAPTER D5

OCEAN VEHICLE DERIVATIVE SUBROUTINE (OVDER)

This chapter presents a programming description of the subroutines for structure-selectively calculating the gradient matrix F in equation D4.2 using equations D4.4 and D2.1. The gradient subroutine (OVDER) is designed to be a companion to the general ocean vehicle subroutine (OVMOD). Many of the inputs and calculation techniques in both are the same, except that OVDER has a much greater computation task to perform than OVMOD. The Fortran IV statements for OVDER and its subroutines are presented in Appendix A8.

D5.1 INPUTS AND OUTPUTS OF SUBROUTINE OVDER

As in OVMOD, all of the inputs and outputs are in vector or constant form to permit variable dimensioning. Numbers listed here are those used in the DSRV studies.

OVDER (ME, U, AV, XF, KF, LF, KB, L, X, KP, LP, P, F, NF, NE)

Inputs:

ME, AV, XF, P, U, NE, X, KB, L - Same as OVMOD, Chapter D2

KF - Integer $0 \leq KF \leq 126$ describing the number of second-degree coefficients to be used in the gradient matrix calculation

LF - Vector describing the indices of the second-degree coefficients used in the gradient

KP - Integer $0 \leq KP \leq 126$ describing the number of parameters of vector P to be used in the gradient matrix calculation

LP - Vector describing the indices of the parameters used in the gradient

Outputs:

F - Output gradient matrix of dimension KB * NF

(KB = number of selected vehicle primary states, maximum of 6 for DSRV, u, v, w, p, q, r) of the form of equation D5.1

$$F = \begin{bmatrix} \frac{\partial X}{\partial u} & \cdots & \frac{\partial X}{\partial r} & \frac{\partial X}{\partial X_{uu}} & \cdots & \frac{\partial X}{\partial N_{rr}} & \frac{\partial X}{\partial p_1} & \cdots & \frac{\partial X}{\partial p_n} \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial N}{\partial u} & \cdots & \frac{\partial N}{\partial r} & \frac{\partial N}{\partial X_{uu}} & \cdots & \frac{\partial N}{\partial N_{rr}} & \frac{\partial N}{\partial p_1} & \cdots & \frac{\partial N}{\partial p_n} \end{bmatrix} \qquad \text{D5.1}$$

NF - Output constant KB $\leq$ NF $\leq$ 200, NF = KB + KF + KP, describing the number of columns in F

D5.2 PROGRAMMING DESCRIPTION OF SUBROUTINE OVDER

The calculations necessary to the computation of the gradient matrix F in equation D5.1 naturally divide themselves into three groups: calculation of partials with respect to primary states, with respect to second-degree coefficients, and with respect to parameters. As in OVMOD, all of the programming in OVDER must be structure selective. However, the structure selectivity in OVDER must apply not only to the states but also to the second-degree coefficients and to the parameters.

The programming necessary to compute the gradient matrix F involves utilizing subroutines XDDER, XFDER, and XPDER to calculate the three groups of partial derivatives in F. Subroutine OVDER begins, as in OVMOD, by calculating the vehicle center of gravity, weight, buoyancy, and center of buoyancy. These values are used to compute the A-matrix which is then inverted selectively depending

upon its dimension.

Next, a series of steps sets up the structure and parameters for calculating the KB * KB derivatives with respect to the primary states by subroutines XDDER, SEDER, PRDER, THDER, and SHDER. These resultant derivatives are then compressed into the vector F.

The following KB * KF derivates with respect to the second-degree coefficients are calculated by XFDER and compressed into F, and the KB * KP derivates with respect to the parameters are calculated by XPDER and compressed into F. The final step in OVDER is then the derivative transformation which consists of multiplying F by A^{-1}.

Subroutine OVDER uses the three general-utility subroutines: A3INV, MINV, and GMPRD. Subroutine A3INV is a self-explanatory routine for inverting a general 3 * 3 matrix. Subroutine MINV is an N * N matrix inverter from the IBM/SSP (I-1), and GMPRD is a matrix multiplier from the IBM/SSP. These IBM/SSP routines are presented, for the convenience of the reader, in Appendix A4.

The following subroutines used by OVDER are also used by OVMOD and are described in Chapters D2 or D3: TANKS, ACALC, XFUNS, SECAL, RGCOM, and UTCOM. The next portions of this chapter describe subroutines XDDER, XFDER, and XPDER - the three main subroutines of OVDER.

D5.3 SUBROUTINES XDDER AND SEDER

Subroutine XDDER calculates the partial derivatives with respect to the primary states of the terms under the double summation in equation D2.1 and furnishes them for inclusion into the first KB * KB terms of F in equation D5.1.

XDDER (AV, XF, P, PC, X, DX, K, M, KB, L)

AV, XF, P, X, KB, L - Same as OVMOD, Chapter D2

PC - Same as PG in XDCAL Chapter D2.3

DX - Output vector of partial derivatives with respect to states

K - Index of the term to be differentiated, $1 \leq K \leq KS$

M - Index of the state with respect to which the partial derivative is taken, $1 \leq M \leq KS$

Subroutine SEDER calculates the partial derivatives of the $\underline{X}_{sec}$ term in equation D2.1 with respect to the primary states. This is done by branching to the proper partial derivative function based upon input index K. These functions represent all of the statewise partial derivatives of the secondary drag functions in Appendix A3.

SEDER (P, M, X, DXS, K)

P,X - Input parameter and state vectors

M - Location of first secondary drag parameter in P, input

DXS - Output of selected derivative of the K-th term

K - Input index of desired secondary drag derivative

The other subroutines used for calculating derivatives with respect to the primary states are the effector routines: PRDER, THDER, and SHDER. These subroutines are described in Chapter D6 on effector gradient subroutines.

D5.4 SUBROUTINE XFDER

Subroutine XFDER calculates the partial derivatives with respect to the second-degree coefficients of the terms under the double

summation in equation D2.1. These KB * KF terms are then compressed by OVDER into the gradient vector F. The XFDER partial derivatives are calculated using a lengthy selection procedure and using subroutine XDFNS which gives the derivatives of the second-degree coefficient functions with respect to the second-degree coefficients. Subroutine SECAL is used for calculating partial derivatives with respect to those two second-degree coefficients used in the secondary drag calculations.

XFDER (AV, XF, P, PC, X, XFD, KF, LF, KB, L, MS)

- AV, XF, P, X, KB, L - Same as OVMOD, Chapter D2
- PC - Same as PG in XDCAL, Chapter D2.3
- KF, LF - Same as OVDER, Chapter D5.1
- MS - Location of first secondary drag parameter in P
- XFD - Output vector of partial derivatives with respect to the second-degree coefficients

XDFNS (P, X, K, I, D, N)

- P, X - Same as OVMOD, Chapter D2
- K - State vector index of desired second-degree coefficient
- I - Same as XF vector in OVMOD, Chapter D2
- D - Output derivative of second-degree coefficient function with respect to the selected second-degree coefficient
- N - Output zero indicator, N = 0 when D = 0., and N = 1, when D $\neq$ 0.

D5.5 SUBROUTINES XPDER AND SEDEP

Subroutine XPDER calculates the partial derivatives with respect to the parameters of the terms in equation D2.1. These KB * KP terms are then compressed by OVDER into the gradient vector F.

Provisions have not been made to calculate derivatives with respect to all of the parameters in vector P. Derivatives are not calculated with respect to the parameters in the A-matrix. This would require implementation of equations D4.5 and D4.6 and has not been done in this thesis. Derivatives are not calculated with respect to the parameters in the second-degree coefficient functions, the tanks parameters, the shroud parameters, the noise parameters, the variable parameters, the mass of the vehicle, or the acceleration of gravity (Appendix A1).

Derivatives are calculated with respect to the secondary drag parameters (SEDEP), the propellor parameters (PRDEP), the thruster parameters (THDEP), and the constant terms parameters. Subroutines PRDEP and THDEP are described in Chapter D6.

XPDER (P, ME, U, NE, X, XPD, KP, LP, KB, L, NT)

P, ME, U, NE, X, KB, L - Same as OVMOD, Chapter D2

XPD - Output vector of partial derivatives with respect to the vehicle parameters

KP, LP - Same as OVDER, Chapter D5.1

NT - Input vector describing thruster derivative numbers of regions in P

Subroutine SEDEP is used by XPDER to calculate the partial derivatives of the untransformed vehicle forces and moments with respect to the secondary drag parameters. These derivatives are calculated by branching to the proper partial derivative function based upon input index K. These functions represent all of the parameterwise partial derivatives of the secondary drag functions in Appendix A3.

SEDEP (P, M, X, XS, K, J)

- P, X - Input parameter and state vectors
- M - Location of first secondary drag parameter in P, input
- XS - Output of selected derivative of the K-th term with respect to the J-th parameter
- K - Input index of the K-th vehicle force or moment term
- J - Input index of the J-th secondary drag parameter

This chapter has presented a basic description of subroutine OVDER and its associated primary subroutines XDDER, XFDER, and XPDER. The Fortran IV statements for OVDER and its associated subroutines described in this chapter are presented in Appendix A8. The next chapter presents a basic description of the effector gradient subroutines which are used by OVDER in calculating the gradient matrix F.

EFFECTOR GRADIENT SUBROUTINES

The subroutines which are called by OVDER as effector gradient subroutines are SHDER, PRDER, THDER, PRDEP, and THDEP. A basic description of each of these subroutines will be presented in this chapter as an aid in understanding and using these subroutines. The Fortran IV statements for these subroutines are presented in Appendix A9.

D6.1 SUBROUTINE SHDER

Subroutine SHDER is mathematically the most complex vehicle subroutine in this thesis. The complexity arises from the two transformations and the nature of the shroud angular functions. SHDER computes the partial derivatives of the shroud forces and moments with respect to the primary vehicle states. These partial derivatives represent, quite literally, the partial derivatives of equations D3.3 through D3.12. A complete description of SHDER is not presented here, but enough of the partial derivative equations are presented to facilitate understanding and utilization of the subroutine when compared with SHCAL in Chapter D3.3.

$$\begin{bmatrix} \frac{\partial X_{sh}}{\partial u} & \frac{\partial X_{sh}}{\partial v} & \cdots & \frac{\partial X_{sh}}{\partial r} \\ \frac{\partial Y_{sh}}{\partial u} & \frac{\partial Y_{sh}}{\partial v} & \cdots & \frac{\partial Y_{sh}}{\partial r} \\ \vdots & \vdots & & \vdots \\ \frac{\partial N_{sh}}{\partial u} & \frac{\partial N_{sh}}{\partial v} & \cdots & \frac{\partial N_{sh}}{\partial r} \end{bmatrix} = XSD \qquad D6.1$$

$$\begin{bmatrix} \frac{\partial X_{sh}}{\partial u} & \frac{\partial X_{sh}}{\partial v} & \dots & \frac{\partial X_{sh}}{\partial r} \\ \frac{\partial Y_{sh}}{\partial u} & \frac{\partial Y_{sh}}{\partial v} & \dots & \frac{\partial Y_{sh}}{\partial r} \\ \frac{\partial Z_{sh}}{\partial u} & \frac{\partial Z_{sh}}{\partial v} & \dots & \frac{\partial Z_{sh}}{\partial r} \end{bmatrix} = T^{-1}_{s22s*} \begin{bmatrix} \frac{\partial F_{us}}{\partial u} & \frac{\partial F_{us}}{\partial v} & \dots & \frac{\partial F_{us}}{\partial r} \\ \frac{\partial F_{vs}}{\partial u} & \frac{\partial F_{vs}}{\partial v} & \dots & \frac{\partial F_{vs}}{\partial r} \\ \frac{\partial F_{ws}}{\partial u} & \frac{\partial F_{ws}}{\partial v} & \dots & \frac{\partial F_{ws}}{\partial r} \end{bmatrix} \tag{D6.2}$$

$$\frac{\partial F_{us}}{\partial x_i} = \frac{\partial LIFT}{\partial x_i} \sin \alpha_s + LIFT \cos \alpha_s \frac{\partial \alpha_s}{\partial x_i} - \frac{\partial DRAG}{\partial x_i} \cos \alpha_s + DRAG \sin \alpha_s \frac{\partial \alpha_s}{\partial x_i} \tag{D6.3}$$

$$\frac{\partial F_{vs}}{\partial x_i} = -\frac{\partial N_f}{\partial x_i} \frac{v_{2s}}{v_{ns}} - N_f \frac{\partial}{\partial x_i} \left(\frac{v_{2s}}{v_{ns}} \right) \tag{D6.4}$$

$$\frac{\partial F_{ws}}{\partial x_i} = -\frac{\partial N_f}{\partial x_i} \frac{w_{2s}}{v_{ns}} - N_f \frac{\partial}{\partial x_i} \left(\frac{w_{2s}}{v_{ns}} \right) \tag{D6.5}$$

$$\frac{\partial N_f}{\partial x_i} = \frac{\partial LIFT}{\partial x_i} \cos \alpha_s - LIFT \sin \alpha_s \frac{\partial \alpha_s}{\partial x_i} + \frac{\partial DRAG}{\partial x_i} \sin \alpha_s + DRAG \cos \alpha_s \frac{\partial \alpha_s}{\partial x_i} \tag{D6.6}$$

$$\frac{\partial}{\partial x_i} \left(\frac{v_{2s}}{v_{ns}} \right) = \frac{1}{v_{ns}} \frac{\partial v_{2s}}{\partial x_i} - \frac{v_{2s}}{v_{ns}^2} \frac{\partial v_{ns}}{\partial x_i} \tag{D6.7}$$

$$\frac{\partial}{\partial x_i} \left(\frac{w_{2s}}{\partial v_{ns}} \right) = \frac{1}{v_{ns}} \frac{\partial w_{2s}}{\partial x_i} - \frac{w_{2s}}{v_{ns}^2} \frac{\partial v_{ns}}{\partial x_i} \tag{D6.8}$$

$$\frac{\partial LIFT}{\partial x_i} = \frac{1}{2}\rho S_r \left[2V \frac{\partial V}{\partial x_i} C_L + V^2 \frac{\partial C_L}{\partial \alpha_s} \frac{\partial \alpha_s}{\partial x_i}\right] \qquad D6.9$$

$$\frac{\partial DRAG}{\partial x_i} = \frac{1}{2}\rho S_r \left[2V \frac{\partial V}{\partial x_i} C_D + V^2 \frac{\partial C_D}{\partial \alpha_s} \frac{\partial \alpha_s}{\partial x_i}\right] \qquad D6.10$$

$$\frac{\partial v_{ns}}{\partial x_i} = v_{ns}^{-\frac{1}{2}} \left(v_{2s} \frac{\partial v_{2s}}{\partial x_i} + w_{2s} \frac{\partial w_{2s}}{\partial x_i}\right) \qquad D6.11$$

$$\frac{\partial \alpha_s}{\partial x_i} = \left[1 + \left(\frac{v_{ns}}{u_{2s}}\right)^2\right]^{-1} \left[\frac{1}{u_{2s}} \frac{\partial v_{ns}}{\partial x_i} - \frac{v_{ns}}{u_{2s}^2} \frac{\partial u_{2s}}{\partial x_i}\right] \qquad D6.12$$

Equations D6.1 through D6.12 in conjunction with equations D3.3 through D3.12 represent the essence of the shroud and shroud derivative calculations. Subroutine ASDER is a grouping and calculation of several of the α_s equations for convenience. Subroutine UTCOM computes the product of the three matrices in equation D3.3. Subroutine RGCOM is used by SHDER to compute the lift and drag coefficients along with their derivatives with respect to α_s. Subroutine A3INV is a general 3 * 3 matrix inverter, and subroutine GMPRD is a general matrix multiplier from the IBM/SSP.

SHDER (P, M, X, U, N, NS, NL, ND, NDL, NDD, XSD, KB, L)

P, X, U, KB, L - Same as OVMOD, Chapter D2

M - Location of the first shroud parameter in P

N - Location of the first shroud angle in U

NS - Number of shroud angles

NL - Number of lift coefficient straight line regions

ND - Number of drag coefficient straight line regions

NDL - Number of lift coefficient derivative straight line regions

NDD - Number of drag coefficient derivative straight line regions

XSD - Output shroud derivative vector, equation D6.1

D6.2 SUBROUTINE PRDER

Subroutine PRDER computes the derivatives with respect to the primary states of the propellor forces and moments. These derivatives are calculated by branching to the proper functional partial derivatives of equations D3.13 through D3.16.

PRDER (P, M, X, U, N, NP, XP, K)

P, M, X, U, N - Same as in SHCAL, Chapter D3, except N = (10)

NP - Number of propellor inputs (2)

XP - Output partial derivative $\partial X^i_{prop} / \partial x_j$

K - Indexes of desired partial derivatives, $1 \leq K \leq 36$,

$(i, j) = K = i + 6(j - 1)$

D6.3 SUBROUTINE THDER

Subroutine THDER computes the derivatives of the thruster forces and moments with respect to the primary states of the vehicle. These derivatives are calculated by branching to the proper partial derivatives of equations D3.17 and D3.18 and then by using subroutine RGCOM to compute the thruster coefficient derivatives as straight line approximations of the curves in reference (B-9).

THDER (P, M, U, N, X, XT, K)

P, M, U, X - Same as SHCAL, Chapter D3

N - Same as NT in XPDER, Chapter D5.5

XT - Output partial derivative $\partial X^{i}_{th}/\partial x_j$

K - Same as PRDER, Chapter D6.2

D6.4 SUBROUTINE PRDEP

Subroutine PRDEP computes the derivatives of the propellor forces and moments with respect to the selected propellor parameters. These derivatives are calculated by branching to the proper functional partial derivatives of equations D3.13 through D3.16.

PRDEP (P, M, X, U, N, NP, XP, K, J)

P, M, X, U, N - Same as in SHCAL, Chapter D3, except N = (10)

NP - Number of propellor inputs (2)

XP - Output partial derivative $\partial X^{k}_{prop}/\partial p_j$

K - Index of force or moment

J - Index of parameter

D6.5 SUBROUTINE THDEP

Subroutine THDEP computes the derivatives of the thruster forces and moments with respect to the selected thruster parameters. These derivatives are calculated by branching to the proper partial derivatives of equations D3.17 and D3.18 or else by using subroutine RPDER to calculate the parametric derivatives of the straight line approximations of the thruster coefficients.

THDEP (P, M, U, N, X, XT, K, J)

P, M, U, N, X - Same as THDER, Chapter D6.3

XT - Output partial derivative $\partial X^{k}_{th}/\partial p_j$

K,J - Same as PRDEP, Chapter D6.4

This chapter has presented a basic description of the DSRV effector gradient subroutines. The Fortran IV statements for the

effector gradient subroutines described in this chapter are presented in Appendix A9. The next chapter describes methods for modifying all of the DSRV subroutines for use with other ocean vehicles and suggests several programming simplifications which could be applied to the DSRV model subroutines.

CHAPTER D7

PROGRAM SIMPLIFICATIONS AND MODIFICATIONS

The purposes of this chapter are to describe the modifications necessary to OVMOD and OVDER for calculation of vehicles other than DSRV and to recognize that these subroutines have several possible program simplifications.

D7.1 PROGRAM SIMPLIFICATIONS

Subroutines OVMOD and OVDER along with their associated subroutines were written and debugged separately, directly, and suboptimally. As written, however, they do their jobs, and so the author has not devoted any extra time to "cleaning up the code," reducing the number of cards, or speeding them up.

The most obvious and potentially profitable work to be done is the combination of OVMOD and OVDER into a single subroutine. Several of the functions computed by each are identical, and many of the variables computed by one can be used by the other. Some provision should be made in the combined subroutine to allow deferring calculation of the gradient when the model is to be used in a model reference framework.

Another set of possible simplifications arises from the fact that all of the equations were written out completely in several cases and not combined by type. An example of this is in ACALC, where AC = 0. appears many times and could be written only once with proper changes in the initial conditional GO TO statement numbers. Several other subroutines to which this simplification would apply are: XFUNS, COCAL, XDFNS, XPDER, and PRDER.

In several subroutines, there are sections containing the same or nearly the same sequence of steps. These could perhaps be simplified by making those sections into subroutines. For example, in subroutine THCAL there are several groups of about 20 statements each which have somewhat the same structure and could be rewritten into a subroutine. The noisy and noiseless sections of OVMOD could also be combined into a single subroutine.

It is now evident to the author that there are several subroutines which contain unnecessary conditional GO TO statements and ISW indicators. An example of this is in subroutine SECAL, where the first conditional GO TO (2,3) ISW is unnecessary if in the two preceeding statements the ISW = 's were changed to GO TO 's. In fact, one of the two preceeding statements could also be dropped. This type of simplification is evident elsewhere in SECAL as well as in several other subroutines.

D7.2 PROGRAM MODIFICATIONS FOR OTHER OCEAN VEHICLES

The keynotes of OVMOD and OVDER as written are structure selectivity, second-degree coefficients, and DSRV effectors. Any potential user of these subroutines who does not require structure selectivity for reducing the parameter space would be well advised to write his own OVMOD/OVDER rather than trying to modify this one. The entire fabric of these two subroutines is woven from structure selectivity statements and a considerable percentage of the computation time (est. 20-30%) is spent selecting structure.

Any potential user who requires linear, third-degree, fourth-degree, etc., coefficients or who does not require second-degree coefficients would have to rewrite XDCAL, XDDER, and XFDER based upon

the requirements of his model. It is anticipated that the programming necessary to include additional coefficients will be straightforeward extension of the techniques used for the second-degree coefficients.

The DSRV effector subroutines will not be applicable to most other ocean vehicles. The TANKS, GRAVT, and COCAL could probably be used directly for any ocean vehicle, but the SHCAL, SECAL, PRCAL, and THCAL (shroud, secondary drag, propellor, thrusters) subroutines are specific to the DSRV. Anyone wishing to use OVMOD/OVDER should write effector subroutines specific to his vehicle and include parameters and calling statements in OVMOD and OVDER.

D7.3 SECTION 5(D) SUMMARY

This section has presented the detailed equations for calculation of the time derivatives and their gradients of the linear and angular velocities of a general ocean vehicle. A subroutine for calculating the linear and angular accelerations (OVMOD) has been presented and its use explained. A subroutine for calculating the gradient with respect to states and parameters of the linear and angular accelerations (OVDER) has also been presented and its use explained. Effector subroutines specific to the DSRV have been included for both OVMOD and OVDER and explained. The Fortran IV statements for these chapters are found in Appendices A6 -- A9.

The next section 6(C) presents the programming and analysis for utilization of these model and gradient subroutines in the identification and identifiability studies of the DSRV coefficients and parameters.

SECTION 6

IDENTIFICATION STUDIES OF THE DSRV DYNAMIC COEFFICIENTS (C)

C1 MAIN 6 * 6 IDENTIFICATION PROGRAM DESCRIPTION AND SEA TRIALS (MAIN)

C2 MODEL REFERENCE CONTOURS FOR SELECTED DSRV 6 * 6 PARAMETERS

C3 DESCRIPTION OF SEVERAL SUBROUTINES FOR USE IN THE MAIN PROGRAM

"THE PURPOSE OF COMPUTING IS INSIGHT, NOT NUMBERS." -

R. W. HAMMING (H-5)

THIS SECTION PRESENTS A DESCRIPTION OF AND SOME RESULTS FROM A STRUCTURE-SELECTIVE SIX DEGREE OF FREEDOM IDENTIFICATION PROGRAM WHICH USES THE DSRV MODEL OF SECTION 5(D). THE COMPUTER RUNS ARE DESIGNED TO SHOW HOW THE PRIMARY VEHICLE BEHAVIOR CHANGES AS MORE DEGREES OF FREEDOM ARE INCLUDED AND TO SHOW THE MODEL REFERENCE CONTOURS OF SEVERAL SELECTED DSRV COEFFICIENTS AND PARAMETERS IN MULTIPLE DEGREE OF FREEDOM MODELS.

CHAPTER C1

MAIN 6 * 6 IDENTIFICATION PROGRAM DESCRIPTION AND SEA TRIALS (MAIN)

The extensive studies of Section 4(P) have shown that the model reference contouring and extended Kalman filtering technqiues are valid for the DSRV single degree of freedom equations. This section shows that the model reference contouring technique is valid for the DSRV in one through six degrees of freedom for several selected coefficients. The DSRV mathematical model has been presented and its subroutines described in Section 5(D); this model is used in the main six degree of freedom, structure-selective identification program developed for the studies of this section.

This chapter presents a description of the DSRV sea trial behavior in 1 through 6 degrees of freedom using a simulated ramp input to the DSRV propellor. A description of the MAIN program for generating this data and for identifying selected vehicle parameters from it using model reference contouring is also presented. The next chapter shows the identification results from using the model reference contouring technique on multiple degree of freedom sea trials and mathematical models.

C1.1 DSRV SEA TRIALS IN 1 THROUGH 6 DEGREES OF FREEDOM

The flexibility of the main program to be described in Chapters C1.2 and C1.3 allows the running of DSRV sea trials using models with 1, 2, 3, 4, 5, or 6 degrees of freedom using the same input function. This offers a useful set of results in that the vehicle coupling and decoupling may be observed. For example, if the surge velocity is the same for a 1 degree of freedom (DOF) model as it is for a 6 DOF model,

then the identifiability characteristics of the directly related parameters in surge for 1 DOF should be applicable to the same parameters in 6 DOF for the same maneuver.

The input function to be used in these runs is a simulated ramp function to the DSRV propellor. This input function is plotted as curve #1 in Figure C1.1, and the propellor angular acceleration is shown by curve #2 in Figure C1.1. This input will remain the same for all of the sea trials described in this chapter.

Using the input in Figure C1.1 and the MAIN program (primarily subroutine SEATR) of Appendices A19 and A17, the DSRV sea trials for 46 seconds, without any control systems or tanks inputs, were calculated using 1, 2, 3, 4, 5, and 6 degree of freedom models. For each of these models the respective DSRV velocities and angles were plotted; these sea trials will be described here in ascending degree of freedom (DOF) order.

<u>1 DOF SURGE (u)</u>

The response of the DSRV mathematical model to the simulated ramp input when the vehicle velocities v, w, p, q, and r are not included is shown in Figure C1.2. As can be seen from this figure, the DSRV accelerates from 0.2 ft/sec to 1.3 ft/sec during the 46 second sea trial. Figure C1.2 also represents the DSRV surge behavior for all of the multiple degree of freedom models to be described in this chapter. The shapes of all of the surge curves are almost identical, and therefore, only the 1 DOF - Figure C1.2 and the 6 DOF - Figure C1.13 surge velocity curves are included in this chapter.

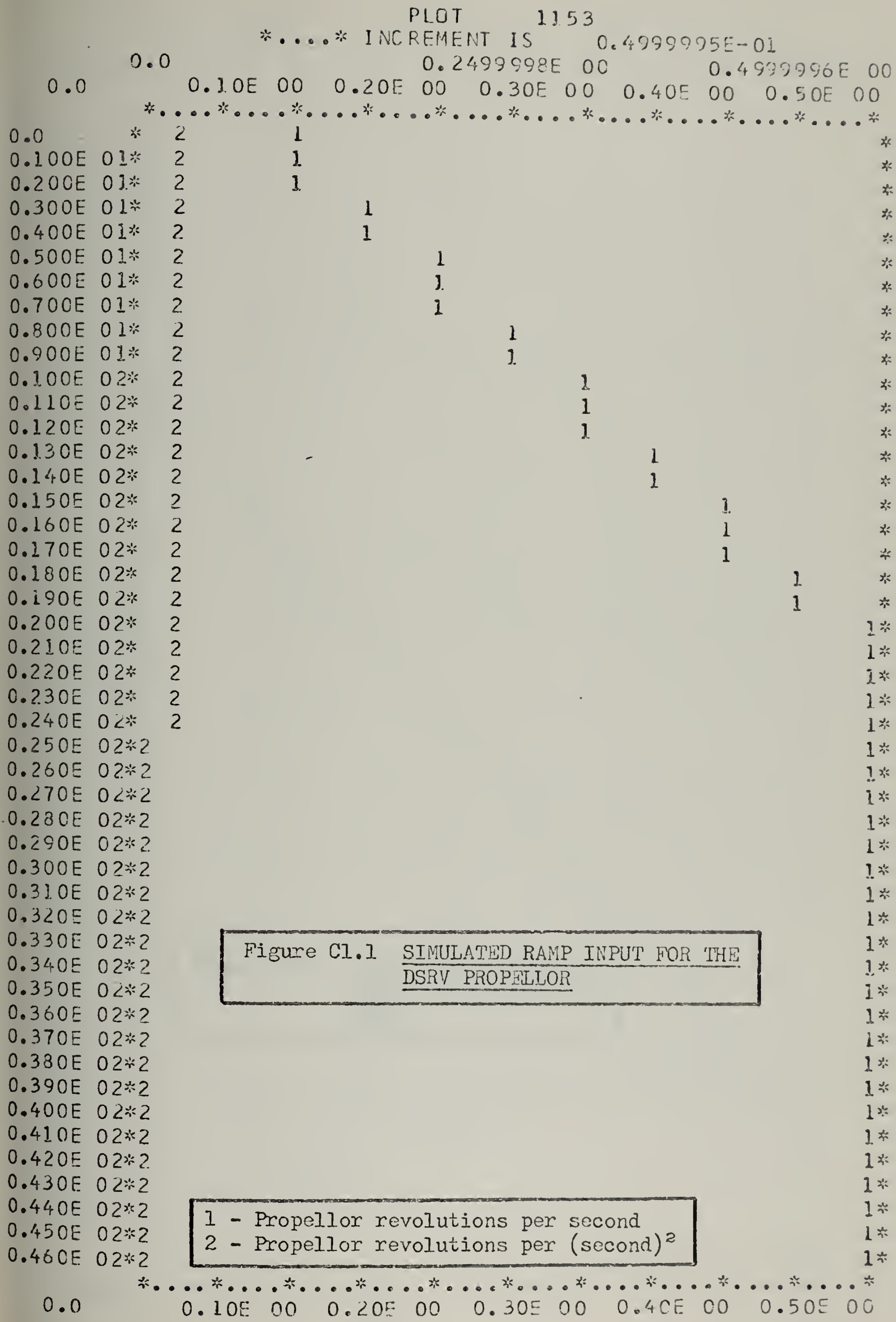


Figure C1.1 SIMULATED RAMP INPUT FOR THE DSRV PROPELLOR

1 - Propellor revolutions per second
2 - Propellor revolutions per (second)^2

```
                              PLOT      1150
                    *....* INCREMENT IS     0.1136613E 00
          0.2013411E 00          0.7696475E 00          0.1337955E 01
     0.20E 00  0.43E 00  0.66E 00  0.88E 00  0.11E 01  0.13E 01
          *....*....*....*....*....*....*....*....*....*....*
0.0      *1                                                  *
0.100E 01*1                                                  *
0.200E 01*1                                                  *
0.300E 01*1                                                  *
0.400E 01*1                                                  *
0.500E 01*1                                                  *
0.600E 01*1                                                  *
0.700E 01* 1                                                 *
0.800E 01* 1                                                 *
0.900E 01*  1                                                *
0.100E 02*  1                                                *
0.110E 02*   1                                               *
0.120E 02*   1                                               *
0.130E 02*    1                                              *
0.140E 02*     1                                             *
0.150E 02*      1                                            *
0.160E 02*       1                                           *
0.170E 02*        1                                          *
0.180E 02*          1                                        *
0.190E 02*           1                                       *
0.200E 02*             1                                     *
0.210E 02*               1                                   *
0.220E 02*                1                                  *
0.230E 02*                  1                                *
0.240E 02*                   1                               *
0.250E 02*                     1                             *
0.260E 02*                       1                           *
0.270E 02*                        1                          *
0.280E 02*                          1                        *
0.290E 02*                           1                       *
0.300E 02*                             1                     *
0.310E 02*                              1                    *
0.320E 02*                                1                  *
0.330E 02*                                 1                 *
0.340E 02*                                  1                *
0.350E 02*                                    1              *
0.360E 02*                                     1             *
0.370E 02*                                      1            *
0.380E 02*                                        1          *
0.390E 02*                                         1         *
0.400E 02*                                          1        *
0.410E 02*                                            1      *
0.420E 02*                                             1     *
0.430E 02*                                              1    *
0.440E 02*                                               1   *
0.450E 02*                                                1  *
0.460E 02*                                                  1*
          *....*....*....*....*....*....*....*....*....*....*
     0.20E 00  0.43E 00  0.66E 00  0.88E 00  0.11E 01  0.13E 01
```

Figure C1.2 DSRV SURGE VELOCITY, 1 DOF, SIMULATED RAMP INPUT

2 DOF SURGE, ROLL (u, p)

The roll response of the DSRV when a 2 DOF model is used and when the roll angular velocity is integrated at each time step to give the roll angle used in the gravity forces calculations is shown in Figure C1.3. The actual DSRV roll angle φ (radians) throughout the sea trial is shown in Figure C1.4. The surge velocity in 2 DOF is almost identical to Figure C1.2.

Figures C1.3 and C1.4 show that for this input the DSRV is almost unstable in its roll response to this sea trial input; the vehicle has a roll rate period of about 10 seconds, a maximum roll rate of 0.5 rad/sec, and reaches a maximum roll of 0.79 radians. The fact that the sinusoidal roll velocity amplitude in Figure C1.3 is increasing at a decreasing exponential rate means that even for the simulated ramp input of Figure C1.1 the roll rate would eventually become either a constant-amplitude or a decreasing-amplitude sinusoidal function for a longer sea trial. Of course in the real DSRV the mercury list tanks and their associated control systems keep this kind of oscillation in roll to a minimum; but for the identification studies of this thesis, this kind of behavior is completely acceptable and, in fact, desirable.

Another important fact which is indicated by the behavior of the surge and roll for this input is that the DSRV roll is not coupled into surge. This means that for the 2 DOF model, the surge and roll parameters may be identified separately.

3 DOF SURGE, HEAVE, PITCH (u, w, q)

The vertical plane behavior of the DSRV for a simulated ramp input is shown by Figures C1.2, C1.5, and C1.6 for the vehicle surge

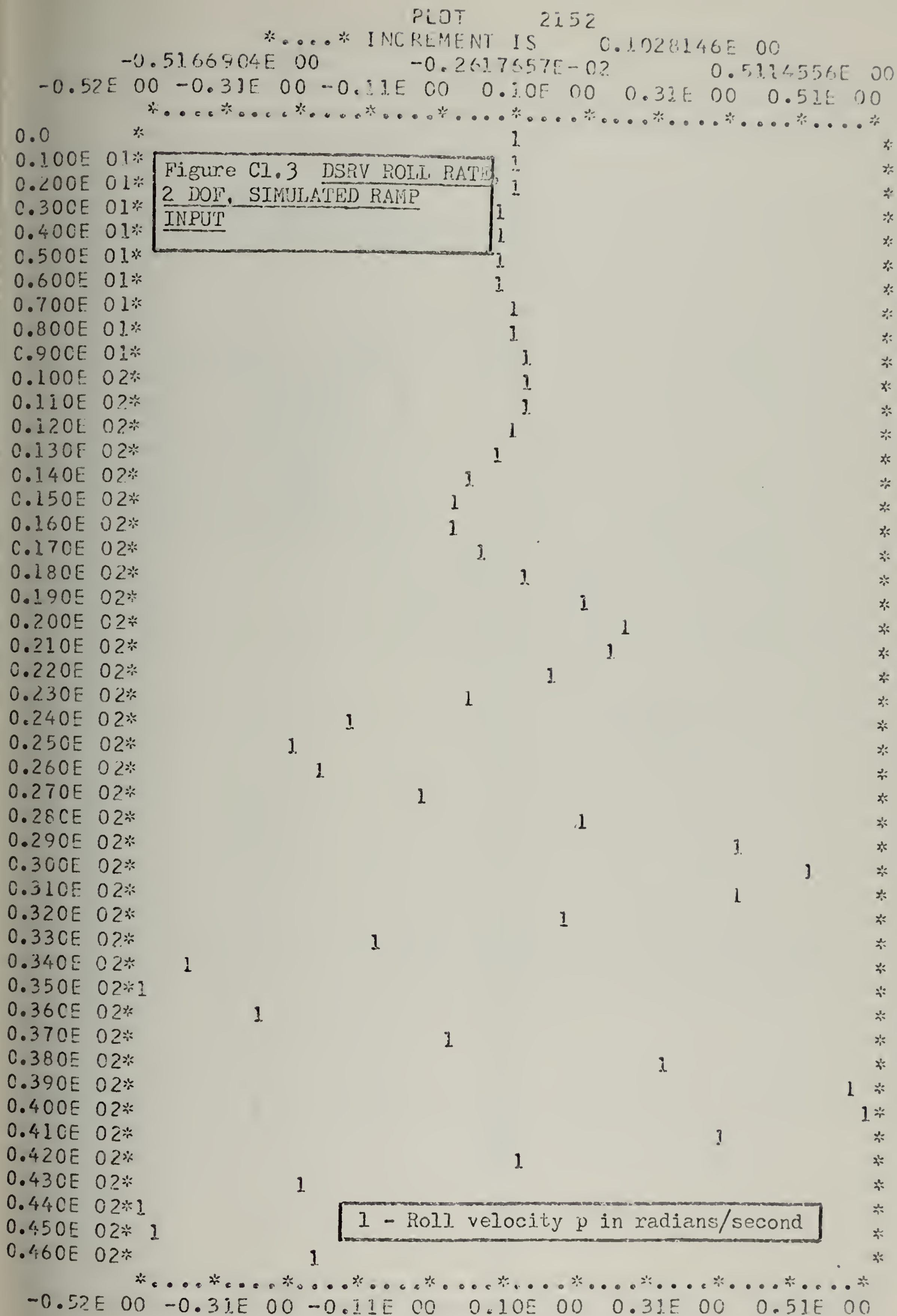
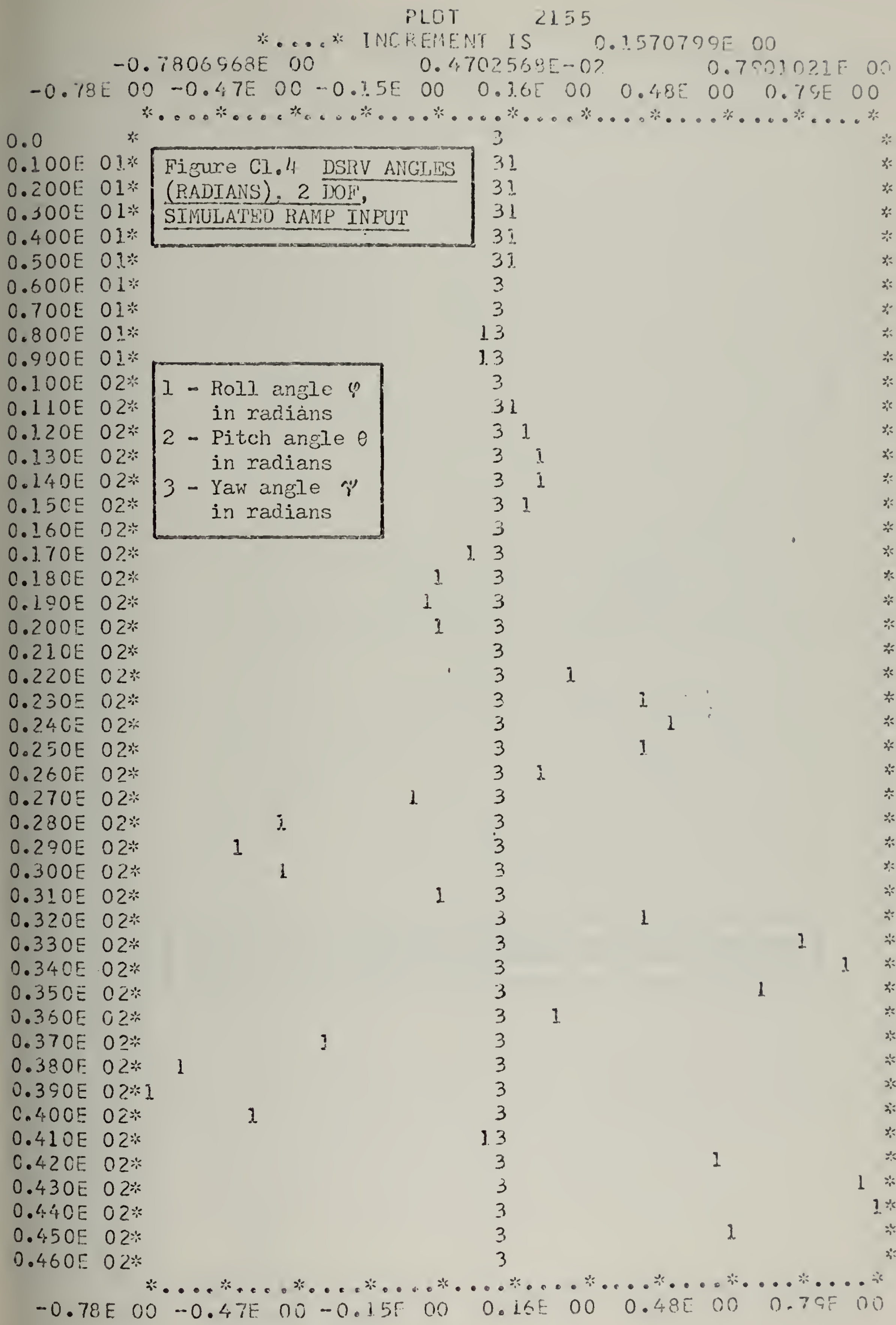


Figure C1.3 DSRV ROLL RATE, 2 DOF, SIMULATED RAMP INPUT


PLOT 2155
.... INCREMENT IS 0.1570799E 00
-0.7806968E 00
0.4702568E-02
0.7901021E 00
-0.78E 00 -0.47E 00 -0.15E 00 0.16E 00 0.48E 00 0.79E 00
Figure C1.4 DSRV ANGLES (RADIANS), 2 DOF, SIMULATED RAMP INPUT
1 - Roll angle φ in radians
2 - Pitch angle θ in radians
3 - Yaw angle ψ in radians
0.0
0.100E 01
0.200E 01
0.300E 01
0.400E 01
0.500E 01
0.600E 01
0.700E 01
0.800E 01
0.900E 01
0.100E 02
0.110E 02
0.120E 02
0.130E 02
0.140E 02
0.150E 02
0.160E 02
0.170E 02
0.180E 02
0.190E 02
0.200E 02
0.210E 02
0.220E 02
0.230E 02
0.240E 02
0.250E 02
0.260E 02
0.270E 02
0.280E 02
0.290E 02
0.300E 02
0.310E 02
0.320E 02
0.330E 02
0.340E 02
0.350E 02
0.360E 02
0.370E 02
0.380E 02
0.390E 02
0.400E 02
0.410E 02
0.420E 02
0.430E 02
0.440E 02
0.450E 02
0.460E 02
-0.78E 00 -0.47E 00 -0.15E 00 0.16E 00 0.48E 00 0.79E 00

PLOT 3151

.... INCREMENT IS 0.1553860E-02

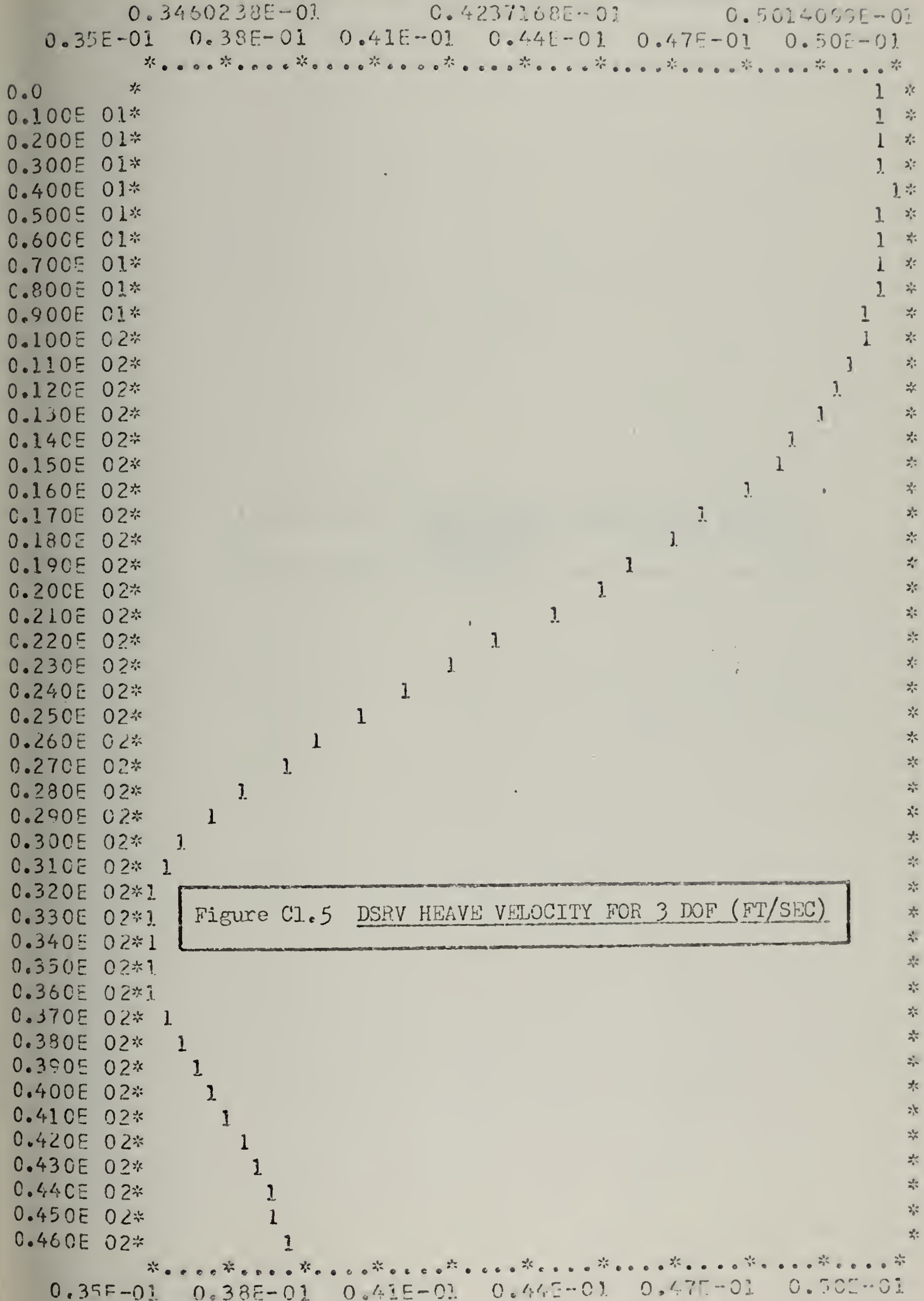


Figure C1.5 DSRV HEAVE VELOCITY FOR 3 DOF (FT/SEC)

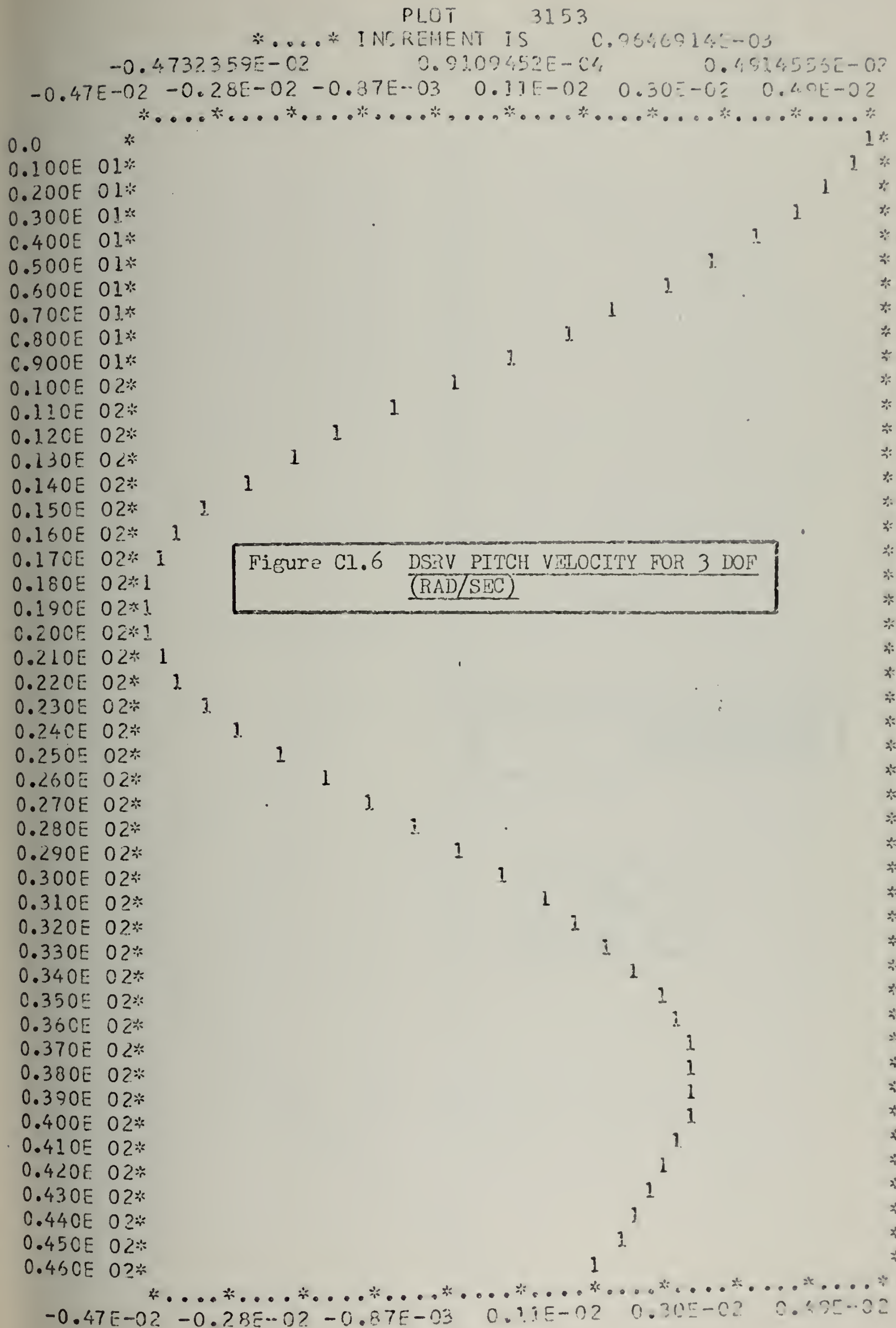


Figure C1.6 DSRV PITCH VELOCITY FOR 3 DOF (RAD/SEC)

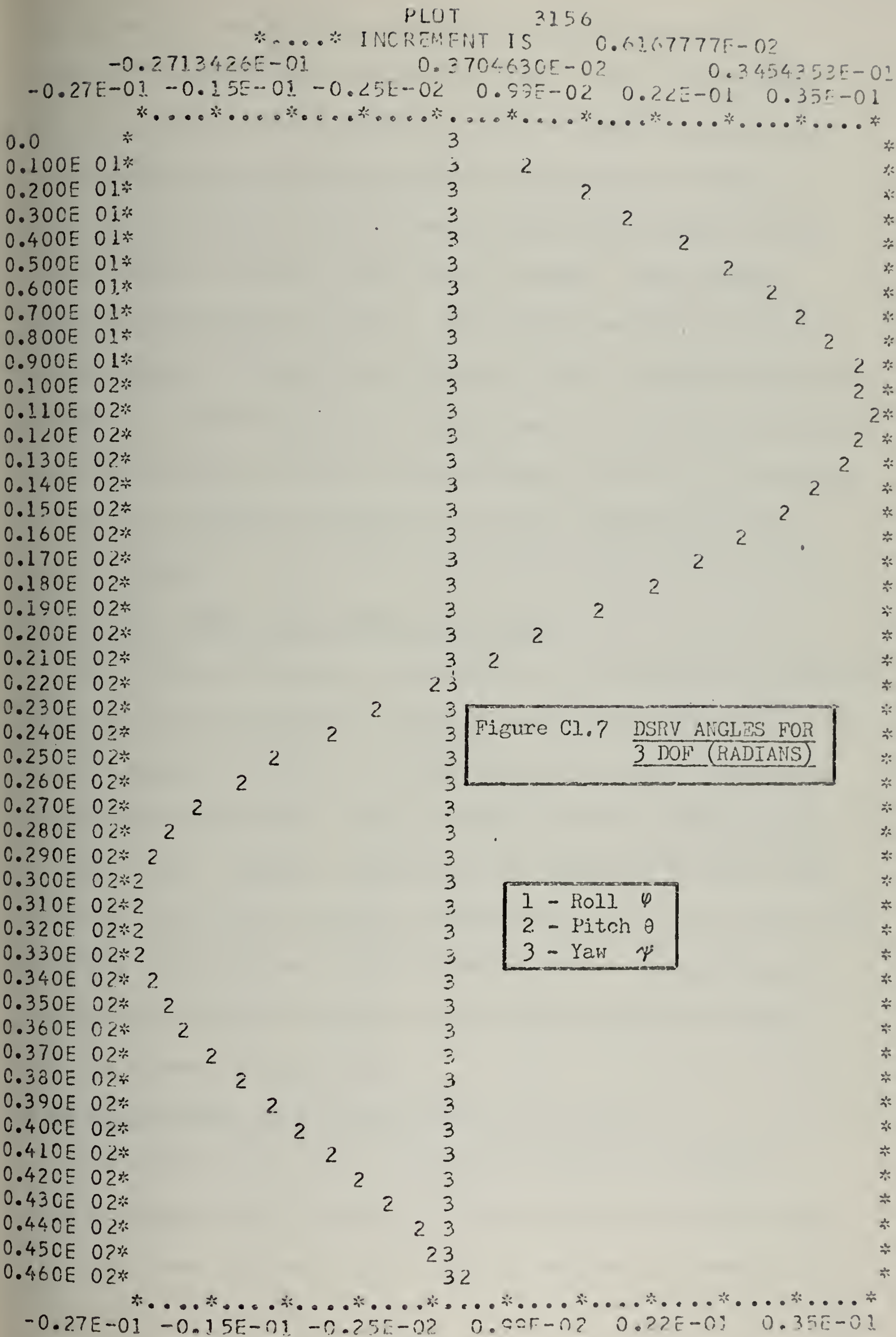


Figure C1.7 DSRV ANGLES FOR 3 DOF (RADIANS)

(u), heave (w), and pitch (q) velocities respectively and by Figure C1.7 for the vehicle angles (φ, θ, Ψ). The initial values for u, w, q were set at 0.2 ft/sec, and 0.005 rad/sec respectively. These curves show that while surging forward in these 3 DOF, the heave and pitch behavior are determined primarily by their initial values and have very small amplitudes of change. This indicates that, for this maneuver, surge is not strongly coupled into either heave or pitch. The fact that Figure C1.2 still represents the surge behavior in 3 DOF indicates that neither heave nor pitch (at least at these small amplitudes) are strongly coupled into surge. The maximum pitch angle throughout the maneuver is seen in Figure C1.7 to be about 2 degrees.

4 DOF SURGE, HEAVE, ROLL, PITCH (u, w, p, q)

The vertical plane plus roll behavior of the DSRV for a simulated ramp input to the propellor is shown by Figures C1.2, C1.8, C1.3, and C1.6 for surge (u), heave (w), roll (p), and heave (q) respectively. The corresponding vehicle angles throughout the sea trial are shown in Figure C1.9. The most significant fact indicated by these 4 DOF curves is that roll is strongly coupled into heave, but not into surge or pitch. Over this sea trial, the vehicle heaves downward up to a maximum of 0.39 ft/sec with some of the roll oscillations coupled into it, as seen in Figure C1.8.

5 DOF SURGE, HEAVE, ROLL PITCH, YAW (u, w, p, q, r)

The vertical plane plus roll and yaw behavior of the DSRV is shown by Figures C1.2, C1.8, C1.3, C1.10, and C1.11 for the vehicle surge (u), heave (w), roll (p), pitch (q), and yaw (r) respectively

```
                             PLOT      4151
                   *....* INCREMENT IS     0.3387600E-01
          0.4826792E-01         0.2176479E 00         0.3870282E 00
    0.48E-01  0.12E 00  0.18E 00  0.25E 00  0.32E 00  0.39E 00
          *....*....*....*....*....*....*....*....*....*....*
0.0       *1                                                 *
0.100E 01 *1                                                 *
0.200E 01 *1                                                 *
0.300E 01 *1                                                 *
0.400E 01 *1                                                 *
0.500E 01 *1                                                 *
0.600E 01 *1                                                 *
0.700E 01 *1                                                 *
0.800E 01 *1                                                 *
0.900E 01 *1                                                 *
0.100E 02 *1                                                 *
0.110E 02 *1                                                 *
0.120E 02 *1                                                 *
0.130E 02 *1                                                 *
0.140E 02 *1                                                 *
0.150E 02 *1                                                 *
0.160E 02 *1                                                 *
0.170E 02 *1                                                 *
0.180E 02 *1                                                 *
0.190E 02 *1                                                 *
0.200E 02 *1                                                 *
0.210E 02 *1                                                 *
0.220E 02 * 1                                                *
0.230E 02 * 1                                                *
0.240E 02 * 1                                                *
0.250E 02 *  1                                               *
0.260E 02 *    1                                             *
0.270E 02 *     1                                            *
0.280E 02 *     1                                            *
0.290E 02 *     1                                            *
0.300E 02 *        1                                         *
0.310E 02 *             1                                    *
0.320E 02 *               1                                  *
0.330E 02 *               1                                  *
0.340E 02 *               1                                  *
0.350E 02 *                    1                             *
0.360E 02 *                          1                       *
0.370E 02 *                            1                     *
0.380E 02 *                           1                      *
0.390E 02 *                            1                     *
0.400E 02 *                                 1                *
0.410E 02 *                                       1          *
0.420E 02 *                                        1         *
0.430E 02 *                                       1          *
0.440E 02 *                                        1         *
0.450E 02 *                                            1     *
0.460E 02 *                                                 1*
          *....*....*....*....*....*....*....*....*....*....*
    0.48E-01  0.12E 00  0.18E 00  0.25E 00  0.32E 00  0.39E 00
```

Figure C1.8 DSRV HEAVE VELOCITY FOR 4 DOF (FT/SEC)

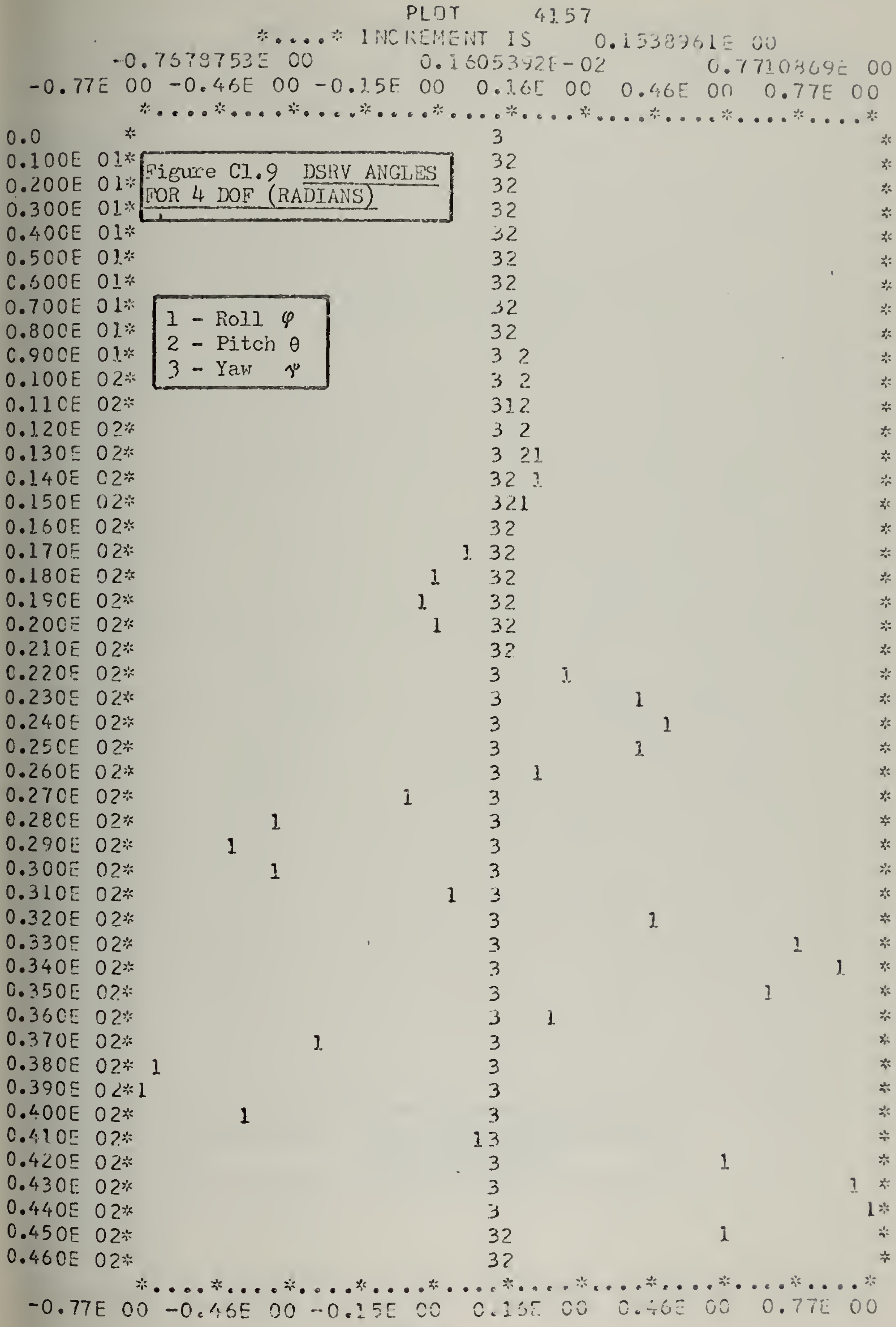


Figure C1.9 DSRV ANGLES FOR 4 DOF (RADIANS)

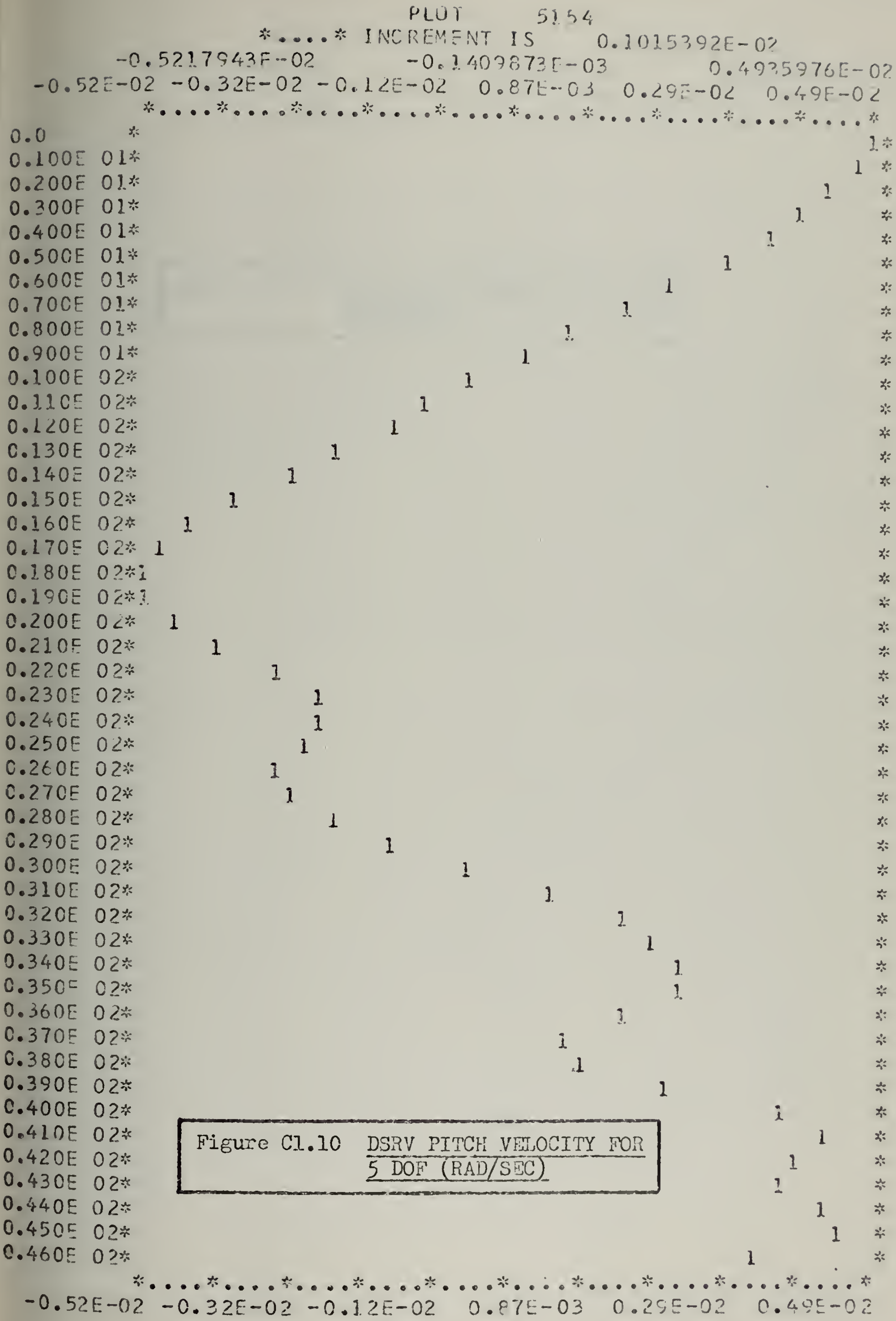


Figure C1.10 DSRV PITCH VELOCITY FOR 5 DOF (RAD/SEC)

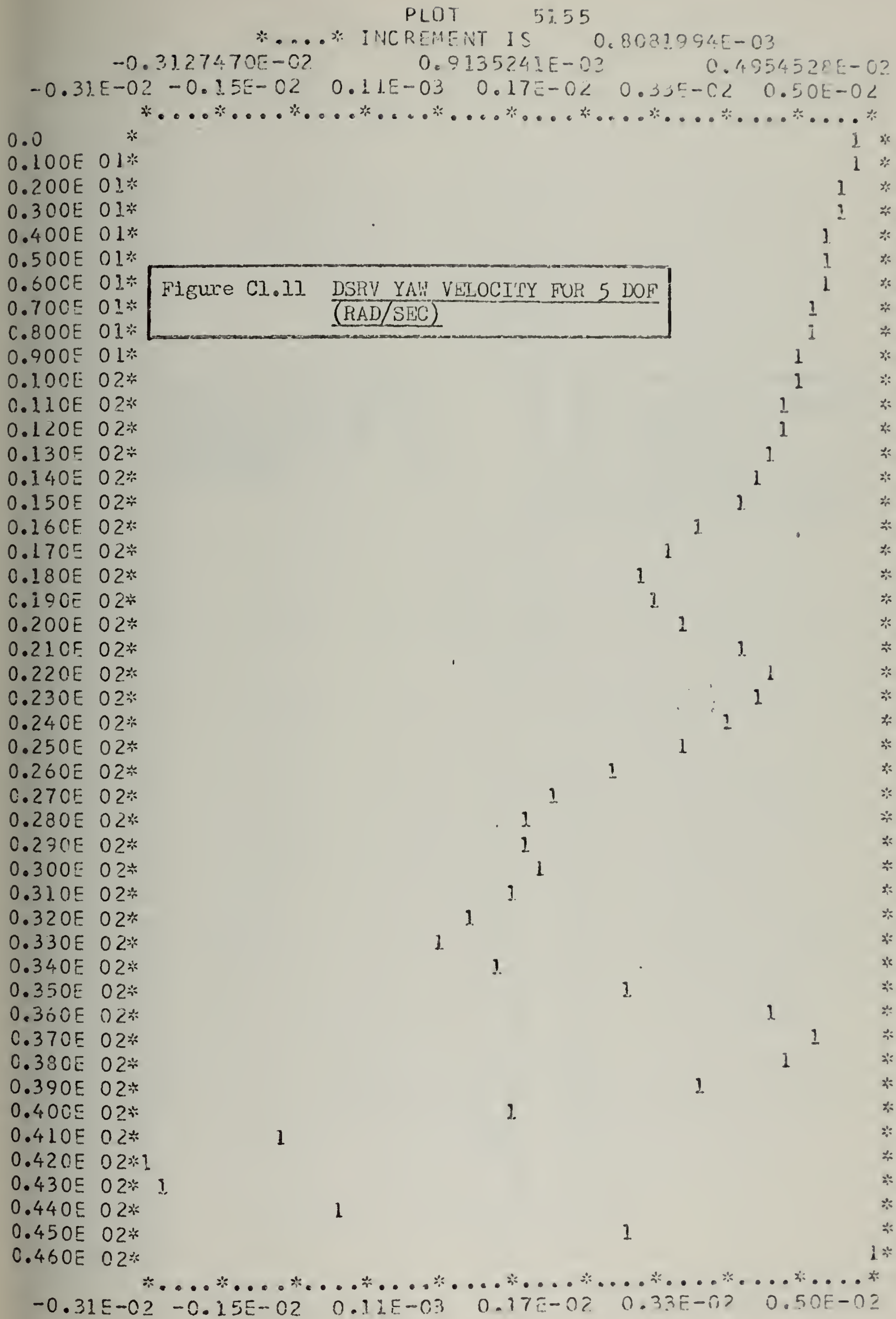


Figure C1.11 DSRV YAW VELOCITY FOR 5 DOF (RAD/SEC)

```
                    PLOT       5158
            *....* INCREMENT IS     0.1537315E 00
   -0.7672982E 00         0.1359463E-02         0.7700175E 00
-0.77E 00 -0.46E 00 -0.15E 00  0.16E 00  0.46E 00  0.77E 00
         *....*....*....*....*....*....*....*....*....*....*
0.0      *                       3                         *
0.100E 01*                        3                        *
0.200E 01*                        3                        *
0.300E 01*                        3                        *
0.400E 01*                        3                        *
0.500E 01*                        3                        *
0.600E 01*                       13                        *
0.700E 01*                       123                       *
0.800E 01*                       123                       *
0.900E 01*                       1 3                       *
0.100E 02*                       1 3                       *
0.110E 02*                        13                       *
0.120E 02*                         3                       *
0.130E 02*                         31                      *
0.140E 02*                        2 3                      *
0.150E 02*                        213                      *
0.160E 02*                        2 3                      *
0.170E 02*                     1  2 3                      *
0.180E 02*                   1    2 3                      *
0.190E 02*                  1     2 3                      *
0.200E 02*                   1    2 3                      *
0.210E 02*                       12 3                      *
0.220E 02*                       2  3 1                    *
0.230E 02*                       2  3      1               *
0.240E 02*                       2   3        1            *
0.250E 02*                       2   3     1               *
0.260E 02*                       2  13                     *
0.270E 02*                 1     2   3                     *
0.280E 02*        1              2   3                     *
0.290E 02*     1                 2   3                     *
0.300E 02*         1             2   3                     *
0.310E 02*                    1 2    3                     *
0.320E 02*                      2    3      1              *
0.330E 02*                      2    3                1    *
0.340E 02*                       2   3                   1 *
0.350E 02*                       2   3             1       *
0.360E 02*                       2   3                     *
0.370E 02*           1           2   3                     *
0.380E 02* 1                     2   3                     *
0.390E 02*1                      2   3                     *
0.400E 02*      1                2    3                    *
0.410E 02*                      12    3                    *
0.420E 02*                       2    3          1         *
0.430E 02*                       2    3                   1*
0.440E 02*                       2    3                    1*
0.450E 02*                       2    3          1         *
0.460E 02*                      1 2   3                    *
         *....*....*....*....*....*....*....*....*....*....*
-0.77E 00 -0.46E 00 -0.15E 00  0.16E 00  0.46E 00  0.77E 00
```

Figure C1.12 DSRV ANGLES FOR 5 DOF (RADIANS)

1 - Roll φ ; + = STBD.
2 - Pitch θ ;
+ = Bow UP
3 - Yaw Ψ ; + = STBD

and by Figure C1.12 for the vehicle angles (φ, θ, ψ). These figures show that surge is primarily unaffected by the other degrees of freedom for this maneuver, that roll is coupled into yaw which is in turn coupled into pitch, and that heave is essentially unaffected by yaw.

<u>6 DOF SURGE, SWAY, HEAVE, ROLL, PITCH, YAW (u, v, w, p, q, r)</u>

The complete DSRV response to the propellor input of Figure C1.1 is shown in Figures C1.13 through C1.19. These curves show that the vehicle is surging forward, swaying to starboard, heaving downward, rolling violently from side to side ($\pm$ 45 degrees), pitching very slightly bow upward, and yawing slightly to starboard (3 degrees) (see Figure T2.1 for nomenclature). These velocity plots show, in addition to the results for lesser degrees of freedom, that roll is strongly coupled into sway. For this maneuver, the most critical degree of freedom from a control standpoint is roll since the greatest motion amplitudes and rates are there. Both the surge and roll dynamics seem to be nearly independent of the other four degrees of freedom, but this is probably because the respective amplitudes of the other four degrees of freedom are small for this maneuver.

There are a large number of other combinations of maneuvers and degrees of freedom which could be used here to learn about the behavior and coupling characteristics of the DSRV. The six cases have been presented above to show how this kind of information may be acquired using the structure-selective mathematical model developed for the DSRV and the program MAIN explained in this chapter.

C1.2 MAIN 6 * 6 IDENTIFICATION PROGRAM DESCRIPTION

The program MAIN presented in Appendix A19 represents an extremely sophisticated version of the model reference contouring program in

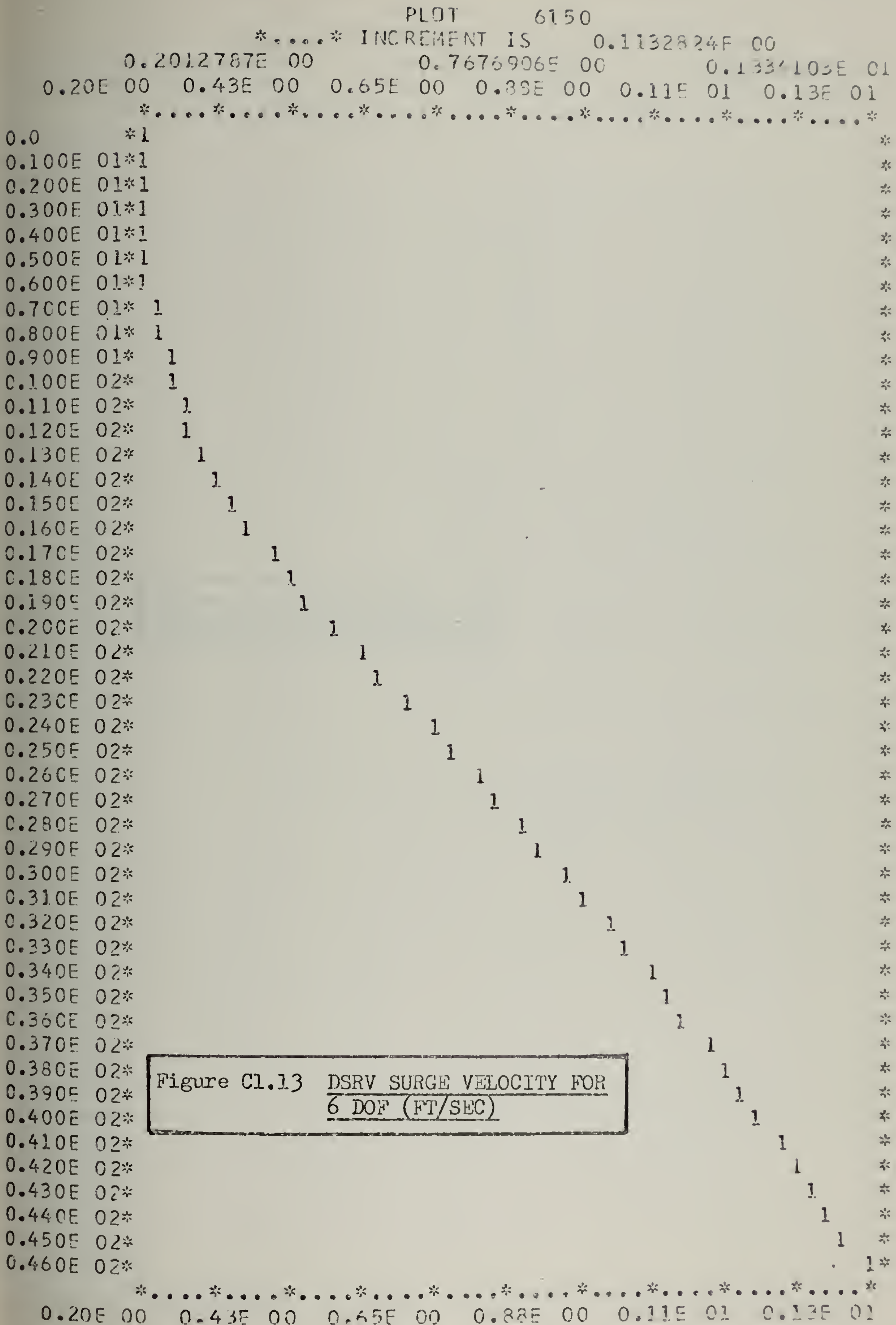


Figure C1.13 DSRV SURGE VELOCITY FOR 6 DOF (FT/SEC)

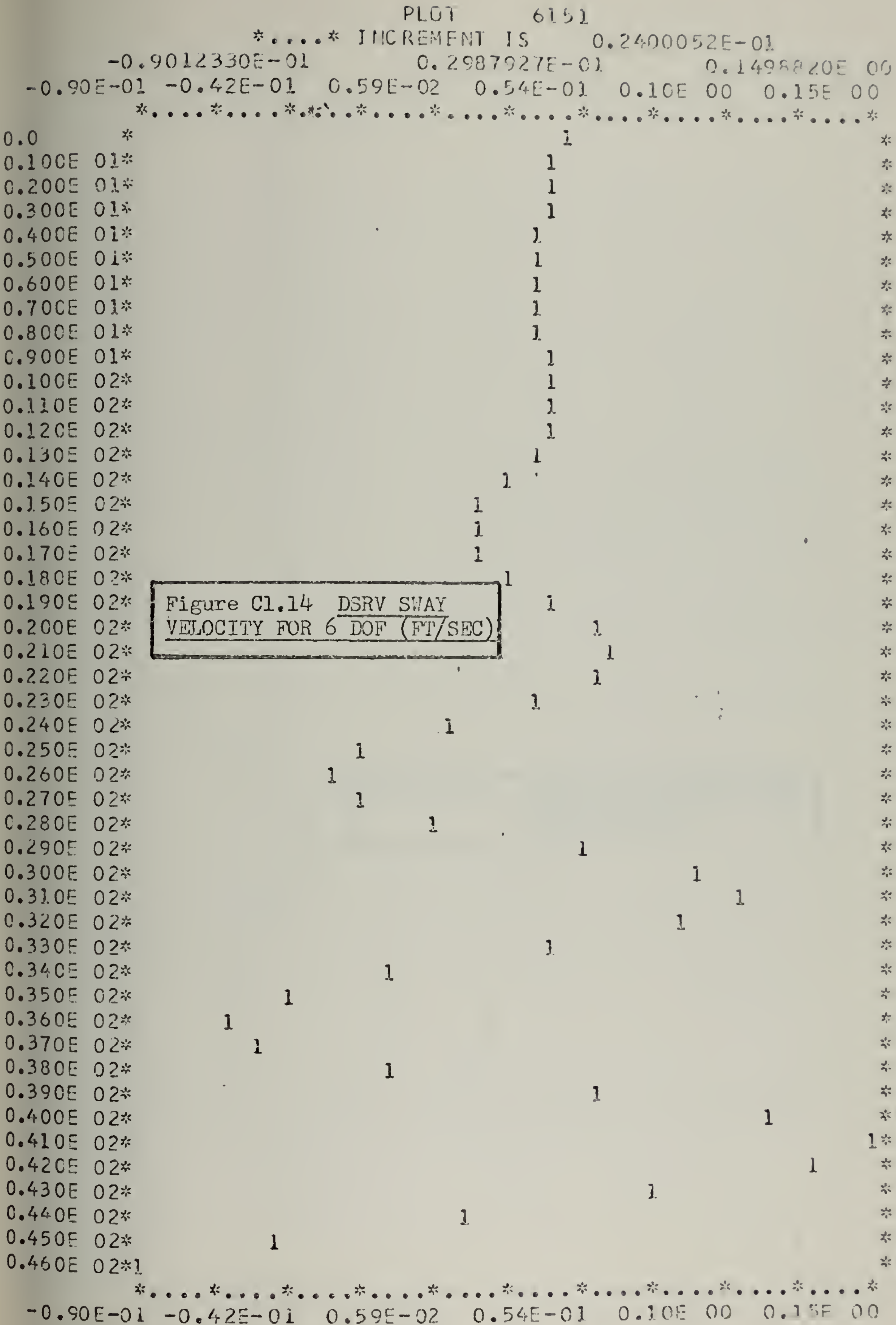


Figure C1.14 DSRV SWAY VELOCITY FOR 6 DOF (FT/SEC)

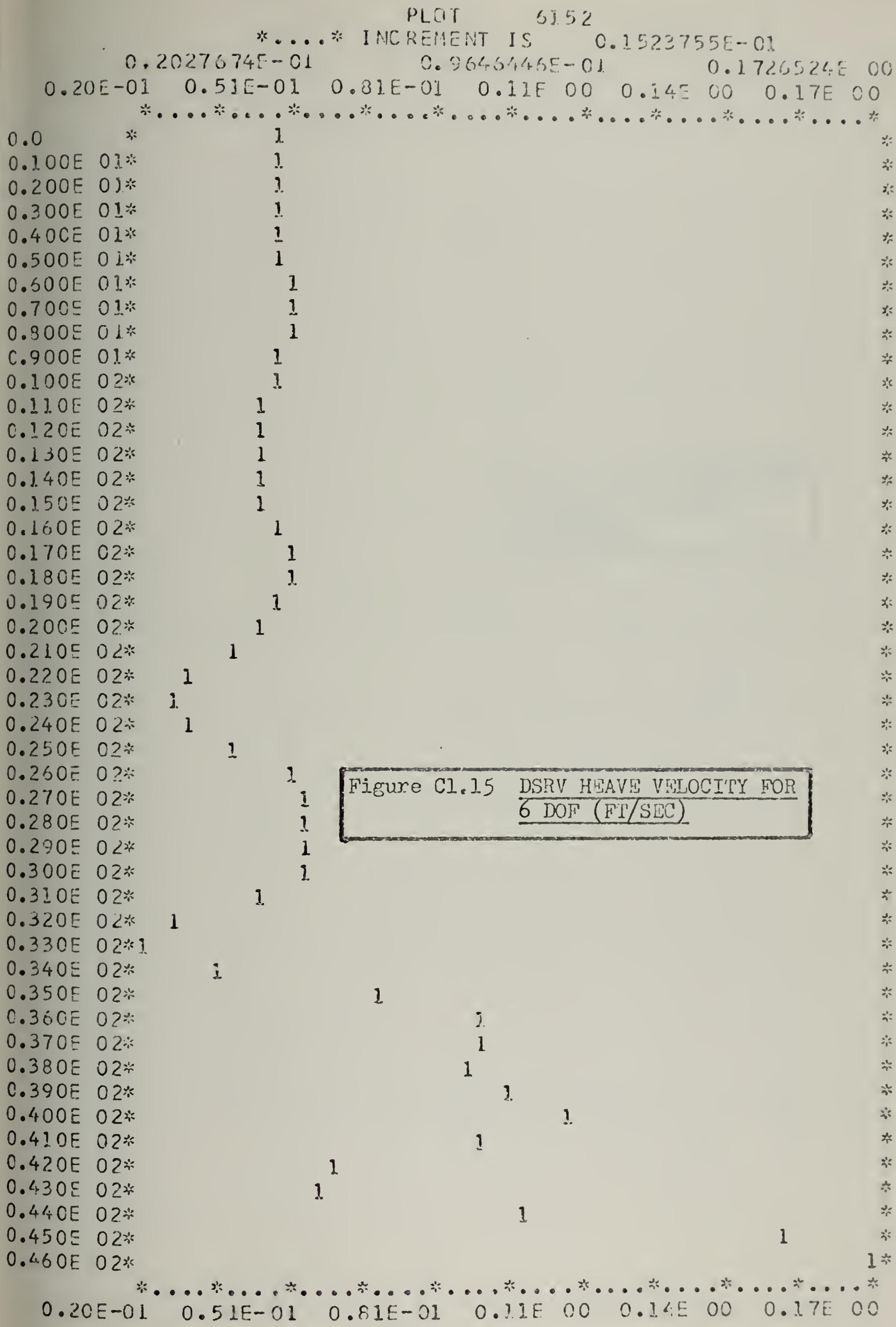


Figure C1.15 DSRV HEAVE VELOCITY FOR 6 DOF (FT/SEC)

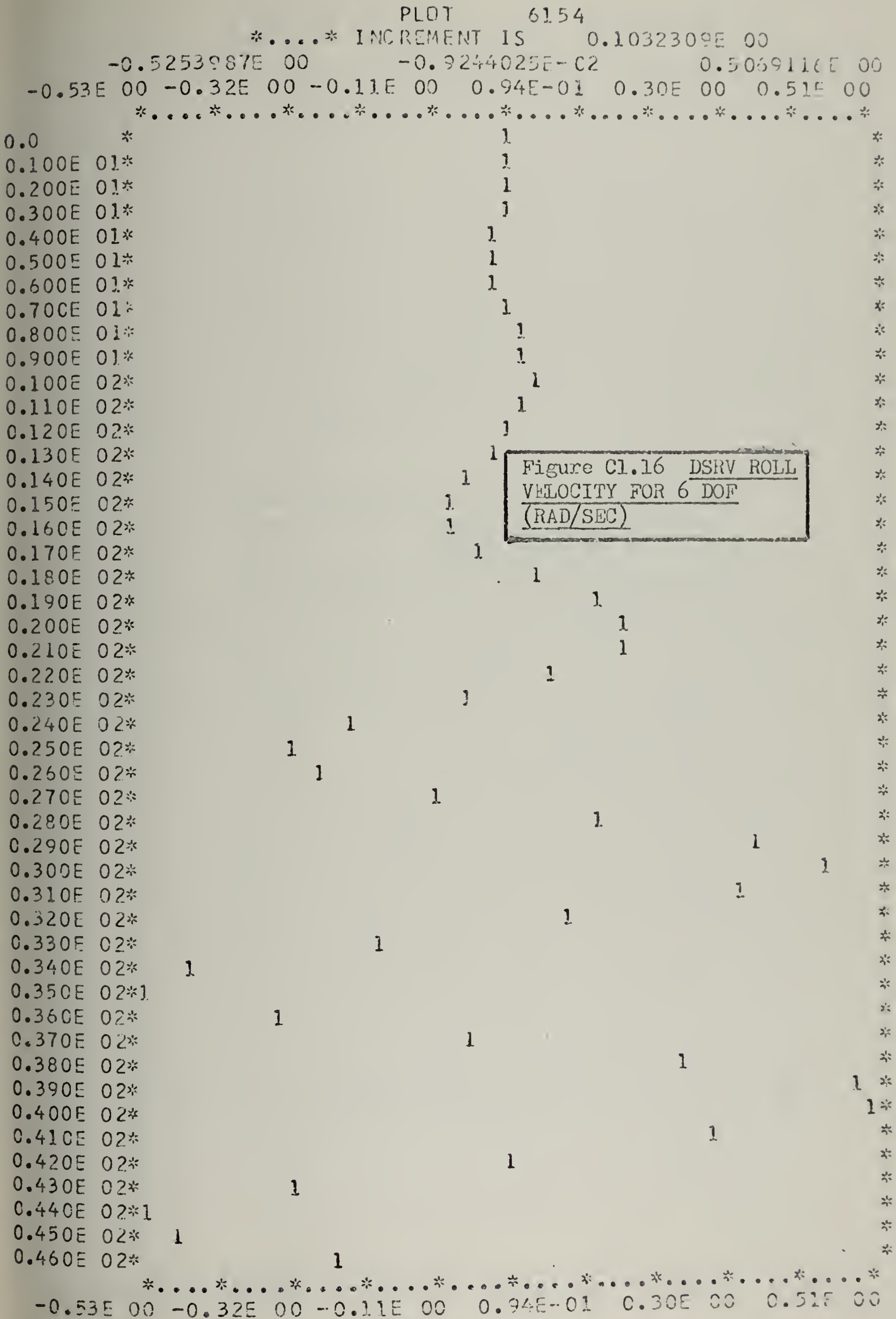


Figure C1.16 DSRV ROLL VELOCITY FOR 6 DOF (RAD/SEC)

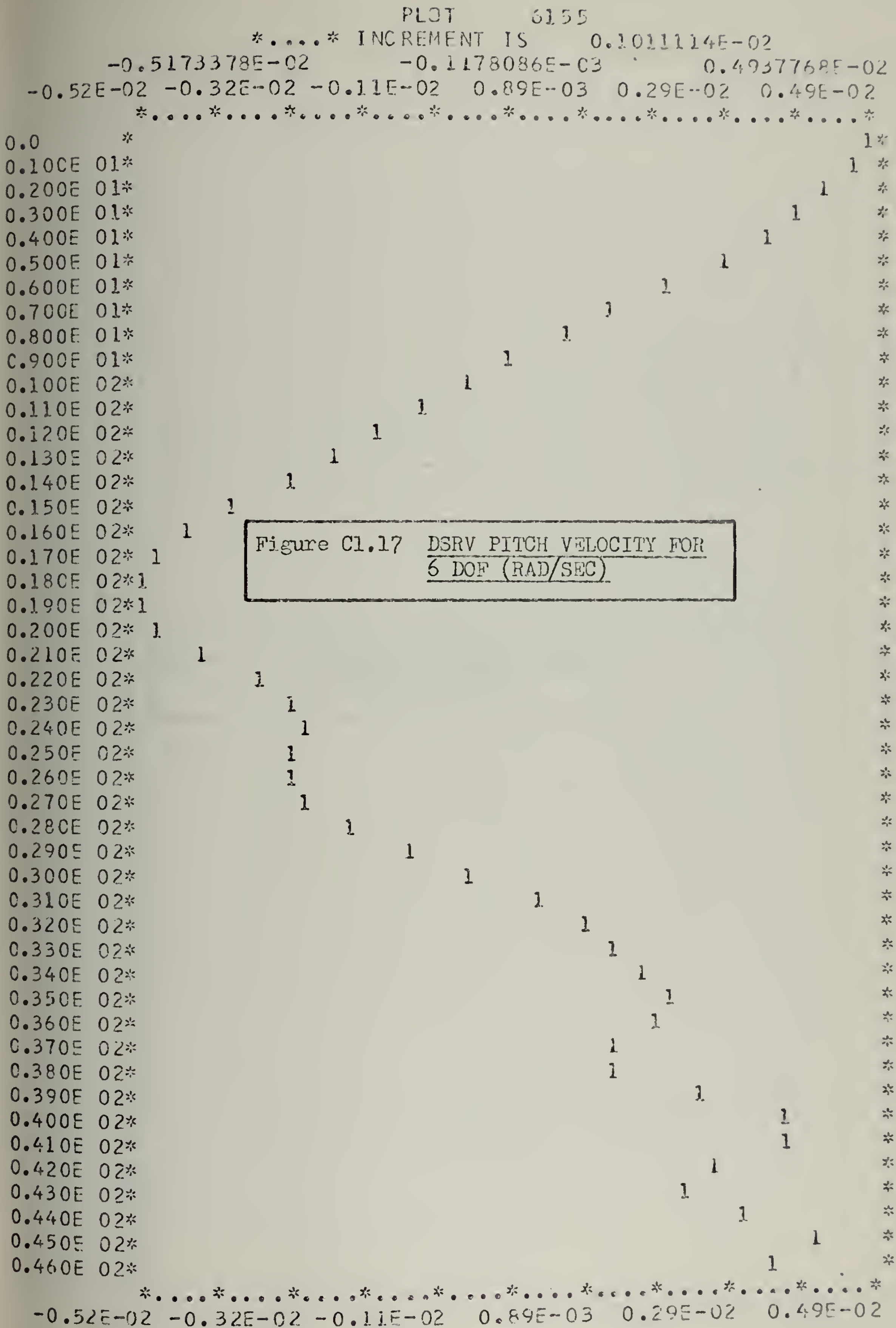


Figure C1.17 DSRV PITCH VELOCITY FOR 6 DOF (RAD/SEC)

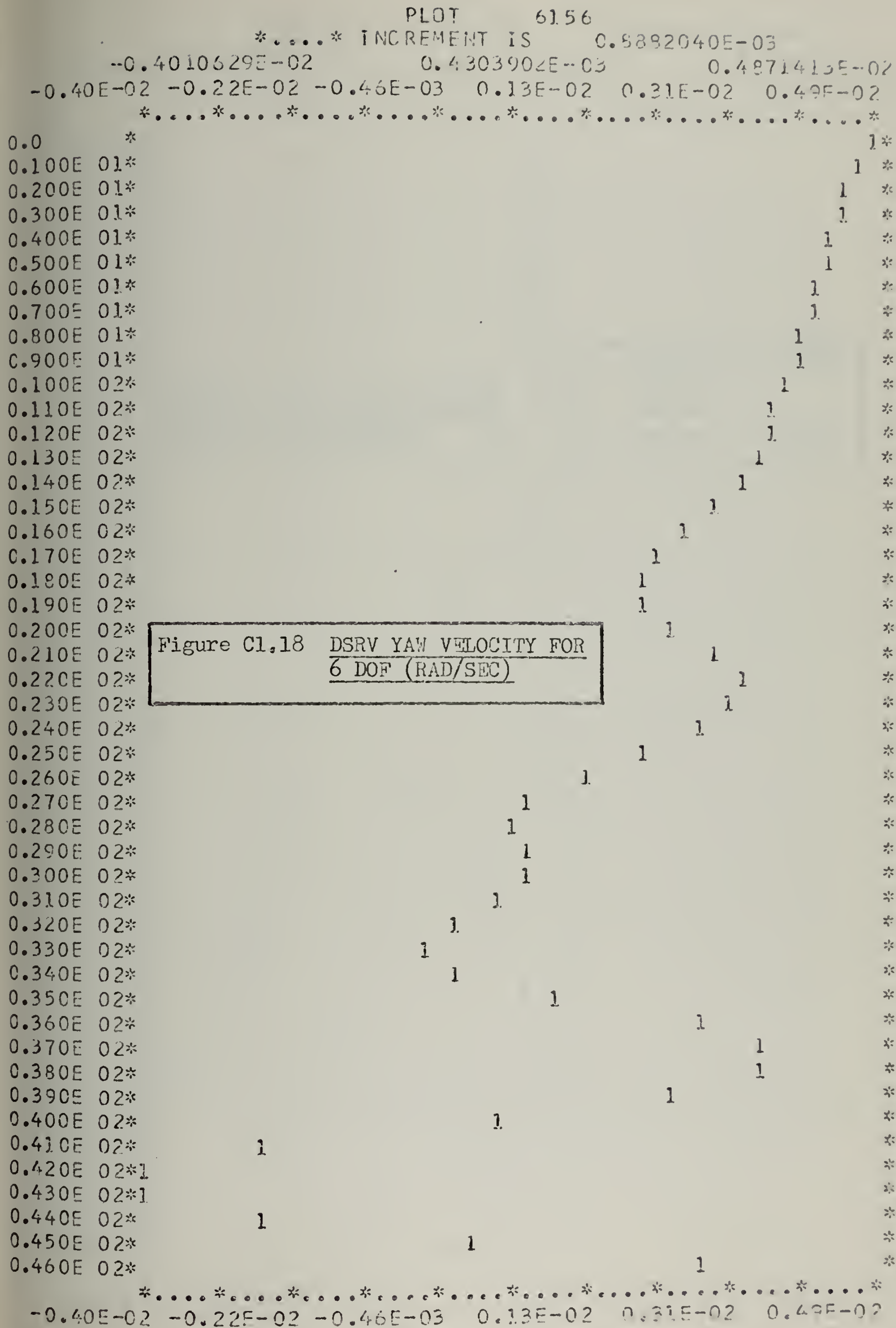


Figure C1.18 DSRV YAW VELOCITY FOR 6 DOF (RAD/SEC)

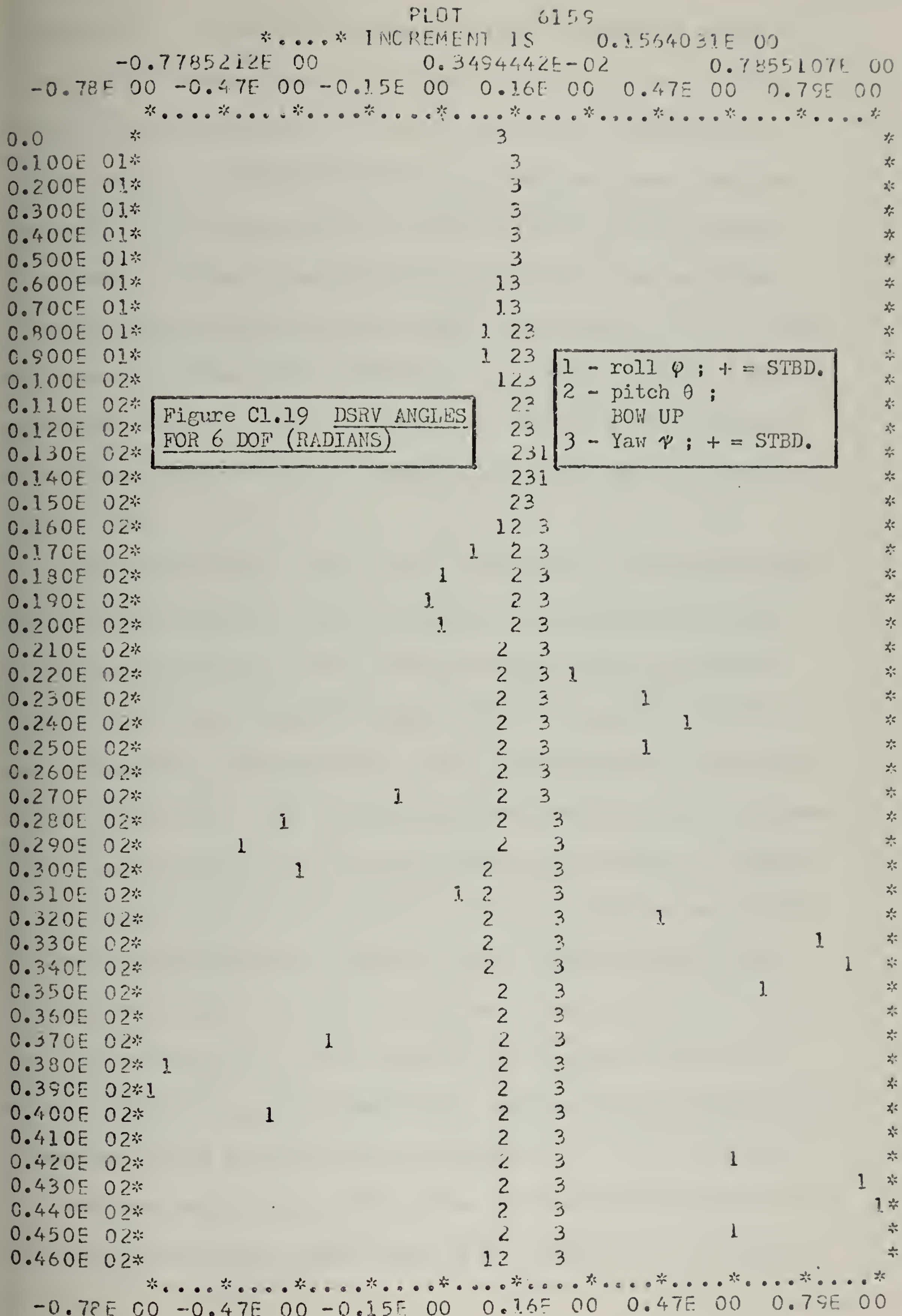


Figure C1.19 DSRV ANGLES FOR 6 DOF (RADIANS)

Appendix A12. Program MAIN is designed to run subroutine OVMOD of Section 5(D) through a sea trial (block), with or without v and w noises, in any combination of six DOF and with any combination of vehicle effectors. Model reference identification passes over the sea trial, while varying any two of the 839 DSRV coefficients and parameters, may then be made with the cost function values being calculated and contoured for each pass. A brief description of MAIN and some of its input data formats will be presented here. A sample input data deck for running the DSRV in 1 through 6 DOF blocks and identifying several selected coefficients in each block is given in Appendix A20.

Program MAIN begins with a list of declaration statements which include some variables (such as EH) which are not used for model reference contouring but which would be required for an extended Kalman filter to be included in MAIN. Next, the program reads in the initial data, sets variables to zero, and writes out the initial data for reference. Then preparations are made for running the block sea trial by initializing selected variables and reading in changes to selected variables for that block. Next, a complete description of the sea trial block is written out; one small portion of this description is shown in Figure C1.20, where the 6 DOF model is to be run with secondary drag, prop, shroud, and no-noise effectors and with coefficient X_{uu} to be identified. Most of the block description is written out in formats which are specific to the DSRV effectors.

The next major step in MAIN is the running and plotting of a sea trial using subroutines SEATR (Chapter C3, Appendix A17) and STOUT.

```
*************************************************************************
*                  BLOCK 6150 CHARACTERISTICS                            *
*                                                                        *
* MODEL  U V W P Q R   EFF TA  SE  SH  PR  TH  NO  CO  CB  XD           *
*        1 1 1 1 1 1        0   1   1   1   0   0   0   0   1           *
*                                                                        *
* DATA PASSES   2   DATA POINTS    47   NO. COEFFS       1               *
* TZERO   0.0           DT   0.1000E 01   NO. PARAM       1              *
*                                                                        *
* INITIALIZING-     M  ME  LF  LP  X  P  E  Q  R  ST  QN  RN  U  O      *
*   O=INITIALIZE   0   0   0   0 0 0 0 0 0  0   0   0  0  0            *
*                                                                        *
* COEFFICIENTS TO BE IDENTIFIED                                          *
*    XUU 1 XUV 0 XUW 0 XUP 0 XUQ 0 XUR 0 XVV 0 XVW 0                   *
*    XVP 0 XVQ 0 XVR 0 XWW 0 XWP 0 XWQ 0 XWR 0 XPP 0                   *
*    XPQ 0 XPR 0 XQQ 0 XQR 0 XRR 0 YUU 0 YUV 0 YUW 0                   *
*    YUP 0 YUQ 0 YUR 0 YVV 0 YVW 0 YVP 0 YVQ 0 YVR 0                   *
*    YWW 0 YWP 0 YWQ 0 YWR 0 YPP 0 YPQ 0 YPR 0 YQQ 0                   *
*    YQR 0 YRR 0 ZUU 0 ZUV 0 ZUW 0 ZUP 0 ZUQ 0 ZUR 0                   *
*    ZVV 0 ZVW 0 ZVP 0 ZVQ 0 ZVR 0 ZWW 0 ZWP 0 ZWQ 0                   *
*    ZWR 0 ZPP 0 ZPQ 0 ZPR 0 ZQQ 0 ZQR 0 ZRR 0 KUU 0                   *
*    KUV 0 KUW 0 KUP 0 KUQ 0 KUR 0 KVV 0 KVW 0 KVP 0                   *
*    KVQ 0 KVR 0 KWW 0 KWP 0 KWQ 0 KWR 0 KPP 0 KPQ 0                   *
*    KPR 0 KQQ 0 KQR 0 KRR 0 MUU 0 MUV 0 MUW 0 MUP 0                   *
*    MUQ 0 MUR 0 MVV 0 MVW 0 MVP 0 MVQ 0 MVR 0 MWW 0                   *
*    MWP 0 MWQ 0 MWR 0 MPP 0 MPQ 0 MPR 0 MQQ 0 MQR 0                   *
*    MRR 0 NUU 0 NUV 0 NUW 0 NUP 0 NUQ 0 NUR 0 NVV 0                   *
*    NVW 0 NVP 0 NVQ 0 NVR 0 NWW 0 NWP 0 NWQ 0 NWR 0                   *
*    NPP 0 NPQ 0 NPR 0 NQQ 0 NQR 0 NRR 0                               *
*                                                                        *
* PARAMETERS TO BE IDENTIFIED                                            *
*     72 0   73 0   74 0   75 0   76 0   77 0   78 0   79 0            *
*     80 0   81 0   82 0   83 0   84 0   85 0   86 0   87 0            *
*     88 0   89 0   90 0   91 0   92 0   93 0   94 0   95 0            *
*     96 0   97 0   98 0   99 0  229 0  230 0  231 0  232 0            *
*    233 0  234 0  235 0  236 0  237 0  238 0  239 0  240 0            *
*    241 0  242 0  243 0  244 0  245 0  246 0  247 0  248 0            *
*    249 0  250 0  251 0  252 0  253 0  254 0  255 0  256 0            *
*    257 0  258 0  259 0  260 0  261 0  262 0  263 0  277 0            *
*    278 0  279 0  280 0  281 0  285 0  286 0  287 0  288 0            *
*    289 0  302 0  303 0  304 0  305 0  306 0  310 0  311 0            *
*    312 0  313 0  320 0  321 0  322 0  323 0  324 0  325 0            *
*    332 0  333 0  334 0  335 0  336 0  337 0  338 0  339 0            *
*    340 0  341 0  352 0  353 0  354 0  355 0  359 0  360 0            *
*    361 0  362 0  374 0  375 0  376 0  377 0  381 0  382 0            *
*    383 0  384 0  386 0  387 0  388 0  389 0  390 0  391 0            *
*    572 0  573 0  574 0  575 0  576 0  577 0                          *
*                                                                        *
*************************************************************************
```

Figure C1.20 SAMPLE PORTION OF BLOCK DESCRIPTION FROM PROGRAM MAIN

The sea trial data may now be used in identification passes by model reference contouring or by extended Kalman filtering. The extended Kalman filtering section of MAIN was not completed prior to the writing of this thesis, but several applicable subroutines are included in Appendix A18.

The model reference contouring steps begin by reading in the indexes of the coefficients or parameters to be varied and contoured along with the weighting matrix for the cost function calculation. After writing out the characteristics of the coefficients or parameters being varied, a mathematical model sea trial is run using the inputs previously generated by subroutine SEATR and using subroutine MOREF (Appendix A17, Chapter C3). The trajectories generated by the "vehicle" in subroutine SEATR and the "model" in MOREF are then used to calculate the weighted integral square cost function. Finally, the values of the cost function are contoured and, if desired, vertical or horizontal slices of the contours are plotted. In addition, the logarithm of the cost function is calculated, contoured, and sliced if IOUT (3) is set to 1.

C1.3 DESCRIPTION OF THE INPUT DATA FOR MAIN

This description goes card by card through the sample input data deck of Appendix A19 showing the formats used and describing the important variables for each card.

Card #1 Format (20I4)

KS - Number of primary states

KME - Number of effector designators

KAV - Number of second degree coefficients

KNE - Number of effector group designators

MP - Location of 1st parameter in P

KMV - Number of the first parameter group index in ME

KMP - Number of parameter group designators in ME

KPF - Number of parameters in the final parameter group

KX - Total number of states

KPG - Number of elements in PG used in OVMOD

KU - Number of input variables in U

NHMAX - Number of input samples; 18 * 47

NIND - Number of integer changes per card

NIN - Number of variable changes per card

NST - Number of step function designators for UINPT

NOUT - Number of output designators

NBLT - Number of letters in word BLOCK

KNP - Number of parameters P

NMRT - Number of letters in word PASS

Card #2 Format (20I4)

MREF - Model reference designator

KFIL - Kalman filter designator

IONE - Initially set to 1

NCN - Number of values of 1st parameter to be contoured

NCD - Number of values of 2nd parameter to be contoured

NMR - Number of data points in model reference calculations

IABO - Initially 1

ITHRE - Initially 3

KUA - Location of angles in input U

Card #3 Format (4I5, 2E10.2)

ISIXT - Initially 15

ISEVT - Initially 16

KEH - Dimension of EH; for Kalman filtering

KEF - Dimension of E and F; for Kalman filtering

EPS - Program zero plus; 0^+

EMAX - Program maximum for the states

Card #4 Format (20I4)

NGPI - Number of elements in MGP(J), NGP(J)

MGP(J) - Locations of parameters to be described in groups

Card #5 Format (20I4)

NGP(J) - Number of parameters in each group to be described in block output

Card #6 Format (40I2)

M(J) - Initial designators for the structure selector

Card #7 Format (20I4)

ME(J) - Effector designators; Chapter D2

Cards #8 - 11 and 12 - 15 Format (40I2)

AV(J), XF(J) - Chapter D2; Appendix A2

Card #16 Format (40I2)

NE(J) - Chapter D2

Cards #17 - 80 Format (10F8.2)

P(J) - Vehicle parameter vector; in groups; Appendix A1

Cards #81 - 106 Format (10F8.2)

X(J) - Vehicle state vector; Appendix A1

Card #107 Format (10F8.2)

PG(J) - Vehicle gravity vector; Chapter D2

Cards #108 - 109 Format (10F8.2)

US(J) - Initial values of the vehicle inputs

Cards #110 - 135 Format (10F8.2)

XINC(J) - Model reference increments of the state variables; Appendix A1

Cards #136 - 194 Format (10F8.2)

PINC(J) - Model reference increments of the vehicle parameters; Appendix A1

Cards #195 - 246 Format 5(I5,F11.4)

J,XS(J) - Starting model reference state variables; Appendix A1

Cards #247 - 364 Format 5(I5,F11.4)

J,PS(J) - Starting model reference parameters; Appendix A1

Card #365 Format (20I4)

NB - Number of blocks to be processed in this computer run

Card #366 Format (80A1)

BLOCK - Test word to make sure that the above data has been read in properly

At this point the actual BLOCK and PASS data is included for the sea trials and model reference passes over the sea trial data. Each block may have multiple passes, but no pass may apply to more than one block. Only the cards for the first block and for the first pass will be described in detail here. The format for the other blocks and passes is the same. The blocks and passes in Appendix A20 are those used to generate the 1 through 6 degree of freedom sea trials of this chapter and many of the identification passes described in the next chapter. The MAIN program, its subroutines, and the input cards of Appendix A20 required 1 hour of IBM 360/65 computation time with

Fortran G (Fortran H won't compile MAIN due to its size.) and generated 30 thousand lines of output descriptions, plots, and contours.

Card #367 Format (20I4)

NBLOK - Block designation number

NPB -- Number of model reference passes in the block

The remainder are initializing integers; 0 = initialize that vector

Card #368 Format 10 (I4,I4)

K,M(K) - Card form for reading changes to the structure-selector vector M(6); this form is used in many of the following vectors and reads changes by index directly into the vector until a 0 index is found either within the format or on a following blank card.

Card #369 Format 10(I4,I4)

K,ME(K) -Changes to the effector selection vector

Card #370 Format (10F8.2)

TZ - Time zero for this block

DT - Time increment for this block

Card #371 Format (20I4)

NDT - Number of time increments in the sea trial

KF - Number of coefficients to be identified

KP -- Number of parameters to be identified

Card #372 Format 10(I4,I4)

K,LF(K)- Changes to the coefficient identification vector

Card #373 Format 10(I4,I4)

K,LP(K)- Changes to the parameter identification vector

Card #374 Format 5(I5,F11.4)

K,X(K) - Changes to the starting state vector for the sea trial

Card #375 Format 5(I5,F11.4)

K,P(K) - Changes to the starting parameter vector for the sea trial

Card #376 Format 5(I5,F11.4)

K,Q(K) - Changes to the w noise generation vector

Card #377 Format 5(I5,F11.4)

K,R(K) - Changes to the v noise generation vector

Card #378 Format 5(I5,F11.4)

K,U(K) - Changes to the initial input vector

Cards #379 - 381 Format 5(I5, F11.4)

K,TST(K) - Changes to the input step function starting times vector

Cards #382 - 384 Format 5(I5,F11.4)

K,UST(K) - Changes to the input step function amplitudes vector

Card #385 Format 5(I5,F11.4)

K,XH(K)- Changes to the starting state vector for model reference contouring or for extended Kalman filtering

Card #386 Format 5(I5,F11.4)

K,PH(K)- Changes to the starting parameter vector

Card #387 Format 5(I5,F11.4)

K,EH(K)- Changes to the starting error covariance matrix

Card #388 Format 5(I5,F11.4)

K,QN(K)- Changes to the extended Kalman filter Q matrix

Card #389 Format 5(I5,F11.4)

K,RN(K)- Changes to the extended Kalman filter R matrix, or to the model reference weighting matrix

Card #390 Format 10(I4,I4)

K,IOUT(K) - Changes to the output designation vector

IOUT(1) - Logarithmic contours

IOUT(2) - Input function plots

IOUT(3) - Contour slices plotted

IOUT(4) - Vehicle angles plotted

IOUT(5) - X, XH, P, PH, EH data listed

Card #391 Format (80A1)

BLOCK - Test card at the end of each set of block data.

The pass data for this block is inserted here. The first pass in each block of Appendix A20 is set up to contour or identify the coefficient X_{uu} and the parameter $X_{\dot{u}}$.

Card #392 Format (20I4)

KF - Number of coefficients to be identified in this pass

KP - Number of parameters to be identified in this pass

Card #393 Format 10(I4,I4)

K,LF(K)- Indexes of the coefficients (part of the state vector X) to be identified in this pass

Card #394 Format 10(I4,I4)

K,LP(K)- Indexes of the parameters to be identified in this pass

Card #395 Format 5(I5,F11.4)

K,RN(K)- Weighting matrix changes for this model reference pass

Card #396 Format (80A1)

PASS - Test card at the end of each set of pass data

This chapter has presented plots and discussions of the dynamic behavior of the DSRV for a simulated ramp input in 1 through 6 degree of freedom models. Then the MAIN program for generating these sea trials and for identifying selected vehicle parameters using model

reference contouring was described briefly with regard to its programming and described in detail with regard to its input data cards. The next chapter discusses the identification results from using the model reference contouring technique on multiple degree of freedom sea trials and mathematical models.

MODEL REFERENCE CONTOURS FOR SELECTED DSRV 6 * 6 PARAMETERS

This chapter is the culmination of the practical results of this thesis, and it shows that the technique of model reference contouring works extremely well for identifying DSRV parameters in models containing as many as six degrees of freedom. The model reference contouring technique used here has been described in detail in Chapters N2 and P3, and the structure-selective DSRV mathematical model has been presented in Section 5(D). The MAIN program and its input data cards have been discussed in Chapter C1.

The coefficient and parameter studies to be described in this chapter represent only a small part of the useful results for the DSRV which could be obtained using program MAIN and subroutine OVMOD. A large portion of the effort in this thesis development was devoted to the writing and debugging of these programs, and the studies made here were done so primarily for the purpose of showing that the method and the programs are valid for the DSRV. The results of these studies are presented in the remainder of this chapter by discussing the parameters one after another starting with those identified in the 1 through 6 DOF sea trials of Chapter C1.1.

X_{uu} and P(229)

DSRV coefficient X_{uu} (-16.7) is the vehicle primary drag coefficient and P(229) (755.) is the main propeller thrust coefficient. Both of these parameters were studied in great detail in Section 4(P) for different input functions and different identification methods. The eleven-by-eleven contours for the identification of X_{uu} and P(229)

using the simulated ramp input function, no noise, and the DSRV 6 DOF model (responses in Chapter C1.1) are shown in Figure C2.1.

There are fewer contour points in Figure C2.1 than there were in the contours of Chapter P3 because each point is more costly to generate for 6 DOF and because it is still possible to identify the parameters using fewer points if logarithmic contours are also plotted. Figure C2.1 has a shape somewhat similar to that of Figure P3.13 for the staircase function input except that the bottom of the contours in Figure C2.1 is much flatter, making X_{uu} hard to identify. Five vertical slices of Figure C2.1 are shown in Figure C2.2, and five horizontal slices of Figure C2.1 are shown in Figure C2.3. These slices show the vertical "trough" shape of the contours in Figure C2.1 clearly, along with the minimum (10^{-9}) and maximum (5.9) values.

One way to greatly accentuate the minimum values for model reference contouring is to contour $\log_e C(\underline{p})$ vs the parameter values as in Figure C2.4 (D-13). Five vertical slices of Figure C2.4 are shown in Figure C2.5, and five horizontal slices of Figure C2.4 are shown in Figure C2.6. The minimum value in Figure C2.4 is seen to be exactly at (-16.7, 755.), whereas in Figure C2.1 only P(229) is identifiable (755.).

Identifications were made of X_{uu} and P(229) using the 1 through 6 DOF runs described in Chapter C1.1. The contours and their logarithms were almost identical in shape for these six cases to Figures C2.1 and C2.4, and the minimum and maximum values corresponded very closely as seen in Table C2.1. These results show conclusively that the identifiability studies of X_{uu} and P(229) for the 1 DOF model may be directly

Figure C2.1 X_{uu} AND P (229) CONTOURS, 6 DOF, SIMULATED RAMP INPUT

```
                          CONTOUR       6152
                    *....* INCREMENT IS       0.7549998E 02
          0.3775000E 03          0.7549998E 03          0.1132500E 04
     0.38E 03  0.53E 03  0.68E 03  0.83E 03  0.98E 03  0.11E 04
             *....*....*....*....*....*....*....*....*....*....*
-0.250E 02*L    E    9    5    3    2    2    3    6    A    G*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.232E 02*L    E    8    5    2    2    2    4    7    B    G*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.214E 02*L    E    8    5    2    2    2    4    7    B    H*
          :                                                  :
          :                                                  :
          :                                                  :
-0.200E 02*L    D    8    5    2    1    2    4    7    B    H*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.182E 02*L    D    8    4    2    1    2    4    7    C    J*
          :                                                  :
          :                                                  :
          :                                                  :
-0.167E 02*L    D    8    4    2    1    2    4    8    C    J*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.149E 02*L    D    8    4    2    1    2    4    8    D    K*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.131E 02*K    D    8    4    2    1    2    5    8    D    K*
          :                                                  :
          :                                                  :
          :                                                  :
-0.116E 02*K    D    8    4    2    2    2    5    9    E    L*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.980E 01*K    C    7    4    2    2    3    5    9    E    M*
          :                                                  :
          :                                                  :
          :                                                  :
-0.835E 01*K    C    7    4    2    2    3    5    9    F    M*
             *....*....*....*....*....*....*....*....*....*....*
     0.38E 03  0.53E 03  0.68E 03  0.83E 03  0.98E 03  0.11E 04
```

```
                              PLOT       6152
                    *....* INCREMENT IS      0.5378589E 00
          0.1363400E-09          0.2939294E 01          0.5878589E 01
   0.14E-09  0.12E 01  0.24E 01  0.35E 01  0.47E 01  0.59E 01
         *....*....*....*....*....*....*....*....*....*....*
-0.250E 02*3   4   2                              5          1   *
          :                                                      :
          :                                                      :
          :                                                      :
          :                                                      :
-0.232E 02*3    4   2                              5          1  *
          :                                                      :
          :                                                      :
          :                                                      :
          :                                                      :
-0.214E 02*3    4 2                                 5        1   *
          :                                                      :
          :                                                      :
          :                                                      :
-0.200E 02*3     42                                   5      1   *
          :                                                      :
          :                                                      :
          :                                                      :
          :                                                      :
-0.182E 02*3     42                                    5     1   *
          :                                                      :
          :                                                      :
          :                                                      :
-0.167E 02*3     42                                     5   1    *
          :                                                      :
          :                                                      :
          :                                                      :
          :                                                      :
-0.149E 02*3     24                                       5 1    *
          :                                                      :
          :                                                      :
          :                                                      :
          :                                                      :
-0.131E 02*3     24                                        5     *
          :                                                      :
          :                                                      :
          :                                                      :
-0.116E 02*3     2 4                                       1 5   *
          :                                                      :
          :                                                      :
          :                                                      :
          :                                                      :
-0.980E 01*3     2 4                                       1   5 *
          :                                                      :
          :                                                      :
          :                                                      :
-0.835E 01*3    2   4                                     1     5*
         *....*....*....*....*....*....*....*....*....*....*
   0.14E-09  0.12E 01  0.24E 01  0.35E 01  0.47E 01  0.59E 01
```

Figure C2.2 FIVE VERTICAL SLICES OF FIGURE C2.1 (LEFT TO RIGHT)

```
                         PLOT      6152
             *....* INCREMENT IS      0.5878589E 00
      0.1363400E-09          0.2939294E 01          0.5878589E 01
  0.14E-09  0.12E 01  0.24E 01  0.35E 01  0.47E 01  0.59E 01
          *....*....*....*....*....*....*....*....*....*....*
0.378E 03*                                              54321   *
         :                                                      :
         :                                                      :
         :                                                      :
         :                                                      :
0.460E 03*                                  5421                *
         :                                                      :
         :                                                      :
         :                                                      :
         :                                                      :
0.542E 03*                     5431                             *
         :                                                      :
         :                                                      :
         :                                                      :
0.607E 03*          5431                                        *
         :                                                      :
         :                                                      :
         :                                                      :
         :                                                      :
0.689E 03* 52                                                   *
         :                                                      :
         :                                                      :
         :                                                      :
0.755E 03*5                                                     *
         :                                                      :
         :                                                      :
         :                                                      :
         :                                                      :
0.837E 03*135                                                   *
         :                                                      :
         :                                                      :
         :                                                      :
         :                                                      :
0.919E 03*    1  34  5                                          *
         :                                                      :
         :                                                      :
         :                                                      :
0.985E 03*             12 34   5                                *
         :                                                      :
         :                                                      :
         :                                                      :
         :                                                      :
0.107E 04*                        1 2  3 4  5                   *
         :                                                      :
         :                                                      :
         :                                                      :
0.113E 04*                                          1   2 3  4     5*
          *....*....*....*....*....*....*....*....*....*....*
  0.14E-09  0.12E 01  0.24E 01  0.35E 01  0.47E 01  0.59E 01
```

Figure C2.3 FIVE HORIZONTAL SLICES OF FIGURE C2.1(TOP TO BOTTOM)

```
                          CONTOUR        6152
                   *....* INCREMENT IS      0.7549998E 02
       0.3775000E 03           0.7549998E 03          0.1132500E 04
   0.38E 03   0.53E 03   0.68E 03   0.83E 03   0.98E 03   0.11E 04
           *....*....*....*....*....*....*....*....*....*....*
-0.250E 02*M     M     M     L     K     G     J     L     L     M     M*
          :                                                           :
          :                                                           :
          :                                                           :
          :                                                           :
-0.232E 02*M     M     M     L     K     G     J     L     L     M     M*
          :                                                           :
          :                                                           :
          :                                                           :
          :                                                           :
-0.214E 02*M     M     M     L     K     G     J     L     L     M     M*
          :                                                           :
          :                                                           :
          :                                                           :
-0.200E 02*M     M     M     L     K     F     K     L     L     M     M*
          :                                                           :
          :                                                           :
          :                                                           :
          :                                                           :
-0.182E 02*M     M     M     L     K     E     K     L     L     M     M*
          :                                                           :
          :                                                           :
          :                                                           :
-0.167E 02*M     M     M     L     K     J     K     L     M     M     M*
          :                                                           :
          :                                                           :
          :                                                           :
          :                                                           :
-0.149E 02*M     M     M     L     K     E     K     L     M     M     M*
          :                                                           :
          :                                                           :
          :                                                           :
          :                                                           :
-0.131E 02*M     M     M     L     K     F     K     L     M     M     M*
          :                                                           :
          :                                                           :
          :                                                           :
-0.116E 02*M     M     M     L     K     G     K     L     M     M     M*
          :                                                           :
          :                                                           :
          :                                                           :
          :                                                           :
-0.980E 01*M     M     L     L     J     G     K     L     M     M     M*
          :                                                           :
          :                                                           :
          :                                                           :
-0.835E 01*M     M     L     L     J     H     K     L     M     M     M*
           *....*....*....*....*....*....*....*....*....*....*
   0.38E 03   0.53E 03   0.68E 03   0.83E 03   0.98E 03   0.11E 04
```

Figure C2.4 LOGARITHMIC CONTOURS OF FIGURE C2.1

```
                                 PLOT      6152
                        *....* INCREMENT IS     0.2448718E 01
        -0.2271587E 02          -0.1047228E 02          0.1771317E 01
  -0.23E 02 -0.18E 02 -0.13E 02 -0.80E 01 -0.31E 01  0.18E 01
            *....*....*....*....*....*....*....*....*....*....*
-0.250E 02*                                            3      42  5 *
          :                                                         :
          :                                                         :
          :                                                         :
          :                                                         :
-0.232E 02*                                           3       42  5 *
          :                                                         :
          :                                                         :
          :                                                         :
          :                                                         :
-0.214E 02*                                          3        42  5 *
          :                                                         :
          :                                                         :
          :                                                         :
-0.200E 02*                                        3          42  5 *
          :                                                         :
          :                                                         :
          :                                                         :
          :                                                         :
-0.182E 02*                                      3            42  5 *
          :                                                         :
          :                                                         :
          :                                                         :
-0.167E 02*3                                                  42  5 *
          :                                                         :
          :                                                         :
          :                                                         :
          :                                                         :
-0.149E 02*                                      3            24  5 *
          :                                                         :
          :                                                         :
          :                                                         :
          :                                                         :
-0.131E 02*                                        3          24  5 *
          :                                                         :
          :                                                         :
          :                                                         :
-0.116E 02*                                          3        24  5 *
          :                                                         :
          :                                                         :
          :                                                         :
          :                                                         :
-0.980E 01*                                            3      24  5 *
          :                                                         :
          :                                                         :
          :                                                         :
-0.835E 01*                                             3     24  15*
            *....*....*....*....*....*....*....*....*....*....*
  -0.23E 02 -0.18E 02 -0.13E 02 -0.80E 01 -0.31E 01  0.18E 01
```

Figure C2.5 FIVE VERTICAL SLICES OF FIGURE C2.4

```
                              PLOT       6152
                    *....* INCREMENT IS     0.2448718E 01
         -0.2271587E 02       -0.1047228E 02        0.1771317E 01
  -0.23E 02 -0.18E 02 -0.13E 02 -0.80E 01 -0.31E 01  0.19E 01
           *....*....*....*....*....*....*....*....*....*....*
0.378E 03*                                                  5 *
         :                                                    :
         :                                                    :
         :                                                    :
         :                                                    :
0.460E 03*                                                 51 *
         :                                                    :
         :                                                    :
         :                                                    :
         :                                                    :
0.542E 03*                                                5   *
         :                                                    :
         :                                                    :
         :                                                    :
0.607E 03*                                             53     *
         :                                                    :
         :                                                    :
         :                                                    :
         :                                                    :
0.689E 03*                                          53        *
         :                                                    :
         :                                                    :
         :                                                    :
0.755E 03*3                                 4   5             *
         :                                                    :
         :                                                    :
         :                                                    :
         :                                                    :
0.837E 03*                                         1245       *
         :                                                    :
         :                                                    :
         :                                                    :
         :                                                    :
0.919E 03*                                             35     *
         :                                                    :
         :                                                    :
         :                                                    :
0.985E 03*                                                45  *
         :                                                    :
         :                                                    :
         :                                                    :
         :                                                    :
0.107E 04*                                                 45 *
         :                                                    :
         :                                                    :
         :                                                    :
0.113E 04*                                                  45*
           *....*....*....*....*....*....*....*....*....*....*
  -0.23E 02 -0.18E 02 -0.13E 02 -0.80E 01 -0.31E 01  0.19E 01
```

Figure C2.6 FIVE HORIZONTAL SLICES OF FIGURE C2.4

DOF	1	2	3	4	5	6
Minimum C(p)	10^{-12}	10^{-12}	10^{-13}	10^{-9}	10^{-11}	10^{-9}
Maximum C(p)	4.9	6.0	4.9	6.1	6.1	5.9

Table C2.1 CONTOUR VALUES FOR X_{uu} AND P(229), 1 THROUGH 6 DOF MODELS

extended to apply to the 6 DOF model. Therefore, if noiseless real DSRV data were available for this maneuver and if the structure of the DSRV model were exactly the same as the real vehicle, then the parameters X_{uu} and P(229) identified by model reference 1 DOF contouring would be the correct ones for the 6 DOF vehicle.

The same conclusions cannot be reached when noises are introduced into multiple degree of freedom models and data. Only a few multiple DOF runs were made with noise for this thesis, and those indicated that for these two parameters and several others the %v and %w noises tend to make the parameters significantly less identifiable in multiple DOF models than in single DOF models due to the noise being coupled through the various states throughout the maneuver. More extensive noise studies could now be made using the programs of this thesis to determine how the noise is coupled into various states and parameters.

$X_{\dot{u}}$ (-144)

The next coefficient studied in detail for multiple DOF models was the DSRV surge "added mass" coefficient $X_{\dot{u}}$. This term enters into the DSRV surge 1 DOF model as shown in equation C2.1. The fact that $X_{\dot{u}}$ is only 3.3% of the vehicle mass m tends to obscure it slightly when

$$(m - X_{\dot{u}})\,\dot{u} = X_{u|u|}\;u|u| + P(229)\;n|n| + P(230)\;n\;u + P(231)u^2 \qquad \text{C2.1}$$

Where: m = DSRV mass; 4363. slugs

$X_{\dot{u}}$ = DSRV added mass in surge; -144. slugs

$X_{u|u|}$ = Primary drag coefficient; -16.7

P(229) = Propellor primary thrust parameter; 755.

P(230) = Propellor cross thrust parameter; -58.

P(231) = Propellor surge thrust reduction parameter; -3.8

n = Propellor revolutions per second

u = Surge velocity ft/sec

attempts are made to identify it, especially for noisy sea trials.

The 6 DOF contours for X_{uu} (-16.7) and $X_{\dot{u}}$ (-144.) for the DSRV are shown in Figure C2.7, where the minimum value is 10^{-9} and the maximum value is 0.03. The 1 through 6 DOF contours are almost identical to Figure C2.7, with the same maximum-minimum values. The very flat bottom of the trough in this figure shows that $X_{\dot{u}}$ is not easily identified, and the partial linear dependence (Figure N4.4, #9) indicates that X_{uu} and $X_{\dot{u}}$ are coupled for this maneuver and could not be properly identified separately. Noise studies have shown that for 3%v and 3%w noises, X_{uu} can be identified to within 3% but $X_{\dot{u}}$ cannot be identified to better than 50% accuracy. However, even at 1.5 times its true value $X_{\dot{u}}$ is still only 5% of m in equation C2.1.

P(230) AND P(231) IN EQUATION C2.1

The secondary propellor thrust parameters P(230) and P(231) in equation C2.1 reveal more dramatically the results of parameter coupling

```
                          CONTOUR      6151
                    *....* INCREMENT IS     0.1440001E 02
      -0.2160000E 03          -0.1440000E 03          -0.7199991E 02
  -0.22E 03 -0.19E 03 -0.16E 03 -0.13E 03 -0.10E 03 -0.72E 02
           *....*....*....*....*....*....*....*....*....*....*
-0.250E 02*J    G    E    C    B    9    8    7    5    5    4*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.232E 02*E    C    B    9    8    6    5    4    4    3    3*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.214E 02*B    9    8    6    5    4    3    3    2    2    2*
          :                                                  :
          :                                                  :
          :                                                  :
-0.200E 02*8    7    5    4    3    3    2    2    2    2    2*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.182E 02*5    4    4    3    2    2    2    2    2    2    2*
          :                                                  :
          :                                                  :
          :                                                  :
-0.167E 02*4    3    2    2    2    1    2    2    2    3    4*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.149E 02*2    2    2    2    2    2    2    3    4    5    6*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.131E 02*2    2    2    2    2    3    4    5    6    7    9*
          :                                                  :
          :                                                  :
          :                                                  :
-0.116E 02*2    2    2    3    4    4    6    7    8    A    C*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.980E 01*3    3    4    5    6    7    8    A    C    E    G*
          :                                                  :
          :                                                  :
          :                                                  :
-0.835E 01*4    5    6    7    8    A    C    E    G    K    M*
           *....*....*....*....*....*....*....*....*....*....*
  -0.22E 03 -0.19E 03 -0.16E 03 -0.13E 03 -0.10E 03 -0.72E 02
```

Figure C2.7 X_{uu} AND $X_{\dot{u}}$ CONTOURS, 6 DOF, SIMULATED RAMP INPUT

(i.e. linear or functional dependence). The 4 DOF contours for these two parameters are presented in Figures C2.8 and C2.9, and both show partially linear coupling between X_{uu} and P(230) and between X_{uu} and P(231). The logarithmic contours in both of these cases permitted perfect identifications of both propellor parameters, but it still may be concluded that for this maneuver the values of X_{uu}, $X_{\dot{u}}$, P(230), and P(231) identified in pairs from noisy data would probably not be the true values. In other words, to find these four parameters when the data is noisy, a scheme (such as extended Kalman filtering) which varies all four parameters at once should be used. However, it is extremely helpful that model reference contouring provides conclusions of this type. Contours identical to Figures C2.8 and C2.9 were also generated using 1, 2, and 3 DOF models, indicating that the above information could have been obtained from 1 DOF studies.

3 DOF IDENTIFICATIONS; SURGE, HEAVE, PITCH; NO NOISE

The DSRV parameters which were identified using the simulated ramp input and 3 DOF model are listed in Table C2.2. The author was somewhat surprised to see the amount of information in terms of parameters which could be extracted from three plots (Figures C1.2, C1.5, and C1.6) containing 46 points each. There are probably several more parameters which could have been identified using this same set of data. The real identification "power" comes from the fact that the structure used to generate the data is exactly (no noises) the same as that in the model and from the fact that the parameters were varied two at a time with the ones not being varied having been set to their "true" values for the identification pass. Another helpful factor

Figure C2.8 X_{uu} AND P (230) CONTOURS, 4 DOF, SIMULATED RAMP INPUT

```
                         CONTOUR        4153
                 *....* INCREMENT IS      0.5799999E 01
        -0.8700000E 02         -0.5800002E 02          -0.2899997E 02
   -0.87E 02 -0.75E 02 -0.64E 02 -0.52E 02 -0.41E 02 -0.29E 02
              *....*....*....*....*....*....*....*....*....*....*
-0.250E 02*H    D    A    7    5    3    2    2    2    2    4*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.232E 02*G    C    9    6    4    3    2    2    2    3    5*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.214E 02*E    A    7    5    3    2    2    2    2    4    6*
          :                                                  :
          :                                                  :
          :                                                  :
-0.200E 02*C    9    6    4    3    2    1    2    3    5    7*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.182E 02*B    8    5    3    2    2    2    2    4    6    9*
          :                                                  :
          :                                                  :
          :                                                  :
-0.167E 02*9    7    4    3    2    1    2    3    5    7    A*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.149E 02*8    6    4    2    2    2    2    4    6    8    C*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.131E 02*7    5    3    2    1    2    3    4    7    A    E*
          :                                                  :
          :                                                  :
          :                                                  :
-0.116E 02*6    4    2    2    2    2    3    6    8    C    G*
          :                                                  :
          :                                                  :
          :                                                  :
          :                                                  :
-0.980E 01*5    3    2    2    2    3    4    7    A    E    K*
          :                                                  :
          :                                                  :
          :                                                  :
-0.835E 01*4    2    2    2    2    3    5    8    C    G    M*
              *....*....*....*....*....*....*....*....*....*....*
   -0.87E 02 -0.75E 02 -0.64E 02 -0.52E 02 -0.41E 02 -0.29E 02
```

Figure C2.9 X_{uu} AND P(231) CONTOURS, 4 DOF, SIMULATED RAMP INPUT

```
                         CONTOUR      4154
                 *....* INCREMENT IS      0.3799999E 00
     -0.5700000E 01         -0.3800001E 01          -0.1899999E 01
  -0.57E 01 -0.49E 01 -0.42E 01 -0.34E 01 -0.27E 01 -0.19E 01
            *....*....*....*....*....*....*....*....*....*....*
-0.250E 02*K    J    G    F    E    D    C    B    A    9    9*
          :                                                   :
          :                                                   :
          :                                                   :
-0.232E 02*E    D    C    B    A    9    8    7    7    6    5*
          :                                                   :
          :                                                   :
          :                                                   :
          :                                                   :
-0.214E 02*A    9    8    7    6    6    5    5    4    4    3*
          :                                                   :
          :                                                   :
          :                                                   :
-0.200E 02*6    6    5    4    4    3    3    3    2    2    2*
          :                                                   :
          :                                                   :
          :                                                   :
          :                                                   :
-0.182E 02*4    3    3    3    2    2    2    2    2    1    1*
          :                                                   :
          :                                                   :
          :                                                   :
-0.167E 02*2    2    2    2    2    1    2    2    2    2    2*
          :                                                   :
          :                                                   :
          :                                                   :
          :                                                   :
-0.149E 02*1    1    2    2    2    2    2    3    3    3    4*
          :                                                   :
          :                                                   :
          :                                                   :
          :                                                   :
-0.131E 02*2    2    2    3    3    4    4    5    5    6    7*
          :                                                   :
          :                                                   :
          :                                                   :
-0.116E 02*3    4    4    5    5    6    7    8    9    9    A*
          :                                                   :
          :                                                   :
          :                                                   :
          :                                                   :
-0.980E 01*6    6    7    8    9    A    B    C    D    E    F*
          :                                                   :
          :                                                   :
          :                                                   :
-0.835E 01*9    A    B    C    D    F    G    H    K    L    M*
            *....*....*....*....*....*....*....*....*....*....*
  -0.57E 01 -0.49E 01 -0.42E 01 -0.34E 01 -0.27E 01 -0.19E 01
```

Parameter	Location	True Value	Comments
x_{shroud}	P(150)	-24.59 ft.	Location of the shroud; perfect identification
$M_{\dot{q}}$	P(29)	$-4.1*10^5$	Perfect identification
X_{wq}	X(19)	-3600.	Perfect identification
Z_q	P(17)	-770.	Perfect identification
X_{uu}	X(7)	-16.7	Perfect identification; used log contours with P(229)
$X_{\dot{u}}$	P(1)	-144.	Perfect identification; highly noise susceptible
P(229)	P(229)	755.	Perfect identification; propellor thrust coefficient
P(230)	P(230)	-58.	Log contour identification; linear dependence with X_{uu}
P(231)	P(231)	-3.8	Log contour identification; linear dependence with X_{uu}
X_{qq}	X(21)	-770.	Perfect identification
$M_{\dot{w}}$	P(27)	-770.	Perfect identification
Z_{uu}	X(49)	2.43	Log contour identification
W_o	P(578)	140369 lbf.	Perfect identification; DSRV equilibrium weight
Z_{uw}	X(52)	-55.9	Perfect identification
Z_{uq}	X(59)	-444.	Perfect identification
Z_{ww}	X(54)	-207.	Perfect identification
M_{uu}	X(91)	-66.	Perfect identification
M_{uw}	X(94)	2760.	Perfect identification
M_{uq}	X(101)	-26600	Perfect identification
z_G	P(581)	0.1335 ft.	Log contour identification; location of C.G. on DSRV
$Z_{\dot{w}}$	P(15)	-3600.	Perfect identification

Table C2.2 DSRV PARAMETERS IDENTIFIED USING 3 DOF; NO NOISE

was that without the very fast roll dynamics of other DOF models, the Euler integrator with a 1 second time step was able to more accurately solve the 3 DOF model. In any case the validity of the technique and of the computer programs has been demonstrated for multiple degree of freedom models.

This chapter has presented identifications in multiple degrees of freedom which validate the model reference contouring technique as a means of studying parametric identifiability for combinations of two parameters at a time. This essentially completes the presentation of practical results from the studies of this thesis. The next chapter describes several of the subroutines used in these multiple degree of freedom studies with either model reference contouring or extended Kalman filtering.

DESCRIPTION OF SEVERAL SUBROUTINES FOR USE IN THE MAIN PROGRAM

This chapter very briefly describes the input-output characteristics of some subroutines developed for use in the main model reference program (written and used in this thesis) and for use in an extended Kalman filtering program (uncompleted).

C3.1 SUBROUTINES FOR MODEL REFERENCE CONTOURING

The detailed statements for subroutines SEATR, UINPT, MOREF, and STOUT are presented in Appendix A17.

SEATR (U, P, X, XN)

This subroutine computes the sea trial trajectories (Z, ZN) for an ocean vehicle OVMOD in a structure-selected combination of 6 degrees of freedom using an Euler integrator. First the inputs U and the time vector T are generated by subroutine UINPT. Then the ocean vehicle primary state vector X and noisy primary state vector XN are propagated from point to point over the length of the sea trial using simple Euler integration. These states are tested after each calculation to ensure stability; if the equation becomes unstable, integer I is set to 0, and the sea trial is terminated. Subroutine SEATR also contains internally the capability (ME (19) = 1) of integrating the resulting vehicle angular velocities to produce the true vehicle angles which are used throughout the sea trial. Noise may be added to the sea trial data both in SEATR (V - noise) and in OVMOD (W - noise).

UINPT (TS, US, UZ, DT, TZ, T, U)

Subroutine UINPT computes a time vector (T) and a set of

combined, ten-step-function inputs U for use in a sea trial. The starting time and input values are designated by TZ and UZ, and the ten-step-function times and amplitudes are designated by TS and US. The time increment is given by DT, and the values of the starting times TS must be greater than or equal to 0 for the steps to be included in the sea trial.

MOREF (I, P, X)

This subroutine is almost identical to subroutine SEATR except that no noise is added to the sea trial data, only one set of trajectories (X) are calculated, and no input functions are computed. These two subroutines could be combined into a single subroutine if the proper selection features were included in the combined subroutine.

STOUT (T, U, Z, ZN, IU, N, NP)

This subroutine is used to generate plots of noisy (ZN) and noiseless (Z) sea trial trajectories, vehicle angles (U), and vehicle input functions (U). These plots are only generated if the corresponding states (M) and effectors (ME) were used in the original sea trial. Several output comments specific to the DSRV are included in this subroutine.

C3.2 SUBROUTINES FOR EXTENDED KALMAN FILTERING

The general, structure-selective, six degree of freedom extended Kalman filtering section of the program MAIN in Chapter C1 was not completed, but the following subroutines which are required for that program were written by the author. These subroutines are included here because they are completely general and may be useful to anyone who is using the extended Kalman filter of Steps N3.9 through N3.16.

The subroutines EHDOT, KGAIN, and UPDAT in Appendix A18 are completely generalized versions of subroutines PROP, EFUN, GAIN, and UPDAT in Appendix A14.

EHDOT (X, U, P, E, XD, ED)

Subroutine EHDOT computes both $\dot{E}$ and $\hat{\dot{\underline{x}}}$ of equations N3.15 and N3.14 using a general ocean vehicle model OVMOD for $\hat{\dot{\underline{x}}}$ = XD and its general state and parameter gradient OVDER for F. The subroutine then combines the compressed versions of F and E (Chapter N3) and the compressed $\underline{w}_n$ noise covariance matrix Q (diagonal) to compute $\dot{E}$ = ED for use in the propagation of the error covariance matrix and the state estimates by the main program.

KGAIN (E, KB, NF, EG)

This subroutine calculates a special form of equation N3.16 in Step N3.12 using the H matrix in equation N3.21 and the diagonal values for the R matrix. This results in the calculation of the general filter gain matrix EG using the error covariance matrix E (compressed), the number of primary states KB, the matrix R (in COMMON), and the total number of primary states, coefficients, and parameters NF. If the matrix to be inverted in equation N3.16 is found to be singular, the diagonal is inverted (modified if necessary) and a message is written stating that this was done.

UPDAT (Z, X, E, EG, XH, EH, KB, NF)

This subroutine computes the updated values of the states XH and error covariance matrix EH based upon the propagated states X, the propagated error covariance matrix E, the noisy measurement of the states Z, and the extended Kalman filter gain matrix EG. The equations solved by this subroutine are equations N3.17 and N3.18 in

Steps N3.13 and N3.14.

This chapter has presented brief explanations of subroutines SEATR, UINPT, MOREF, and STOUT for use in model reference contouring and of subroutines EHDOT, KGAIN, and UPDAT for use in extended Kalman filtering. This section has presented the one through six degree of freedom studies of the identifiability characteristics of several selected DSRV coefficients and parameters using the MAIN program of Appendix A19 and the DATA of Appendix A20. These studies have shown that for the DSRV the most significant degree of freedom is roll and that the model reference contouring technique is valid for up through six degree of freedom motions of the DSRV. The next section presents a brief summary of this thesis and a description of several areas for further study.

SECTION 7

SUMMARY AND CONCLUSION (S)

S1 THESIS SUMMARY AND CONCLUSIONS

S2 AREAS FOR FURTHER STUDY

"NO PLEASURE IS COMPARABLE TO THE STANDING UPON THE VANTAGEGROUND OF TRUTH." FRANCIS BACON (1561 - 1626)

THE TECHNIQUES OF MODEL REFERENCE CONTOURING AND EXTENDED KALMAN FILTERING ARE VALID AND USEFUL FOR IDENTIFYING COEFFICIENTS AND PARAMETERS IN OCEAN VEHICLE NONLINEAR DYNAMIC MATHEMATICAL MODELS USING NOISY INPUT-OUTPUT DATA.

CHAPTER S1

THESIS SUMMARY AND CONCLUSIONS

The studies and results of this thesis are most applicable to the general area of motion control of ocean vehicles. The ability to mathematically simulate or predict the behavior of ocean vehicles in response to their effectors is usually helpful and sometimes essential in designing or utilizing vehicle control systems. The use of a vehicle mathematical model permits the running of a mathematical version of the vehicle in a computer rather than running the real vehicle in water.

A mathematical model for simulating the motion of an ocean vehicle consists of a set of differential equations, called the structure, and a set of undetermined variables, called parameters. The equation structure is usually developed by theoretical and experimental investigations using some of the basic principles of physics, mathematics, and ocean engineering. The parameters in the mathematical model are usually determined by observing the behavior of the vehicle, a physical model of the vehicle, or a mathematical model of the vehicle. The central purpose of this thesis is the development, presentation, DSRV utilization and analysis of techniques for the identification of parameters in an ocean vehicle dynamic mathematical model using noisy input-output data. The primary conclusion here is that the techniques of model reference contouring and extended Kalman filtering are valid and useful for accomplishing this identification in as many as six degrees of freedom.

Model reference contouring is essentially a comparison technique to determine a "best" set of parameters for the mathematical model

which causes it to behave similar to the vehicle. As a general procedure, model reference identification runs the model, with the same inputs as to the system, for a large number of different parameters; and then it selects the specific set of parameters which result in the model output which is "closest" to the vehicle output.

Extended Kalman filtering is a data processing technique which uses known characteristics of the noise in the vehicle data to estimate the true (noiseless) behavior of the vehicle and its parameters throughout a maneuver. Kalman filtering is a linear technique with a firm theoretical foundation which, when extended to nonlinear systems such as ocean vehicles, loses its theoretical foundation but sometimes works extremely well.

Both of these identification techniques require mathematical models which are in "state space" form. This means that in order to use these techniques, the usual structure adopted for ocean vehicle dynamics must be converted to state space form. For these equations the state space format is simply a redefinition of the primary explicit variables as a set of ordered and indexed variables, but this redefinition process greatly facilitates both the computation of the equations and the inclusion of such aspects of ocean vehicle behavior as fluidic memory states and higher order derivatives of the primary states.

The studies of this thesis begin with the development of general state space models for the dynamic motions of ocean vehicles. Then the detailed equations for the two identification techniques are presented and their use is described in detail. Next, the vehicle equations and

the identification technique equations are combined for in-depth studies of the DSRV single degree of freedom mathematical models. Then the complete 6 degree of freedom DSRV model, its gradient, and all of its computer programs are described and several selected coefficients and parameters are identified using 1, 2, 3, 4, 5, and 6 degree of freedom simulated sea trials, and the results are discussed and compared. Finally, all of the computer programs used are listed, and a bibliography of references from the areas of system identification and ocean vehicle dynamics is included.

Several significant conclusions may be drawn from the single and multiple degree of freedom DSRV studies of this thesis. First of all, both model reference contouring and extended Kalman filtering are valid for the single degree of freedom models of the DSRV. For model reference contouring the best input function is found to be a sinusoidal function with a period near the "natural frequency" or "break point frequency" of the system; and detailed noise studies show that for the DSRV surge equation, 10% process and 10% measurement noises permit about 10% identification accuracy of the parameters. In addition, the shapes of the model reference contours show the identifiability characteristics of the parameters in addition to actually permitting their identification.

Regarding the extended Kalman filtering technique, it has been found that it is much more difficult to "tune" the filter and that the integration technique must be more accurate than for model reference contouring. However, once the filter is adjusted and given reasonably good initial estimates of the parameters, it is a far more

accurate identification technique in the presence of large (20% or greater) amounts of noise than is the model reference contouring technique. In addition, the extended Kalman filter in its general form may be used for the identification of many parameters at once, whereas the model reference contouring technique looks at the parameters two at a time. The multiple degree of freedom (DOF) extended Kalman filter presented in the computer programs of this thesis is incomplete, but the 1 DOF filter is shown to be valid for the six individual single degree of freedom DSRV equations.

Finally, and most significantly, the model reference contouring technique is shown to be valid for selected parameters in 1 through 6 degree of freedom DSRV mathematical models. In several cases, it is shown that 1 DOF parameter identifiability studies may be directly applied to those same parameters in 6 DOF for the same maneuver.

CHAPTER S2

AREAS FOR FURTHER STUDY

The fact that the parametric identification techniques of model reference contouring and extended Kalman filtering are valid and useful techniques for the evaluation of ocean vehicle model parameters opens up many possible areas for further study. These areas are described here in somewhat of a simplified "idea" format with references to the applicable sections of this thesis and to entries in the Bibliography in Chapter B1.

S2.1 Develop and analyze general and specific measurement functions for ocean vehicles (Chapter M3.1). Study the parameters for their identifiability characteristics, especially in combination with the vehicle parameters, and perform measurement parameter identifications from noisy sea trial data using the model reference or EKF techniques.

S2.2 Develop the general state space equations of motion for a vehicle with a time varying mass or with an accelerating center of gravity (Chapter M1.2). How do the identifiability characteristics of the parameters change?

S2.3 There exists a wealth of data reduction techniques for finding parameters in ocean vehicle models, techniques which have been used for many years in the field of Naval Architecture and Marine Engineering. A comparison should be made of these techniques with the techniques of Modern Control Theory to determine the advantages and limitations of each. (Chapter N1.1)

S2.4 Instead of using a Taylor series expansion of the hydrodynamic forces and moments in Chapter M2.3, use a Fourier, Bessel, Hermite, or other type of expansion for the specification of this structure. Then make studies of the coefficient identifiability and determine the kinds of ocean vehicles best modeled by such expansions and the reasons why.

S2.5 Develop the equations and study the coefficients required for some implicit solutions to equation M2.24. What parameters are required, how identifiable are they, and how much better is the mathematical model with them included?

S2.6 Relate the general theory of coefficient expansions of state space ocean vehicle mathematical models to the general theory of covariant and contravariant tensors (Chapter M2.4). Develop general relationships for the numbers of n'th degree coefficients with and without symmetry. Develop general expressions for efficiently and rapidly addressing and storing n'th degree coefficients in computer programs (Equation D1.16 for example).

S2.7 Develop the equations for and study the identifiability of the parameters in "fluidic memory" mathematical models for specific ocean vehicles (Chapter M3.2). Formulate simple examples to show the memory effects, and use single or double degree of freedom models to identify fluidic memory coefficients. Formulate general conditions and vehicle constraints which may be used to reduce the numbers of fluidic memory coefficients. Set up and evaluate multiple degree of freedom fluidic memory models for specific ocean vehicles. The author considers the incorporation of fluidic memory states and their associated parameters and the incorporation of higher order derivative states

and their associated parameters to be the two most potentially profitable areas for further studies using the techniques of this thesis.

S2.8 Study the feasibility, expense, and accuracy of using analog or hybrid computation and continuous parametric identification techniques for ocean vehicle mathematical models. This may or may not require decoupled models or a structure-selective multiple degree of freedom model.

S2.9 Can a list be made of most of the significant types of structure (such as absolute square in Chapter M3.3) for a class of ocean vehicles (for example, surface ships)? If this is possible, then perhaps the identifiability characteristics of the parameters in each of these structural types may be determined separately using simple single degree of freedom models and combined in some manner to reveal the identifiability characteristics of more sophisticated models which involve combinations of these significant types of structure.

S2.10 Investigate model reference contouring using different configurations from that of Figure N2.1 (see Astrom) (A-13).

S2.11 There are some techniques in Modern Control Theory which are applicable to specific problems of parametric identification in distributed systems or partial differential equations. Investigate general partial differential equation ocean vehicle models, perhaps including continuous fluidic memory, and determine techniques applicable to identifying parameters and to determining their identifiability characteristics.

S2.12 One possible reduction of the general state-space models of Section 2(M) which contains linear plus second degree coefficients is given by equation S2.1. This equation may or may not behave like a specific ocean vehicle, but its nonlinear characteristics may relate to those of the general models (especially the second degree coefficients in A_2) for certain maneuvers. Study this equation in general using the results and techniques of Modern Control Theory.

$$\dot{\underline{x}} = A_1 \underline{x} + A_2 \underline{x}\,\underline{x}^T \underline{a} + B_1 \underline{u} + B_2 \underline{u}\,\underline{u}^T \underline{b} + C \underline{x}\,\underline{u}^T \underline{c} \qquad \text{S2.1}$$

Where: $\underline{x}$ = state vector

$\underline{u}$ = control vector

$\underline{a}$, $\underline{b}$, $\underline{c}$ = known weighting or non-dimensionalizing vectors

A_1, A_2, B_1, B_2, C = system matrices containing parameters which must be identified

Can a general state transition matrix be found for this equation? What are the conditions on the equation for there to be stability, observability, controllability, identifiability, invertability, etc.? How does the behavior of this equation correspond to known ocean vehicle behavior? What is indicated by the fact that equation S2.1 contains no absolute square law behavior?

S2.13 The effects of vehicle control systems have not been included in the models of Section 2(M) (for example, equation M2.32) or in the programming of Section 5(D). Investigate the effects on parametric identifiability for specific ocean vehicles when the control is a linear function of the state given by equation S2.2 or a nonlinear one given by equation S2.3.

$$\underline{u} = -K\,\underline{x} \quad ; \quad K = \text{control gain matrix} \qquad S2.2$$

$$\underline{u} = -k\,(\underline{x}) \quad ; \quad \underline{k} = \text{control function structure} \qquad S2.3$$

S2.14 It is possible to include higher order derivatives of the states in the general mathematical models of Section 2(M). With these multiple derivatives (md), included in the hydrodynamic structure, equation M2.24 becomes equation S2.4. Now, if a new state

$$A_r\,\dot{\underline{x}} - \underline{X}_{hyd}\,(\underline{x}, \dot{\underline{x}}, \ddot{\underline{x}}, \dddot{\underline{x}}, \ldots, \overset{(\cdot)^n}{\underline{x}}) = \underline{f}_r(\underline{x}) + \underline{X}_{eff} \qquad S2.4$$

vector $\underline{x}_{md}$ is defined as in equation S2.5 to include the multiple

$$\underline{x}_{md} = \begin{bmatrix} \underline{x} \\ \dot{\underline{x}} \\ \vdots \\ \overset{(\cdot)^{n-1}}{\underline{x}} \end{bmatrix} \quad ; \quad \dot{\underline{x}}_{md} = \begin{bmatrix} \dot{\underline{x}} \\ \ddot{\underline{x}} \\ \vdots \\ \overset{(\cdot)^{n}}{\underline{x}} \end{bmatrix} \qquad S2.5$$

derivative terms, then equation S2.4 may be written in terms of the multiple derivative state vector $\underline{x}_{md}$ as in equation S2.6. The forms

$$\boxed{A_{md}\,\dot{\underline{x}}_{md} - \underline{X}_{md,hyd}\,(\underline{x}_{md}, \dot{\underline{x}}_{md}) = \underline{f}_{md}\,(\underline{x}_{md}) + \underline{X}_{md,eff}\,(\underline{x}_{md}, \underline{u}_{md}, \underline{p}) \qquad S2.6}$$

for the matrix A_{md} and the vector $\underline{f}_{md}$ are given in equations S2.7 and S2.8.

$$A_{md} = \left[\begin{array}{c|c} A_r & 0 \\ \hline 0 & 0 \end{array}\right] \qquad S2.7$$

$$\underline{f}_{md} = \begin{bmatrix} f_r \\ 0 \\ | \\ | \\ 0 \end{bmatrix} \qquad \text{S2.8}$$

$$(\underset{\cdot}{\underline{x}})^n = \underline{X}_1 \left(\underline{x}, \dot{\underline{x}}, \cdots, (\underset{\cdot}{x})^{n-1}, A_r, \underline{f}_r, \underline{X}_{eff}\right) \qquad \text{S2.9}$$

Equation S2.6 is somewhat misleading in that it does not directly display the n-th order differential equation behavior in the original equation S2.4. One way of directly showing this behavior is to assume that $\underline{X}_{hyd}$ in equation S2.4 can be solved for the highest state vector derivative by the function $\underline{X}_1$ as in equation S2.9. In this case, the general mathematical model could be written as the first order vector differential equation S2.10. The

$$\dot{\underline{x}}_{md} = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ & & & & \\ 0 & \cdots & 0 & I & 0 \\ 0 & \cdots & 0 & 0 & I \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix} \underline{x}_{md} + \begin{bmatrix} 0 \\ | \\ | \\ | \\ 0 \\ \underline{X}_1(\underline{x}_{md}, A_r, f_r, \underline{X}_{eff}) \end{bmatrix}$$

Where: I = Identity Matrix S2.10

multiple derivative state vector $\underline{x}_{md}$ used here can also be redefined to include the fluidic memory states of Chapter M3.2 and their multiple derivates, with the appropriate modifications to $\underline{X}_1$.

Can a set of criteria be developed which will tell when the mathematical model for an ocean vehicle should include fluidic memory states or multiple derivative states? (See also Chapter S2.7) It is expected that, as in the case of fluidic memory, the multiple

derivatives need not apply for many derivatives or for more than a few degrees of freedom. Therefore, simple models may be developed which include the basic effects of multiple derivatives and which contain multiple derivative coefficients and parameters which may be identified using the techniques of this thesis.

S2.15 The gravity forces presented and discussed in Chapter M1.3 were included in the general category of effector forces. These gravity forces are functions of the vehicle orientation angles (φ, θ, ψ) which are in turn functions (integrals) of the primary vehicle velocities (states). In the later state space models of Chapter M2 these angles do not appear directly (They do appear directly in the model of Section 5(D) and in the computer programs.) but rather appear implicitly as an effector in $\underline{X}_{eff}$ with its own state variables whose input-output behavior is all that appears in the model. This has the effect of making a nine state variable $(u\ v\ w\ p\ q\ r\ \varphi\ \theta\ \psi)^T$ problem appear as a six state variable $(u\ v\ w\ p\ q\ r)^T$ problem.

Formulate the state space relationships for the direct inclusion of the integrals of the primary velocities $\int \underline{x}\, dt$ in the general mathematical models of Section 2(M). Investigate the parameters required and their identifiability characteristics.

S2.16 Complete the extended Kalman filtering programming for Section 6(C). This will require a subroutine of the form of the MAIN program of Appendix A14 but somewhat larger and able to manipulate the general subroutines of Appendix A18. These subroutines represent, respectively, the general versions of subroutines PROP, GAIN, and UPDAT of Appendix A14. An output subroutine to plot the outputs of the desired

states, parameters, and error covariances would also have to be written. The result of this programming would be an extended Kalman filter capable of identifying (estimating) up to 200 parameters at once for a general nonlinear ocean vehicle of the form of subroutines OVMOD and OVDER described in Section 5(D).

S2.17 Investigate the use of a time-sharing digital computation program with the model reference configuration to allow an operator at a typewriter to "visualize" the behavior in C ($\underline{p}$) of more than two parameters at once. The operator could start with the initial values of the parameters, make variations in those directions which reduced C ($\underline{p}$), and thereby "learn" the shapes of n-dimensional contours and the identifiability characteristics of many parameters at once. (S-6)

S2.18 The 6 * 6 model reference contouring program of Section 6(C) is so flexible and general that the author's studies of the DSRV did not even begin to exhaust the wealth of behavioral and parametric studies which could be conducted on the DSRV model using these programs directly as written. Someone interested in further studying these aspects of the DSRV could use this "tool" very effectively and beneficially.

S2.19 By changing only the input data and the characteristics of OVMOD, another ocean vehicle could be included in the 6 * 6 model reference contouring program of Section 6(C). This vehicle could then have the identifiability characteristics of its parameters studied in pairs as was done for the DSRV.

S2.20 For a specific and sophisticated ocean vehicle which requires extremely precise control, perhaps the "model within the ship" idea

of Chapter 13 could be applied. What predictive capabilities are required for a specified precision of control? What kind of input-output data and what size computation facility must be on-board to permit the self-identification of the vehicle? What kinds of behavioral cost functions must be specified for the vehicle to complete its missions?

S2.21 Investigate the use of the Pade approximation technique (C-14, 8-103) for modeling fluidic memory in state space ocean vehicle models.

S2.22 One identification technique which offers potential for application to ocean vehicle mathematical models is the "maximum likelihood estimation" technique (S-6) (R-7) (K-14) (J-5) (I-3) (I-4) (G-6) (C-3) (A-13). This technique has more general applicability than either model reference contouring or extended Kalman filtering and combines several aspects of both of these techniques. Investigate the use of the maximum likelihood technique for simple or complex ocean vehicle models with different combinations and configurations of system noises.

S2.23 Investigate the theoretical and practical considerations which lead to the application of single DOF identifiability results to selected multiple DOF models and to the application of dual parameter identifiability results to the multiple parameter models and identifiability studies.

SECTION 8

APPENDICES (A)

A1 NUMERICAL VALUES OF DSRV PARAMETERS AND COEFFICIENTS

A2 DSRV COEFFICIENT FUNCTIONS AND SELECTION CONSTANTS

A3 SECONDARY DRAG EQUATIONS

A4 IBM/SSP SUBROUTINES USED IN THIS THESIS

A5 CONTOURING AND PLOTTING SUBROUTINES

A6 CHAPTER D2 SUBROUTINES (OVMOD)

A7 CHAPTER D3 SUBROUTINES (EFFECTORS)

A8 CHAPTER D5 SUBROUTINES (OVDER, GRADIENT)

A9 CHAPTER D6 SUBROUTINES (EFFECTOR GRADIENT)

A10 THE MEAN OF A SQUARED GAUSSIAN RANDOM VARIABLE

A11 A NONLINEAR OBSERVABILITY CRITERION APPLIED TO THE DSRV SINGLE DEGREE OF FREEDOM EQUATION

A12 CHAPTER P3 MODEL REFERENCE SINGLE DEGREE OF FREEDOM PROGRAM AND SUBROUTINES

A13 MODEL REFERENCE STUDIES OF A LINEAR SURGE EQUATION

A14 CHAPTER P4 EXTENDED KALMAN FILTER SINGLE DEGREE OF FREEDOM PROGRAM AND SUBROUTINES

A15 PUTTING A "WHAMMY" ON NEGATIVE E IN EXTENDED KALMAN FILTERING

A16 SUBROUTINES PROP AND RKNL WITH RUNGE-KUTTA INTEGRATION

A17 SUBROUTINES USED IN 6 * 6 MODEL REFERENCE CONTOURING

A18 SUBROUTINES USED IN 6 * 6 EXTENDED KALMAN FILTERING

A19 MAIN 6 * 6 IDENTIFICATION PROGRAM STATEMENTS IN FORTRAN IV (MAIN)

A20 SAMPLE DSRV INPUT DATA DECK FOR APPENDIX A19

"ROUND NUMBERS ARE ALWAYS FALSE." SAMUEL JOHNSON (1709-1784)

THIS SECTION CONTAINS THE FORTRAN IV COMPUTER PROGRAMS DEVELOPED FOR AND USED THROUGHOUT THIS THESIS. IN ADDITION, SEVERAL EXTENSIONS OF AND DEVELOPMENTS FOR THE RESULTS OF EARLIER SECTIONS ARE INCLUDED.

APPENDIX A1

NUMERICAL VALUES OF DSRV PARAMETERS AND COEFFICIENTS

The DSRV states and coefficients are contained in the augmented state vector X(258).

X(1) - X(6) = Primary states

X(7) - X(132) = Second-degree coefficients

X(133) - X(258) = Reverse-mode second-degree coefficients

The DSRV parameters are contained in the parameter vector P(587).

P(1) - P(36) = Added mass terms $X_{\dot{u}}$ to $N_{\dot{r}}$ in the order of $X^{i}_{\dot{x}_j}$ located at j + 6(i - 1)

P(37) - P(42) = Cross products of inertia $I_{x_i x_i}$ located at i + 36

P(43) = DSRV reverse mode region determination constant

P(44) - P(71) = Tanks parameters; 44-50 = equilibrium loadings of 7 tanks, 51-57 = x-locations of 7 tanks, 58-64 = y-locations of 7 tanks, 65-71, z-locations of 7 tanks

P(72) - P(102) = Secondary drag parameters; 72-99 = C_{i2}, C_{i3}, C_{i5}, C_{i6}, i = 1, 7 in groups of 7, P(100) = L_s, P(101) = L_1, P(102) = $L_1 - L_s$

P(103) - P(228) = Shroud parameters

103 - 118 = Lift parameters; 103-108 = ranges, 109-113 = lift slopes, 114-118 = lift intercepts

119 - 149 = Drag parameters; 119-129 = ranges, 130-139 = drag slopes, 140-149 = drag intercepts

150 = x_s, shroud location

151 = S_r, shroud area

152	=	ρ_W, specific gravity of water
153	=	EPS or 0^+ for SHCAL
154	=	Overflow indicator for SHCAL
155 - 182	=	Lift derivative parameters; 155-164 = ranges, 165-173 = lift derivative slopes, 174-182 = lift derivative intercepts
183 - 228	=	Drag derivative parameters; 183-198 = ranges, 199-213 = drag derivative slopes, 214-228 = drag derivative intercepts
P(229) - P(265)	=	Propellor Parameters
P(266) - P(559)	=	Thruster Parameters
266 - 290	=	T_1^* ranges, slopes, intercepts, (8)
291 - 315	=	T_2^* ranges, slopes, intercepts, (8)
316 - 325	=	T_3^* ranges, slopes, intercepts (3)
326 - 341	=	T_4^* ranges, slopes, intercepts (5)
342 - 363	=	M_1^* ranges, slopes, intercepts, (7)
364 - 385	=	M_2^* ranges, slopes, intercepts, (7)
386 - 391	=	Thruster coefficients
392	=	Thruster EPS or 0^+
393 - 394	=	Maximum ranges for $\pm$ u/|n|
395 - 428	=	DT_1^* ranges, slopes, intercepts, (11)
429 - 456	=	DT_2^* ranges, slopes, intercepts, (9)
457 - 472	=	DT_3^* ranges, slopes, intercepts, (5)
473 - 500	=	DT_4^* ranges, slopes, intercepts, (9)
501 - 531	=	DM_1^* ranges, slopes, intercepts, (10)
532 - 559	=	DM_2^* ranges, slopes, intercepts, (9)

P(560) - P(571) = Noise means (6) and variances (6)

P(573) - P(577) = Constant forces and moments

P(578) - P(585) = Variable parameters, W, x_G, y_G, z_G, B, x_B, y_B, z_B

P(586) - P(587) = Constants; DSRV mass and acceleration of gravity, G

The following pages list these states, coefficients, and parameters as used in the DSRV simulations and identifications in this thesis. The start values are designed to place a $\pm$ 50% region around the central value of the state, coefficient, or parameter. The increments are set for producing this $\pm$ 50% region by using the start value and 46 increments for a total of 47 specific values with the true value at location 24.

KIJ	IJK	X=STATE VECTOR	XS=START VALUE	XINC=INCREMENT
1	1	0.0000	0.0000	0.0000
2	2	0.0000	0.0000	0.0000
3	3	0.0000	0.0000	0.0000
4	4	0.0000	0.0000	0.0000
5	5	0.0000	0.0000	0.0000
6	6	0.0000	0.0000	0.0000
7	1	-16.7000	-25.0500	0.3340
8	2	0.0000	0.0000	0.0000
9	3	0.0000	0.0000	0.0000
10	4	0.0000	0.0000	0.0000
11	5	0.0000	0.0000	0.0000
12	6	0.0000	0.0000	0.0000
13	7	0.0000	0.0000	0.0000
14	8	0.0000	0.0000	0.0000
15	9	0.0000	0.0000	0.0000
16	10	0.0000	0.0000	0.0000
17	11	0.0000	0.0000	0.0000
18	12	0.0000	0.0000	0.0000
19	13	-3600.0000	-5400.0000	72.0000
20	14	0.0000	0.0000	0.0000
21	15	-770.0000	-1155.0000	15.4000
22	16	0.0000	0.0000	0.0000
23	17	3720.0000	1860.0000	74.4000
24	18	0.0000	0.0000	0.0000
25	19	-793.0000	-1189.5000	15.8600
26	20	0.0000	0.0000	0.0000
27	21	-830.0000	-1245.0000	16.6000
28	22	0.0000	0.0000	0.0000
29	23	-124.0000	-186.0000	2.4800
30	24	-346.0000	-519.0000	6.9200
31	25	0.0000	0.0000	0.0000
32	26	-112.0000	-168.0000	2.2400
33	27	0.0000	0.0000	0.0000
34	28	264.0000	132.0000	5.2800
35	29	0.0000	0.0000	0.0000
36	30	3600.0000	1800.0000	72.0000
37	31	0.0000	0.0000	0.0000
38	32	0.0000	0.0000	0.0000
39	33	0.0000	0.0000	0.0000
40	34	0.0000	0.0000	0.0000
41	35	770.0000	385.0000	15.4000
42	36	0.0000	0.0000	0.0000
43	37	1040.0000	520.0000	20.8000
44	38	0.0000	0.0000	0.0000
45	39	0.0000	0.0000	0.0000
46	40	0.0000	0.0000	0.0000
47	41	0.0000	0.0000	0.0000
48	42	0.0000	0.0000	0.0000
49	43	2.4300	1.2150	0.0486
50	44	0.0000	0.0000	0.0000

KIJ	IJK	X=STATE VECTOR	XS=START VALUE	XINC=INCREMENT
51	45	-195.0000	-292.5000	3.9000
52	46	-55.9000	-83.8500	1.1180
53	47	0.0000	0.0000	0.0000
54	48	-207.0000	-310.5000	4.1400
55	49	0.0000	0.0000	0.0000
56	50	-3720.0000	-5580.0000	74.4000
57	51	0.0000	0.0000	0.0000
58	52	793.0000	396.5000	15.8600
59	53	-444.0000	-666.0000	8.8800
60	54	0.0000	0.0000	0.0000
61	55	0.0000	0.0000	0.0000
62	56	0.0000	0.0000	0.0000
63	57	0.0000	0.0000	0.0000
64	58	0.0000	0.0000	0.0000
65	59	0.0000	0.0000	0.0000
66	60	0.0000	0.0000	0.0000
67	61	830.0000	415.0000	16.6000
68	62	0.0000	0.0000	0.0000
69	63	0.0000	0.0000	0.0000
70	64	0.0000	0.0000	0.0000
71	65	264.0000	132.0000	5.2800
72	66	142.0000	71.0000	2.8400
73	67	0.0000	0.0000	0.0000
74	68	638.0000	319.0000	12.7600
75	69	0.0000	0.0000	0.0000
76	70	-1870.0000	-2805.0000	37.4000
77	71	0.0000	0.0000	0.0000
78	72	-793.0000	-1189.5000	15.8600
79	73	-33000.0000	-49500.0000	659.9998
80	74	0.0000	0.0000	0.0000
81	75	592.0000	296.0000	11.8400
82	76	0.0000	0.0000	0.0000
83	77	0.0000	0.0000	0.0000
84	78	0.0000	0.0000	0.0000
85	79	-1390.0000	-2085.0000	27.8000
86	80	0.0000	0.0000	0.0000
87	81	-592.0000	-888.0000	11.8400
88	82	0.0000	0.0000	0.0000
89	83	0.0000	0.0000	0.0000
90	84	0.0000	0.0000	0.0000
91	85	-66.0000	-99.0000	1.3200
92	86	0.0000	0.0000	0.0000
93	87	-3360.0000	-5040.0000	67.2000
94	88	2760.0000	1380.0000	55.2000
95	89	0.0000	0.0000	0.0000
96	90	0.0000	0.0000	0.0000
97	91	0.0000	0.0000	0.0000
98	92	-830.0000	-1245.0000	16.6000
99	93	0.0000	0.0000	0.0000
100	94	0.0000	0.0000	0.0000

KIJ	IJK	X=STATE VECTOR	XS=START VALUE	XINC=INCREMENT
101	95	-26600.0000	-39900.0000	531.9998
102	96	0.0000	0.0000	0.0000
103	97	0.0000	0.0000	0.0000
104	98	0.0000	0.0000	0.0000
105	99	0.0000	0.0000	0.0000
106	100	0.0000	0.0000	0.0000
107	101	793.0000	396.5000	15.8600
108	102	0.0000	0.0000	0.0000
109	103	406000.0000	203000.0000	8119.9960
110	104	0.0000	0.0000	0.0000
111	105	0.0000	0.0000	0.0000
112	106	0.0000	0.0000	0.0000
113	107	-3600.0000	-5400.0000	72.0000
114	108	0.0000	0.0000	0.0000
115	109	0.0000	0.0000	0.0000
116	110	598.0000	299.0000	11.9600
117	111	0.0000	0.0000	0.0000
118	112	-935.0000	-1402.5000	18.7000
119	113	0.0000	0.0000	0.0000
120	114	-770.0000	-1155.0000	15.4000
121	115	0.0000	0.0000	0.0000
122	116	0.0000	0.0000	0.0000
123	117	-793.0000	-1189.5000	15.8600
124	118	0.0000	0.0000	0.0000
125	119	-406000.0000	-609000.0000	8119.9960
126	120	0.0000	0.0000	0.0000
127	121	-32000.0000	-48000.0000	639.9998
128	122	0.0000	0.0000	0.0000
129	123	0.0000	0.0000	0.0000
130	124	0.0000	0.0000	0.0000
131	125	0.0000	0.0000	0.0000
132	126	0.0000	0.0000	0.0000
133	127	-19.6000	-29.4000	0.3920
134	128	0.0000	0.0000	0.0000
135	129	0.0000	0.0000	0.0000
136	130	0.0000	0.0000	0.0000
137	131	0.0000	0.0000	0.0000
138	132	0.0000	0.0000	0.0000
139	133	0.0000	0.0000	0.0000
140	134	0.0000	0.0000	0.0000
141	135	0.0000	0.0000	0.0000
142	136	0.0000	0.0000	0.0000
143	137	0.0000	0.0000	0.0000
144	138	0.0000	0.0000	0.0000
145	139	0.0000	0.0000	0.0000
146	140	0.0000	0.0000	0.0000
147	141	0.0000	0.0000	0.0000
148	142	0.0000	0.0000	0.0000
149	143	0.0000	0.0000	0.0000
150	144	0.0000	0.0000	0.0000

KIJ	IJK	X=STATE VECTOR	XS=START VALUE	XINC=INCREMENT
151	145	0.0000	0.0000	0.0000
152	146	0.0000	0.0000	0.0000
153	147	0.0000	0.0000	0.0000
154	148	0.0000	0.0000	0.0000
155	149	-241.0000	-361.5000	4.8200
156	150	0.0000	0.0000	0.0000
157	151	0.0000	0.0000	0.0000
158	152	0.0000	0.0000	0.0000
159	153	0.0000	0.0000	0.0000
160	154	642.0000	321.0000	12.8400
161	155	0.0000	0.0000	0.0000
162	156	0.0000	0.0000	0.0000
163	157	0.0000	0.0000	0.0000
164	158	0.0000	0.0000	0.0000
165	159	0.0000	0.0000	0.0000
166	160	0.0000	0.0000	0.0000
167	161	0.0000	0.0000	0.0000
168	162	0.0000	0.0000	0.0000
169	163	9240.0000	4620.0000	184.8000
170	164	0.0000	0.0000	0.0000
171	165	0.0000	0.0000	0.0000
172	166	0.0000	0.0000	0.0000
173	167	0.0000	0.0000	0.0000
174	168	0.0000	0.0000	0.0000
175	169	-2.4300	-3.6450	0.0486
176	170	0.0000	0.0000	0.0000
177	171	0.0000	0.0000	0.0000
178	172	-158.0000	-237.0000	3.1600
179	173	0.0000	0.0000	0.0000
180	174	0.0000	0.0000	0.0000
181	175	0.0000	0.0000	0.0000
182	176	0.0000	0.0000	0.0000
183	177	0.0000	0.0000	0.0000
184	178	0.0000	0.0000	0.0000
185	179	-10400.0000	-15600.0000	208.0000
186	180	0.0000	0.0000	0.0000
187	181	0.0000	0.0000	0.0000
188	182	0.0000	0.0000	0.0000
189	183	0.0000	0.0000	0.0000
190	184	0.0000	0.0000	0.0000
191	185	0.0000	0.0000	0.0000
192	186	0.0000	0.0000	0.0000
193	187	0.0000	0.0000	0.0000
194	188	0.0000	0.0000	0.0000
195	189	0.0000	0.0000	0.0000
196	190	0.0000	0.0000	0.0000
197	191	642.0000	321.0000	12.8400
198	192	0.0000	0.0000	0.0000
199	193	0.0000	0.0000	0.0000
200	194	0.0000	0.0000	0.0000

KIJ	IJK	X=STATE VECTOR	XS=START VALUE	XINC=INCREMENT
201	195	0.0000	0.0000	0.0000
202	196	-3670.0000	-5505.0000	73.4000
203	197	0.0000	0.0000	0.0000
204	198	0.0000	0.0000	0.0000
205	199	0.0000	0.0000	0.0000
206	200	0.0000	0.0000	0.0000
207	201	0.0000	0.0000	0.0000
208	202	0.0000	0.0000	0.0000
209	203	0.0000	0.0000	0.0000
210	204	0.0000	0.0000	0.0000
211	205	-4290.0000	-6435.0000	85.8000
212	206	0.0000	0.0000	0.0000
213	207	0.0000	0.0000	0.0000
214	208	0.0000	0.0000	0.0000
215	209	0.0000	0.0000	0.0000
216	210	0.0000	0.0000	0.0000
217	211	-66.0000	-99.0000	1.3200
218	212	0.0000	0.0000	0.0000
219	213	0.0000	0.0000	0.0000
220	214	-3000.0000	-4500.0000	60.0000
221	215	0.0000	0.0000	0.0000
222	216	0.0000	0.0000	0.0000
223	217	0.0000	0.0000	0.0000
224	218	0.0000	0.0000	0.0000
225	219	0.0000	0.0000	0.0000
226	220	0.0000	0.0000	0.0000
227	221	-32000.0000	-48000.0000	639.9998
228	222	0.0000	0.0000	0.0000
229	223	0.0000	0.0000	0.0000
230	224	0.0000	0.0000	0.0000
231	225	0.0000	0.0000	0.0000
232	226	0.0000	0.0000	0.0000
233	227	0.0000	0.0000	0.0000
234	228	0.0000	0.0000	0.0000
235	229	0.0000	0.0000	0.0000
236	230	0.0000	0.0000	0.0000
237	231	0.0000	0.0000	0.0000
238	232	0.0000	0.0000	0.0000
239	233	2880.0000	1440.0000	57.6000
240	234	0.0000	0.0000	0.0000
241	235	0.0000	0.0000	0.0000
242	236	0.0000	0.0000	0.0000
243	237	0.0000	0.0000	0.0000
244	238	-4290.0000	-6435.0000	85.8000
245	239	0.0000	0.0000	0.0000
246	240	0.0000	0.0000	0.0000
247	241	0.0000	0.0000	0.0000
248	242	0.0000	0.0000	0.0000
249	243	0.0000	0.0000	0.0000
250	244	0.0000	0.0000	0.0000

KIJ	IJK	X=STATE VECTOR	XS=START VALUE	XINC=INCREMENT
251	245	0.0000	0.0000	0.0000
252	246	0.0000	0.0000	0.0000
253	247	-49700.0000	-74550.0000	993.9998
254	248	0.0000	0.0000	0.0000
255	249	0.0000	0.0000	0.0000
256	250	0.0000	0.0000	0.0000
257	251	0.0000	0.0000	0.0000
258	252	0.0000	0.0000	0.0000

KIJ	P=PARAM VECTOR	PS=START VALUE	PINC=INCREMENT
1	-144.0000	-216.0000	2.8800
2	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000
6	0.0000	0.0000	0.0000
7	0.0000	0.0000	0.0000
8	-4320.0000	-6480.0000	86.4000
9	0.0000	0.0000	0.0000
10	793.0000	396.5000	15.8600
11	0.0000	0.0000	0.0000
12	8300.0000	4150.0000	166.0000
13	0.0000	0.0000	0.0000
14	0.0000	0.0000	0.0000
15	-3600.0000	-5400.0000	72.0000
16	0.0000	0.0000	0.0000
17	-770.0000	-1155.0000	15.4000
18	0.0000	0.0000	0.0000
19	0.0000	0.0000	0.0000
20	917.0000	458.5000	18.3400
21	0.0000	0.0000	0.0000
22	-4530.0000	-6795.0000	90.6000
23	0.0000	0.0000	0.0000
24	0.0000	0.0000	0.0000
25	0.0000	0.0000	0.0000
26	0.0000	0.0000	0.0000
27	-770.0000	-1155.0000	15.4000
28	0.0000	0.0000	0.0000
29	-410000.0000	-615000.0000	8199.9960
30	0.0000	0.0000	0.0000
31	0.0000	0.0000	0.0000
32	830.0000	415.0000	16.6000
33	0.0000	0.0000	0.0000
34	0.0000	0.0000	0.0000
35	0.0000	0.0000	0.0000
36	-410000.0000	-615000.0000	8199.9960
37	37800.0000	18900.0000	755.9998
38	0.0000	0.0000	0.0000
39	452000.0000	226000.0000	9039.9960
40	0.0000	0.0000	0.0000
41	0.0000	0.0000	0.0000
42	450000.0000	225000.0000	8999.9960
43	0.1730	0.0865	0.0035
44	2404.0000	1202.0000	48.0800
45	93.0000	46.5000	1.8600
46	93.0000	46.5000	1.8600
47	210.0000	105.0000	4.2000
48	210.0000	105.0000	4.2000
49	200.0000	100.0000	4.0000
50	200.0000	100.0000	4.0000

KIJ	P=PARAM VECTOR	PS=START VALUE	PINC=INCREMENT
51	-3.3700	-5.0550	0.0674
52	-3.3700	-5.0550	0.0674
53	-3.3700	-5.0550	0.0674
54	18.9000	9.4500	0.3780
55	-18.4000	-27.6000	0.3680
56	12.7500	6.3750	0.2550
57	-12.3000	-18.4500	0.2460
58	0.0000	0.0000	0.0000
59	2.2100	1.1050	0.0442
60	-2.2100	-3.3150	0.0442
61	0.0000	0.0000	0.0000
62	0.0000	0.0000	0.0000
63	0.0000	0.0000	0.0000
64	0.0000	0.0000	0.0000
65	3.0800	1.5400	0.0616
66	-2.2100	-3.3150	0.0442
67	-2.2100	-3.3150	0.0442
68	-2.8400	-4.2600	0.0568
69	-2.5000	-3.7500	0.0500
70	-2.4600	-3.6900	0.0492
71	-2.2800	-3.4200	0.0456
72	1.0000	0.5000	0.0200
73	-0.1500	-0.2250	0.0030
74	183.6975	91.8487	3.6739
75	0.0142	0.0071	0.0003
76	-0.0032	-0.0048	0.0001
77	23.4752	11.7376	0.4695
78	-1.7606	-2.6409	0.0352
79	1.0000	0.5000	0.0200
80	0.1500	0.0750	0.0030
81	137.7731	68.8866	2.7555
82	0.0142	0.0071	0.0003
83	0.0032	0.0016	0.0001
84	23.4752	11.7376	0.4695
85	1.7606	0.8803	0.0352
86	3234.3220	1617.1610	64.6864
87	17.6063	8.8032	0.3521
88	-0.0036	-0.0053	0.0001
89	3.5213	1.7606	0.0704
90	376.3950	188.1975	7.5279
91	0.0750	0.0375	0.0015
92	41.3311	20.6655	0.8266
93	3234.3220	1617.1610	64.6864
94	17.6063	8.8032	0.3521
95	-0.0036	-0.0053	0.0001
96	-0.0000	-0.0000	0.0000
97	0.0036	0.0018	0.0001
98	-0.0750	-0.1125	0.0015
99	-41.3311	-61.9966	0.8266
100	-46.9500	-70.4250	0.9390

KIJ	P=PARAM VECTOR	PS=START VALUE	PINC=INCREMENT
101	-23.4000	-35.1000	0.4680
102	-23.5500	-35.3250	0.4710
103	0.0000	0.0000	0.0000
104	15.0000	7.5000	0.3000
105	40.0000	20.0000	0.8000
106	135.0000	67.5000	2.7000
107	170.0000	85.0000	3.4000
108	180.0000	90.0000	3.6000
109	0.0800	0.0400	0.0016
110	-0.0080	-0.0120	0.0002
111	-0.0179	-0.0269	0.0004
112	-0.0029	-0.0043	0.0001
113	0.0800	0.0400	0.0016
114	0.0000	0.0000	0.0000
115	1.3200	0.6600	0.0264
116	1.7150	0.8575	0.0343
117	-0.3140	-0.4710	0.0063
118	-14.4000	-21.6000	0.2880
119	0.0000	0.0000	0.0000
120	10.0000	5.0000	0.2000
121	50.0000	25.0000	1.0000
122	70.0000	35.0000	1.4000
123	85.0000	42.5000	1.7000
124	90.0000	45.0000	1.8000
125	95.0000	47.5000	1.9000
126	110.0000	55.0000	2.2000
127	130.0000	65.0000	2.6000
128	170.0000	85.0000	3.4000
129	180.0000	90.0000	3.6000
130	0.0050	0.0025	0.0001
131	0.0350	0.0175	0.0007
132	0.0150	0.0075	0.0003
133	-0.0732	-0.1098	0.0015
134	-0.0200	-0.0300	0.0004
135	0.0200	0.0100	0.0004
136	0.0732	0.0366	0.0015
137	-0.0150	-0.0225	0.0003
138	-0.0325	-0.0487	0.0006
139	-0.0100	-0.0150	0.0002
140	0.0500	0.0250	0.0010
141	-0.2500	-0.3750	0.0050
142	0.7500	0.3750	0.0150
143	6.9300	3.4650	0.1386
144	2.4000	1.2000	0.0480
145	-1.2000	-1.8000	0.0240
146	-6.2700	-9.4050	0.1254
147	3.4500	1.7250	0.0690
148	5.7250	2.8625	0.1145
149	1.9000	0.9500	0.0380
150	24.5900	12.2950	0.4918

KIJ	P=PARAM VECTOR	PS=START VALUE	PINC=INCREMENT
151	14.6400	7.3200	0.2928
152	1.9940	0.9970	0.0399
153	0.0000	0.0000	0.0000
154	0.0000	0.0000	0.0000
155	0.0000	0.0000	0.0000
156	10.0000	5.0000	0.2000
157	20.0000	10.0000	0.4000
158	35.0000	17.5000	0.7000
159	45.0000	22.5000	0.9000
160	130.0000	65.0000	2.6000
161	140.0000	70.0000	2.8000
162	165.0000	82.5000	3.3000
163	175.0000	87.5000	3.5000
164	180.0000	90.0000	3.6000
165	0.0000	0.0000	0.0000
166	-0.0088	-0.0132	0.0002
167	0.0000	0.0000	0.0000
168	-0.0010	-0.0015	0.0000
169	0.0000	0.0000	0.0000
170	0.0015	0.0007	0.0000
171	0.0000	0.0000	0.0000
172	0.0083	0.0041	0.0002
173	0.0000	0.0000	0.0000
174	0.0800	0.0400	0.0016
175	0.1680	0.0840	0.0034
176	-0.0080	-0.0120	0.0002
177	0.0267	0.0133	0.0005
178	-0.0179	-0.0269	0.0004
179	-0.2129	-0.3193	0.0043
180	-0.0029	-0.0043	0.0001
181	-1.3708	-2.0561	0.0274
182	0.0800	0.0400	0.0016
183	0.0000	0.0000	0.0000
184	5.0000	2.5000	0.1000
185	15.0000	7.5000	0.3000
186	45.0000	22.5000	0.9000
187	55.0000	27.5000	1.1000
188	65.0000	32.5000	1.3000
189	75.0000	37.5000	1.5000
190	80.0000	40.0000	1.6000
191	100.0000	50.0000	2.0000
192	105.0000	52.5000	2.1000
193	115.0000	57.5000	2.3000
194	125.0000	62.5000	2.5000
195	135.0000	67.5000	2.7000
196	165.0000	82.5000	3.3000
197	175.0000	87.5000	3.5000
198	180.0000	90.0000	3.6000
199	0.0000	0.0000	0.0000
200	0.0030	0.0015	0.0001

KIJ	P=PARAM VECTOR	PS=START VALUE	PINC=INCREMENT
201	0.0000	0.0000	0.0000
202	-0.0020	-0.0030	0.0000
203	0.0000	0.0000	0.0000
204	-0.0088	-0.0132	0.0002
205	0.0000	0.0000	0.0000
206	0.0073	0.0037	0.0001
207	0.0000	0.0000	0.0000
208	-0.0088	-0.0132	0.0002
209	0.0000	0.0000	0.0000
210	-0.0018	-0.0026	0.0000
211	0.0000	0.0000	0.0000
212	0.0022	0.0011	0.0000
213	0.0000	0.0000	0.0000
214	0.0050	0.0025	0.0001
215	-0.0100	-0.0150	0.0002
216	0.0350	0.0175	0.0007
217	0.1250	0.0625	0.0025
218	0.0150	0.0075	0.0003
219	0.5883	0.2941	0.0118
220	-0.0732	-0.1098	0.0015
221	-0.6588	-0.9882	0.0132
222	0.0732	0.0366	0.0015
223	1.0993	0.5497	0.0220
224	-0.0150	-0.0225	0.0003
225	0.2038	0.1019	0.0041
226	-0.0325	-0.0487	0.0006
227	-0.4038	-0.6056	0.0081
228	-0.0100	-0.0150	0.0002
229	755.0000	377.5000	15.1000
230	-58.0000	-87.0000	1.1600
231	-3.8000	-5.7000	0.0760
232	365.0000	182.5000	7.3000
233	-172.0000	-258.0000	3.4400
234	-45.0000	-67.5000	0.9000
235	755.0000	377.5000	15.1000
236	60.0000	30.0000	1.2000
237	22.0000	11.0000	0.4400
238	365.0000	182.5000	7.3000
239	-13.0000	-19.5000	0.2600
240	22.0000	11.0000	0.4400
241	-30.0000	-45.0000	0.6000
242	-12.0000	-18.0000	0.2400
243	-30.0000	-45.0000	0.6000
244	-12.0000	-18.0000	0.2400
245	530.0000	265.0000	10.6000
246	-6.5000	-9.7500	0.1300
247	-4.0500	-6.0750	0.0810
248	468.0000	234.0000	9.3600
249	-155.0000	-232.5000	3.1000
250	-38.0000	-57.0000	0.7600

KIJ	P=PARAM VECTOR	PS=START VALUE	PINC=INCREMENT
251	530.0000	265.0000	10.6000
252	80.0000	40.0000	1.6000
253	15.2000	7.6000	0.3040
254	468.0000	234.0000	9.3600
255	-8.0000	-12.0000	0.1600
256	15.2000	7.6000	0.3040
257	-765.0000	-1147.5000	15.3000
258	-306.0000	-459.0000	6.1200
259	765.0000	382.5000	15.3000
260	306.0000	153.0000	6.1200
261	26.0000	13.0000	0.5200
262	22.0000	11.0000	0.4400
263	131.0000	65.5000	2.6200
264	-0.2100	-0.3150	0.0042
265	-25.5000	-38.2500	0.5100
266	-1.0000	-1.5000	0.0200
267	-0.6000	-0.9000	0.0120
268	-0.4000	-0.6000	0.0080
269	-0.1000	-0.1500	0.0020
270	0.0000	0.0000	0.0000
271	0.1000	0.0500	0.0020
272	0.5000	0.2500	0.0100
273	0.7000	0.3500	0.0140
274	1.0000	0.5000	0.0200
275	0.1250	0.0625	0.0025
276	1.7500	0.8750	0.0350
277	6.6670	3.3335	0.1333
278	2.5000	1.2500	0.0500
279	-4.2000	-6.3000	0.0840
280	-8.6500	-12.9750	0.1730
281	-2.3500	-3.5250	0.0470
282	-0.5000	-0.7500	0.0100
283	2.2250	1.1125	0.0445
284	3.2000	1.6000	0.0640
285	5.1667	2.5834	0.1033
286	4.7500	2.3750	0.0950
287	4.7500	2.3750	0.0950
288	5.1700	2.5850	0.1034
289	2.0450	1.0225	0.0409
290	0.7500	0.3750	0.0150
291	-1.0000	-1.5000	0.0200
292	-0.6000	-0.9000	0.0120
293	-0.5000	-0.7500	0.0100
294	-0.1000	-0.1500	0.0020
295	0.0000	0.0000	0.0000
296	0.1000	0.0500	0.0020
297	0.4000	0.2000	0.0080
298	0.6000	0.3000	0.0120
299	1.0000	0.5000	0.0200
300	0.5000	0.2500	0.0100

KIJ	P=PARAM VECTOR	PS=START VALUE	PINC=INCREMENT
301	3.5000	1.7500	0.0700
302	8.7500	4.3750	0.1750
303	4.0000	2.0000	0.0800
304	-2.5000	-3.7500	0.0500
305	-6.6670	-10.0005	0.1333
306	-1.7500	-2.6250	0.0350
307	-0.1250	-0.1875	0.0025
308	0.8000	0.4000	0.0160
309	2.6000	1.3000	0.0520
310	5.2250	2.6125	0.1045
311	4.7500	2.3750	0.0950
312	4.7500	2.3750	0.0950
313	5.1667	2.5834	0.1033
314	3.2000	1.6000	0.0640
315	2.2250	1.1125	0.0445
316	-1.0000	-1.5000	0.0200
317	-0.2000	-0.3000	0.0040
318	0.2000	0.1000	0.0040
319	1.0000	0.5000	0.0200
320	-3.3750	-5.0625	0.0675
321	-5.7500	-8.6250	0.1150
322	-3.3750	-5.0625	0.0675
323	0.4750	0.2375	0.0095
324	0.0000	0.0000	0.0000
325	-0.4750	-0.7125	0.0095
326	-1.0000	-1.5000	0.0200
327	-0.6000	-0.9000	0.0120
328	-0.1000	-0.1500	0.0020
329	0.1000	0.0500	0.0020
330	0.6000	0.3000	0.0120
331	1.0000	0.5000	0.0200
332	-3.5000	-5.2500	0.0700
333	-0.3000	-0.4500	0.0060
334	-1.5000	-2.2500	0.0300
335	-0.3000	-0.4500	0.0060
336	-3.5000	-5.2500	0.0700
337	-1.8000	-2.7000	0.0360
338	0.1200	0.0600	0.0024
339	0.0000	0.0000	0.0000
340	-0.1200	-0.1800	0.0024
341	1.8000	0.9000	0.0360
342	-1.0000	-1.5000	0.0200
343	-0.8000	-1.2000	0.0160
344	-0.4500	-0.6750	0.0090
345	-0.1000	-0.1500	0.0020
346	0.0000	0.0000	0.0000
347	0.1000	0.0500	0.0020
348	0.5000	0.2500	0.0100
349	1.0000	0.5000	0.0200
350	270.0000	135.0000	5.4000

KIJ	P=PARAM VECTOR	PS=START VALUE	PINC=INCREMENT
351	48.5714	24.2857	0.9714
352	240.0000	120.0000	4.8000
353	90.0000	45.0000	1.8000
354	-90.0000	-135.0000	1.8000
355	-72.5000	-108.7500	1.4500
356	-6.0000	-9.0000	0.1200
357	199.0000	99.5000	3.9800
358	21.8571	10.9286	0.4371
359	108.0000	54.0000	2.1600
360	93.0000	46.5000	1.8600
361	93.0000	46.5000	1.8600
362	93.2500	46.6250	1.8650
363	60.0000	30.0000	1.2000
364	-1.0000	-1.5000	0.0200
365	-0.5000	-0.7500	0.0100
366	-0.1000	-0.1500	0.0020
367	0.0000	0.0000	0.0000
368	0.1000	0.0500	0.0020
369	0.4500	0.2250	0.0090
370	0.7000	0.3500	0.0140
371	1.0000	0.5000	0.0200
372	-8.0000	-12.0000	0.1600
373	-72.5000	-108.7500	1.4500
374	-40.0000	-60.0000	0.8000
375	80.0000	40.0000	1.6000
376	217.1430	108.5715	4.3429
377	64.0000	32.0000	1.2800
378	163.3333	81.6666	3.2667
379	-57.0000	-85.5000	1.1400
380	-86.7500	-130.1250	1.7350
381	-84.0000	-126.0000	1.6800
382	-84.0000	-126.0000	1.6800
383	-97.7143	-146.5714	1.9543
384	-28.8000	-43.2000	0.5760
385	-98.3333	-147.4999	1.9667
386	1.0600	0.5300	0.0212
387	1.1300	0.5650	0.0226
388	-0.6960	-1.0440	0.0139
389	0.9800	0.4900	0.0196
390	1.0400	0.5200	0.0208
391	-0.6960	-1.0440	0.0139
392	0.0000	0.0000	0.0000
393	-1.0000	-1.5000	0.0200
394	1.0000	0.5000	0.0200
395	-1.0000	-1.5000	0.0200
396	-0.6500	-0.9750	0.0130
397	-0.5500	-0.8250	0.0110
398	-0.4500	-0.6750	0.0090
399	-0.3500	-0.5250	0.0070
400	-0.1500	-0.2250	0.0030

KIJ	P=PARAM VECTOR	PS=START VALUE	PINC=INCREMENT
401	0.1500	0.0750	0.0030
402	0.4500	0.2250	0.0090
403	0.5500	0.2750	0.0110
404	0.6500	0.3250	0.0130
405	0.7500	0.3750	0.0150
406	1.0000	0.5000	0.0200
407	0.0000	0.0000	0.0000
408	16.2500	8.1250	0.3250
409	0.0000	0.0000	0.0000
410	49.1700	24.5850	0.9834
411	0.0000	0.0000	0.0000
412	-51.0567	-76.5851	1.0211
413	0.0000	0.0000	0.0000
414	63.0000	31.5000	1.2600
415	0.0000	0.0000	0.0000
416	16.0000	8.0000	0.3200
417	0.0000	0.0000	0.0000
418	0.1250	0.0625	0.0025
419	10.5775	5.2887	0.2115
420	1.7500	0.8750	0.0350
421	23.8765	11.9382	0.4775
422	6.6670	3.3335	0.1333
423	-0.9917	-1.4875	0.0198
424	-8.6500	-12.9750	0.1730
425	-37.0000	-55.5000	0.7400
426	-2.3500	-3.5250	0.0470
427	-12.7500	-19.1250	0.2550
428	-0.7500	-1.1250	0.0150
429	-1.0000	-1.5000	0.0200
430	-0.6500	-0.9750	0.0130
431	-0.4500	-0.6750	0.0090
432	-0.1500	-0.2250	0.0030
433	0.1500	0.0750	0.0030
434	0.3500	0.1750	0.0070
435	0.4500	0.2250	0.0090
436	0.5500	0.2750	0.0110
437	0.6500	0.3250	0.0130
438	1.0000	0.5000	0.0200
439	0.0000	0.0000	0.0000
440	41.2500	20.6250	0.8250
441	0.0000	0.0000	0.0000
442	-51.3900	-77.0850	1.0278
443	0.0000	0.0000	0.0000
444	49.1700	24.5850	0.9834
445	0.0000	0.0000	0.0000
446	16.2500	8.1250	0.3250
447	0.0000	0.0000	0.0000
448	0.5000	0.2500	0.0100
449	27.3125	13.6563	0.5462
450	8.7500	4.3750	0.1750

KIJ	P=PARAM VECTOR	PS=START VALUE	PINC=INCREMENT
451	1.0415	0.5208	0.0208
452	-6.6670	-10.0005	0.1333
453	-23.8765	-35.8147	0.4775
454	-1.7500	-2.6250	0.0350
455	-10.6875	-16.0313	0.2137
456	-0.1250	-0.1875	0.0025
457	-1.0000	-1.5000	0.0200
458	-0.2500	-0.3750	0.0050
459	-0.1500	-0.2250	0.0030
460	0.1500	0.0750	0.0030
461	0.2500	0.1250	0.0050
462	1.0000	0.5000	0.0200
463	0.0000	0.0000	0.0000
464	-23.7500	-35.6250	0.4750
465	0.0000	0.0000	0.0000
466	23.7500	11.8750	0.4750
467	0.0000	0.0000	0.0000
468	-3.3750	-5.0625	0.0675
469	-9.3125	-13.9688	0.1862
470	-5.7500	-8.6250	0.1150
471	-9.3125	-13.9688	0.1862
472	-3.3750	-5.0625	0.0675
473	-1.0000	-1.5000	0.0200
474	-0.6500	-0.9750	0.0130
475	-0.5500	-0.8250	0.0110
476	-0.1500	-0.2250	0.0030
477	-0.0500	-0.0750	0.0010
478	0.0500	0.0250	0.0010
479	0.1500	0.0750	0.0030
480	0.5500	0.2750	0.0110
481	0.6500	0.3250	0.0130
482	1.0000	0.5000	0.0200
483	0.0000	0.0000	0.0000
484	32.0000	16.0000	0.6400
485	0.0000	0.0000	0.0000
486	-12.0000	-18.0000	0.2400
487	0.0000	0.0000	0.0000
488	12.0000	6.0000	0.2400
489	0.0000	0.0000	0.0000
490	-32.0000	-48.0000	0.6400
491	0.0000	0.0000	0.0000
492	-3.5000	-5.2500	0.0700
493	16.3000	8.1500	0.3260
494	-0.3000	-0.4500	0.0060
495	-2.1000	-3.1500	0.0420
496	-1.5000	-2.2500	0.0300
497	-2.1000	-3.1500	0.0420
498	-0.3000	-0.4500	0.0060
499	16.3000	8.1500	0.3260
500	-3.5000	-5.2500	0.0700

KIJ	P=PARAM VECTOR	PS=START VALUE	PINC=INCREMENT
501	-1.0000	-1.5000	0.0200
502	-0.8500	-1.2750	0.0170
503	-0.7500	-1.1250	0.0150
504	-0.5000	-0.7500	0.0100
505	-0.4000	-0.6000	0.0080
506	-0.1500	-0.2250	0.0030
507	0.0500	0.0250	0.0010
508	0.1500	0.0750	0.0030
509	0.4500	0.2250	0.0090
510	0.5500	0.2750	0.0110
511	1.0000	0.5000	0.0200
512	0.0000	0.0000	0.0000
513	-2214.2900	-3321.4350	44.2858
514	0.0000	0.0000	0.0000
515	1914.2900	957.1450	38.2858
516	0.0000	0.0000	0.0000
517	-1650.0000	-2475.0000	33.0000
518	175.0000	87.5000	3.5000
519	0.0000	0.0000	0.0000
520	665.0000	332.5000	13.3000
521	0.0000	0.0000	0.0000
522	270.0000	135.0000	5.4000
523	-1612.1390	-2418.2090	32.2428
524	48.5714	24.2857	0.9714
525	1005.7140	502.8569	20.1143
526	240.0000	120.0000	4.8000
527	-7.5000	-11.2500	0.1500
528	-98.7500	-148.1250	1.9750
529	-72.5000	-108.7500	1.4500
530	-371.7500	-557.6250	7.4350
531	-6.0000	-9.0000	0.1200
532	-1.0000	-1.5000	0.0200
533	-0.5500	-0.8250	0.0110
534	-0.4500	-0.6750	0.0090
535	-0.0500	-0.0750	0.0010
536	0.1500	0.0750	0.0030
537	0.4000	0.2000	0.0080
538	0.5000	0.2500	0.0100
539	0.6500	0.3250	0.0130
540	0.7500	0.3750	0.0150
541	1.0000	0.5000	0.0200
542	0.0000	0.0000	0.0000
543	-645.0000	-967.5000	12.9000
544	0.0000	0.0000	0.0000
545	1448.2150	724.1074	28.9643
546	0.0000	0.0000	0.0000
547	-1531.4290	-2297.1440	30.6286
548	0.0000	0.0000	0.0000
549	993.3333	496.6665	19.8667
550	0.0000	0.0000	0.0000

KIJ	P=PARAM VECTOR	PS=START VALUE	PINC=INCREMENT
551	-8.0000	-12.0000	0.1600
552	-362.7500	-544.1250	7.2550
553	-72.5000	-108.7500	1.4500
554	-0.0867	-0.1301	0.0017
555	217.1430	108.5715	4.3429
556	829.7151	414.8574	16.5943
557	64.0000	32.0000	1.2800
558	-581.6670	-872.5005	11.6333
559	163.3333	81.6666	3.2667
560	0.0000	0.0000	0.0000
561	0.0000	0.0000	0.0000
562	0.0000	0.0000	0.0000
563	0.0000	0.0000	0.0000
564	0.0000	0.0000	0.0000
565	0.0000	0.0000	0.0000
566	2.0000	1.0000	0.0400
567	2.0000	1.0000	0.0400
568	2.0000	1.0000	0.0400
569	50.0000	25.0000	1.0000
570	50.0000	25.0000	1.0000
571	50.0000	25.0000	1.0000
572	0.0000	0.0000	0.0000
573	0.0000	0.0000	0.0000
574	0.0000	0.0000	0.0000
575	0.0000	0.0000	0.0000
576	0.0000	0.0000	0.0000
577	0.0000	0.0000	0.0000
578	140369.0000	70184.5000	2807.3790
579	0.0000	0.0000	0.0000
580	0.0000	0.0000	0.0000
581	0.1335	0.0667	0.0027
582	140369.0000	70184.5000	2807.3790
583	0.0000	0.0000	0.0000
584	0.0000	0.0000	0.0000
585	0.0000	0.0000	0.0000
586	4363.0000	2181.5000	87.2600
587	32.1726	16.0863	0.6435

APPENDIX A2

DSRV COEFFICIENT FUNCTIONS AND SELECTION CONSTANTS

The DSRV model uses 59 of the possible 126 second-degree coefficients. Of these 59, 33 are functions calculated by XFUNS, 24 are constants, some of which have regional peculiarities, and 2 are functions calculated by SECAL.

The functions and constants are selected in XFUNS using the vector (listed across page) XF. Absolute values are selected in XDCAL using the vector AV. The coefficient functions may be determined by looking at the corresponding branch of the initial conditional GO TO statement in subroutine XFUNS based upon the value of XF of the desired index.

XF(126) =	35	0	0	0	0	0	0	0	0	0	0	0	1	2
	3	0	4	0	5	0	6	0	35	0	0	36	0	35
	0	7	8	0	0	0	9	0	10	0	0	0	11	12
	35	0	37	35	0	0	0	13	0	14	15	0	0	0
	16	0	34	0	17	18	0	0	35	34	0	36	34	35
	19	20	34	21	34	0	0	0	22	0	34	0	23	0
	35	0	37	35	0	0	0	24	0	0	25	0	26	0
	0	0	27	0	28	0	34	0	35	0	0	36	0	35
	0	29	0	0	34	30	31	0	32	33	0	0	0	0
AV(126) =	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	-1	0	0	1	0	-1
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1	0	0	-1	0	0	0	0	0	0	0	0	0	0

(Contd.)

AV(126) = (Contd.)

$$
\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & 0 & -1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}
$$

APPENDIX A3

SECONDARY DRAG EQUATIONS

The secondary drag equations presented here are the state space form of the similar equations in Lockheed's model (B-9).

Variables	u	v	w	p	q	r
Indexes (K)	1	2	3	4	5	6
(KM)		6	5		3	2

Region Determination

Region 1 $\quad T < L_1 - L_S$ or $T > L_1$

Region 2 $\quad L_1 - L_S \leq T \leq L_1$

Values of T

v, r $\quad T = v/-r \qquad$ Indexes 2, 6

w, q $\quad T = w/q \qquad$ Indexes 3, 5

Values of L for DSRV

$L_1 = 23.40$, $L_S = 46.95$, $L_1 - L_S = -23.55$

Equations (KM = 8 - K)

Region 1

$$K = 2,3 \quad XC = C_{1K}\, x_K\, |x_K| + C_{2K}\, x_{KM}\, |x_K| + C_{3K}\, \text{Sign}\,(x_K)\, x_{KM}^2$$

$$K = 5,6 \quad XC = C_{6K}\, x_{KM}\, |x_{KM}| + C_{5K}\, x_K\, |x_{KM}| + C_{7K}\, \text{Sign}\,(x_K)\, x_K^2$$

Region 2

$$K = 2,3 \quad XC = \text{Sign}\,(x_{KM})\, x_K^2\, (C_{4K}\, x_K/x_{KM} + C_{5K}) + C_{6K}\, x_K\, |x_{KM}| + C_{7K}\, x_{KM}\, |x_{KM}|$$

$$K = 5,6 \quad XC = C_{1K}\, x_K\, |x_K| + \mathrm{Sign}\,(x_K)\, x_K^2 \left[C_{3K}\, (x_{KM}/x_K)^2 + C_{2K}\right] + C_{4K}\, x_{KM}\, |x_K|$$

Values of Forces and Moments

$K = 2,\ 6 \quad X = YC * Y_{vv} \qquad Y_{vv} = x\ (30) = X(24)$

$K = 3,\ 5 \quad X = XC * Z_{ww} \qquad Z_{ww} = x\ (54) = X(48)$

Values of Parameters C_{IK}

There are 28 values of these parameters $1 \leq I \leq 7$, $K = 2, 3, 5, 6$ in the vector P starting at P (72) = C_{12} and running in groups of 7 in the order of K.

IBM/SSP SUBROUTINES USED IN THIS THESIS

```
      SUBROUTINE GAUSS(IX,S,AM,V)
      A=0.0
      DO 50 I=1,12
      CALL RANDU(IX,IY,Y)
      IX=IY
   50 A=A+Y
      V=(A-6.0)*S+AM
      RETURN
      END
```

```
      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*65539
      IF(IY)5,6,6
    5 IY=IY+2147483647+1
    6 YFL=IY
      YFL=YFL*.4656613E-9
      RETURN
      END
```

```
      SUBROUTINE GMPRD(A,B,R,N,M,L)
      DIMENSION A(1),B(1),R(1)
      IR=0
      IK=-M
      DO 10 K=1,L
      IK=IK+M
      DO 10 J=1,N
      IR=IR+1
      JI=J-N
      IB=IK
      R(IR)=0.
      DO 10 I=1,M
      JI=JI+N
      IB=IB+1
   10 R(IR)=R(IR)+A(JI)*B(IB)
      RETURN
      END
```

```
      SUBROUTINE SIMQ(A,B,N,KS)
      DIMENSION A(1),B(1)
C
C        FORWARD SOLUTION
C
      TOL=0.0
      KS=0
      JJ=-N
      DO 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0
      IT=JJ-J
      DO 30 I=J,N
C
C        SEARCH FOR MAXIMUM COEFFICIENT IN COLUMN
C
      IJ=IT+I
      IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
   20 BIGA=A(IJ)
      IMAX=I
   30 CONTINUE
C
C        TEST FOR PIVOT LESS THAN TOLERANCE (SINGULAR MATRIX)
C
      IF(ABS(BIGA)-TOL) 35,35,40
   35 KS=1
      RETURN
C
C        INTERCHANGE ROWS IF NECESSARY
C
   40 I1=J+N*(J-2)
      IT=IMAX-J
      DO 50 K=J,N
      I1=I1+N
      I2=I1+IT
      SAVE=A(I1)
      A(I1)=A(I2)
      A(I2)=SAVE
C
C        DIVIDE EQUATION BY LEADING COEFFICIENT
C
   50 A(I1)=A(I1)/BIGA
      SAVE=B(IMAX)
      B(IMAX)=B(J)
      B(J)=SAVE/BIGA
C
C        ELIMINATE NEXT VARIABLE
C
      IF(J-N) 55,70,55
   55 IQS=N*(J-1)
      DO 65 IX=JY,N
      IXJ=IQS+IX
```

```
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
C
C        BACK SOLUTION
C
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END
```

```
      SUBROUTINE MINV(A,N,D,L,M)
      DIMENSION A(1),L(1),M(1)
C
C        SEARCH FOR LARGEST ELEMENT
C
      D=1.0
      NK=-N
      DO 80 K=1,N
      NK=NK+N
      L(K)=K
      M(K)=K
      KK=NK+K
      BIGA=A(KK)
      DO 20 J=K,N
      IZ=N*(J-1)
      DO 20 I=K,N
      IJ=IZ+I
   10 IF( ABS(BIGA)- ABS(A(IJ))) 15,20,20
   15 BIGA=A(IJ)
      L(K)=I
      M(K)=J
   20 CONTINUE
C
C        INTERCHANGE ROWS
C
      J=L(K)
      IF(J-K) 35,35,25
   25 KI=K-N
      DO 30 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
C
C        INTERCHANGE COLUMNS
C
   30 A(JI) =HOLD
   35 I=M(K)
      IF(I-K) 45,45,38
   38 JP=N*(I-1)
      DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
   40 A(JI) =HOLD
C
C        DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
C        CONTAINED IN BIGA)
C
   45 IF(BIGA) 48,46,48
   46 D=0.0
      RETURN
```

```
   48 DO 55 I=1,N
      IF(I-K) 50,55,50
   50 IK=NK+I
      A(IK)=A(IK)/(-BIGA)
   55 CONTINUE
C
C        REDUCE MATRIX
C
      DO 65 I=1,N
      IK=NK+I
      HOLD=A(IK)
      IJ=I-N
      DO 65 J=1,N
      IJ=IJ+N
      IF(I-K) 60,65,60
   60 IF(J-K) 62,65,62
   62 KJ=IJ-I+K
      A(IJ)=HOLD*A(KJ)+A(IJ)
   65 CONTINUE
C
C        DIVIDE ROW BY PIVOT
C
      KJ=K-N
      DO 75 J=1,N
      KJ=KJ+N
      IF(J-K) 70,75,70
   70 A(KJ)=A(KJ)/BIGA
   75 CONTINUE
C
C        PRODUCT OF PIVOTS
C
      D=D*BIGA
C
C        REPLACE PIVOT BY RECIPROCAL
C
      A(KK)=1.0/BIGA
   80 CONTINUE
C
C        FINAL ROW AND COLUMN INTERCHANGE
C
      K=N
  100 K=(K-1)
      IF(K) 150,150,105
  105 I=L(K)
      IF(I-K) 120,120,108
  108 JQ=N*(K-1)
      JR=N*(I-1)
      DO 110 J=1,N
      JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
      A(JK)=-A(JI)
  110 A(JI) =HOLD
```

```
120 J=M(K)
    IF(J-K) 100,100,125
125 KI=K-N
    DO 130 I=1,N
    KI=KI+N
    HOLD=A(KI)
    JI=KI-K+J
    A(KI)=-A(JI)
130 A(JI) =HOLD
    GO TO 100
150 RETURN
    END
```

CONTOURING AND PLOTTING SUBROUTINES

```
      SUBROUTINE CONTUR(NO,X,Y,Z,N,M,NS)
C
C     ..................................................................
C
C        SUBROUTINE CONTUR
C
C        PURPOSE
C           PLOT SEVERAL CONTOURS OF A Z VARIABLE VERSUS TWO
C           BASE VARIABLES X AND Y IN A FORM SUITABLE FOR
C           THESIS USE
C
C        USAGE
C           CALL CONTUR(NO,X,Y,Z,N,M,NS)
C
C        DESCRIPTION OF PARAMETERS
C           NO - CONTOUR NUMBER OF .LE. 3 DIGITS
C           X  - N VECTOR OF BASE VARIABLES, VERTICAL
C           Y  - M VECTOR OF BASE VARIABLES, HORIZONTAL
C           Z  - N*M VECTOR OF CONTOUR VARIABLES STORED
C                IN COLUMNWISE VECTOR FORM
C           N  - NUMBER OF ROWS IN Z. N MUST BE .LE.  47
C           M  - NUMBER OF COLUMNS IN Z. M MUST BE .LE.  51
C           NS - CODE FOR DATA SORTING IN ASCENDING ORDER
C                  0 - NO SORTING
C                  1 - SORT X
C                  2 - SORT Y
C                  3 - SORT BOTH X AND Y
C
      DIMENSION X(1),Y(1),Z(1)
C        REMARKS
C           (1)- THE CONTOUR IS CONSTRAINED BY THE FORMATS
C                TO A 47 LINE BY 51 SPACE ARRAY FOR THESIS
C                USAGE
C           (2)- THIS PROGRAM REPRESENTS A HIGHLY MODIFIED
C                VERSION OF SUBROUTINE PLOT IN THE IBM 360
C                SCIENTIFIC SUBROUTINE PACKAGE
C           (3)- SUBROUTINE CONTUR HAS TWO OUTPUTS
C                  1 - BOX LISTING THE CONTOUR PARAMETERS
C                      ON ONE PAGE
C                  2 - CONTOUR PLOTTED ON THE NEXT PAGE
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        AUTHOR
C           MICHAEL N. HAYES ,RM.5-333,MIT X6807
C
C     ..................................................................
C
      DIMENSION OUT(51),YPR(6),YPT(3)
      DIMENSION IANG(21),ZD(21)
     0INTEGER IANG/'1','2','3','4','5','6','7','8','9','A',
     1'B','C','D','E','F','G','H','J','K','L','M'/
```

```
      INTEGER OUT
C     FORMAT STATEMENTS FOR THESIS USE
    1 FORMAT(1H1,27X,7HCONTOUR,1X,I8)
    2 FORMAT(1H ,E10.3,'*',51A1,'*')
    3 FORMAT(1H ,10X,':',51X,':')
   40FORMAT(1H1,6X,36H************************************,
     115H***************)
   50FORMAT(1H ,5X,'*',12X,7HCONTOUR,1X,I3,12H  PARAMETERS,
     116X,'*')
   60FORMAT(1H ,5X,'*',10H X RANGE :,E11.4,4H TO ,E11.4,
     14H DX=,E11.4,'*')
   70FORMAT(1H ,11X,36H*oooo*oooo*oooo*oooo*oooo*oooo*oooo*,
     115Hoooo*oooo*oooo*)
   80FORMAT(1H ,3X,E9.2,1X,E9.2,1X,E9.2,1X,E9.2,1X,E9.2,1X,
     1E9.2)
   90FORMAT(1H ,5X,'*',10H Y RANGE :,E11.4,4H TO ,E11.4,
     14H DY=,E11.4,'*')
  130FORMAT(1H ,5X,'*',10H Z DOMAIN:,E11.4,4H TO ,E11.4,
     14H DZ=,E11.4,'*')
  170FORMAT(1H ,5X,'*',27H Z DOMAINS FOR THE CONTOURS,
     121H :MAX VALUES FOR EACH,3X,'*')
  200FORMAT(1H ,5X,'*',4H NO.,I2,E11.4,4H NO.,I2,
     1E11.4,4H NO.,I2,E11.4,'*')
  230FORMAT(1H ,6X,36H************************************,
     115H***************)
   60 FORMAT(1H ,18X,' *oooo* INCREMENT IS  ',E15.7)
   61 FORMAT(1H ,8X,E15.7,5X,E15.7,5X,E15.7)
      NL=47
      NTH=51
      ZCON=1.99
      NCONT=21
      NLL=NL
C     SORTING ROUTINES
      NSKIP=0
      IF(NS-1)105,101,102
  102 IF(NS-2)105,103,104
  104 NSKIP=1
C     SORT X
  101 DO 15 I=1,N
      DO 14 J=I,N
      IF(X(I)-X(J))14,14,11
   11 F=X(I)
      X(I)=X(J)
      X(J)=F
      L=I-N
      LL=J-N
      DO 12 K=1,M
      L=L+N
      LL=LL+N
      F=Z(L)
      Z(L)=Z(LL)
      Z(LL)=F
   12 CONTINUE
```

```
   14 CONTINUE
   15 CONTINUE
      IF(NSKIP)105,105,103
C     SORT Y
  103 DO 25 I=1,M
      DO 24 J=I,M
      IF(Y(I)-Y(J))24,24,21
   21 F=Y(I)
      Y(I)=Y(J)
      Y(J)=F
      L=(I-1)*N
      LL=(J-1)*N
      DO 22 K=1,N
      L=L+1
      LL=LL+1
      F=Z(L)
      Z(L)=Z(LL)
      Z(LL)=F
   22 CONTINUE
   24 CONTINUE
   25 CONTINUE
C     FIND BASE VARIABLE SCALES
  105 XSCAL=(X(N)-X(1))/(FLOAT(NLL-1))
      YSCAL=(Y(M)-Y(1))/(FLOAT(NTH-1))
C     SET BLANK
      BLANK=0
C     FIND CONTOUR VARIABLE SCALE
      ZMIN=1.0E75
      ZMAX=-1.0E75
      M1=1
      M2=N*M
      DO 40 J=M1,M2
      IF(Z(J).GT.ZMAX)ZMAX=Z(J)
      IF(Z(J).LT.ZMIN)ZMIN=Z(J)
   40 CONTINUE
C     LINEAR INTERPOLATION FOR NCONT CONTOURS
      ZSCAL=(ZMAX-ZMIN)/FLOAT(NCONT-1)
      IF(ZSCAL.EQ.0.)ZSCAL=1.0E-40
C     DEVELOP AND PRINT CONTOUR PARAMETER BOX
      XMIN=X(1)
      XMAX=X(N)
      YMIN=Y(1)
      YMAX=Y(M)
      WRITE(6,4)
      WRITE(6,5)NO
      WRITE(6,6)XMIN,XMAX,XSCAL
      WRITE(6,9)YMIN,YMAX,YSCAL
      WRITE(6,13)ZMIN,ZMAX,ZSCAL
      WRITE(6,17)
      ZD(1)=ZMIN+(2.0-ZCON)*ZSCAL
      NZCAL=NCONT-2
      DO 18 IZ=1,NZCAL
      ZD(IZ+1)=ZD(1)+FLOAT(IZ)*ZSCAL
```

```
   18 CONTINUE
      ZD(NCONT)=ZMAX
      IP=7
      DO 19 IZ=1,IP
      N1=IZ
      N2=N1+IP
      N3=N2+IP
      WRITE(6,20)N1,ZD(N1),N2,ZD(N2),N3,ZD(N3)
   19 CONTINUE
      WRITE(6,23)
C     CALCULATE THE Y SCALE VARIABLES
      YPR(1)=Y(1)
      DO 90 KN=1,4
      YPR(KN+1)=YPR(KN)+YSCAL*10.0
   90 CONTINUE
      YPR(6)=Y(M)
      YPT(1)=YMIN
      YPT(2)=YMIN+YSCAL*25.0
      YPT(3)=YMAX
      YSTAR=YSCAL*5.0
C     PRINT HEADING
      WRITE(6,1)NO
      WRITE(6,60)YSTAR
      WRITE(6,61)(YPT(IP),IP=1,3)
      WRITE(6,8)(YPR(IP),IP=1,6)
      WRITE(6,7)
C     FIND THE X SCALE PRINT POSITION
      XB=X(1)
      L=1
      I=1
      XEPS=XSCAL/FLOAT(2*(NLL-1))
      YEPS=YSCAL/FLOAT(2*(NTH-1))
   45 F=FLOAT(I-1)
      XPR=XB+F*XSCAL
      XDIF=X(L)-XPR-XEPS
      IF(XDIF)50,50,70
C     FIND THE Y SCALE PRINT POSITION
   50 DO 55 IX=1,NTH
      OUT(IX)=BLANK
   55 CONTINUE
      K=1
      LM=1
      YB=Y(1)
   35 G=FLOAT(K-1)
      YP=YB+G*YSCAL
   37 YDIF=Y(LM)-YP-YEPS
      IF(YDIF)30,30,31
C     FIND CONTOUR POSITION AND MAGNITUDE
   30 JZ=(LM-1)*N+L
      JP=IFIX(((Z(JZ)-ZMIN)/ZSCAL)+ZCON)
      OUT(K)=IANG(JP)
      LM=LM+1
   31 K=K+1
```

```
      IF(K-NTH)35,34,36
   34 YP=Y(M)
      GO TO 37
C     PRINT THE LINE
   36 WRITE(6,2)XPR,(OUT(IZ),IZ=1,NTH)
      L=L+1
      GO TO 80
C     SKIP THE LINE
   70 WRITE(6,3)
   80 I=I+1
      IF(I-NLL)45,84,86
   84 XPR=X(N)
      GO TO 50
C     PRINT BOTTOM AND Y VARIABLE SCALE
   86 WRITE(6,7)
      WRITE(6,8)(YPR(IP),IP=1,6)
      RETURN
      END
```

```
      SUBROUTINE PLOT(NO,A,N,M,NS)
C
C     ******************************************************************
C
C        SUBROUTINE PLOT
C
C        PURPOSE
C           PLOT SEVERAL CROSS VARIABLES Y VERSUS A BASE
C           VARIABLE X IN A FORMAT SUITABLE FOR THESIS USE
C
C        USAGE
C           CALL PLOT(NO,A,N,M,NS)
C
C        DESCRIPTION OF PARAMETERS
C           NO - PLOT NUMBER OF .LE.  3 DIGITS
C           A  - MATRIX OF DATA TO BE PLOTTED. MUST BE IN
C                STANDARD SINGLE COLUMN FORM. FIRST COLUMN
C                REPRESENTS BASE VARIABLE AND SUCCESSIVE
C                COLUMNS ARE THE CROSS VARIABLES (MAXIMUM IS
C                NINE). BASE VARIABLE IS VERTICAL.
C           N  - NUMBER OF ROWS IN MATRIX A. N MUST BE
C                .LE.  47
C           M  - NUMBER OF COLUMNS IN MATRIX A. M MUST BE
C                .LE   10
C           NS - CODE FOR SORTING THE BASE VARIABLE DATA IN
C                ASCENDING ORDER
C                  0  SORTING IS NOT NECESSARY(ALREADY IN
C                     ASCENDING ORDER)
C                  1  SORTING IS NECESSARY
C
      DIMENSION OUT(51),YPR(6),IANG(9),A(1),YPT(3)
C
C        REMARKS
C           (1)- THE PLOT IS CONSTRAINED BY THE FORMATS TO
C                A 47 LINE BY 51 SPACE ARRAY FOR THESIS USE
C           (2)- THIS PROGRAM REPRESENTS A SIGNIFICANTLY
C                MODIFIED VERSION OF ITS NAMESAKE IN THE
C                IBM 360 SCIENTIFIC SUBROUTINE PACKAGE
C           (3)- THE PLOT STARTS IN THE UPPER LEFT CORNER
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        AUTHOR
C           MICHAEL N. HAYES ,RM.5-333,MIT X6807
C
C     ******************************************************************
C
      INTEGER IANG/'1','2','3','4','5','6','7','8','9'/
      INTEGER OUT
C     FORMAT STATEMENTS FOR THESIS USE
    1 FORMAT(1H1,27X,7H  PLOT ,I8)
    2 FORMAT(1H ,E10.3,'*',51A1,'*')
```

```
    3 FORMAT(1H ,10X,':',51X,':')
    4 FORMAT(1H ,18X,' *....* INCREMENT IS  ',E15.7)
    5 FORMAT(1H ,8X,E15.7,5X,E15.7,5X,E15.7)
   70FORMAT(1H ,11X,36H*....*....*....*....*....*....*....*,
    115H....*....*....*)
   80FORMAT(1H ,3X,E9.2,1X,E9.2,1X,E9.2,1X,E9.2,1X,E9.2,1X,
    1E9.2)
    9 FORMAT(1H )
      NL=47
      NTH=51
      NLL=NL
      IF(NS)16,16,10
C     SORTING ROUTINE
   10 DO 15 I=1,N
      DO 14 J=I,N
      IF(A(I)-A(J))14,14,11
   11 L=I-N
      LL=J-N
      DO 12 K=1,M
      L=L+N
      LL=LL+N
      F=A(L)
      A(L)=A(LL)
   12 A(LL)=F
   14 CONTINUE
   15 CONTINUE
C     SET BLANK
   16 BLANK=0
C     FIND BASE AND CROSS VARIABLE SCALES
      XSCAL=(A(N)-A(1))/(FLOAT(NLL-1))
      M1=N+1
      YMAX=-1.0E75
      YMIN=1.0E75
      M2=M*N
      DO 40 J=M1,M2
      IF (A(J) .GT. YMAX) YMAX=A(J)
      IF (A(J) .LT. YMIN) YMIN=A(J)
   40 CONTINUE
      YSCAL=(YMAX-YMIN)/50.0
      IF(YSCAL.EQ.0.)YSCAL=1.0E-40
      YPR(1)=YMIN
      DO 90 KN=1,4
      YPR(KN+1)=YPR(KN)+YSCAL*10.0
   90 CONTINUE
      YPR(6)=YMAX
      YPT(1)=YMIN
      YSTAR=YSCAL*5.0
      YPT(2)=YMIN+YSCAL*25.0
      YPT(3)=YMAX
C     PRINT HEADING AND CROSS VARIABLE SCALE
      WRITE(6,1)NO
      WRITE(6,4)YSTAR
      WRITE(6,5)(YPT(IP),IP=1,3)
```

```
      WRITE(6,8)(YPR(IP),IP=1,6)
      WRITE(6,7)
C     FIND BASE VARIABLE PRINT POSITION
      XB=A(1)
      L=1
      MY=M-1
      I=1
      XEPS=XSCAL/FLOAT(2*(NLL-1))
   45 F=FLOAT(I-1)
      XPR=XB+F*XSCAL
      XDIF=A(L)-XPR-XEPS
      IF(XDIF)50,50,70
C     FIND CROSS VARIABLES
   50 DO 55 IX=1,NTH
      OUT(IX)=BLANK
   55 CONTINUE
      DO 60 J=1,MY
      LL=L+J*N
      JP=((A(LL)-YMIN)/YSCAL)+1.0
      OUT(JP)=IANG(J)
   60 CONTINUE
C     PRINT LINE AND CLEAR, OR SKIP
      WRITE(6,2)XPR,(OUT(IZ),IZ=1,NTH)
      L=L+1
      GO TO 80
   70 WRITE(6,3)
   80 I=I+1
      IF(I-NLL)45,84,86
   84 XPR=A(N)
      GO TO 50
C     PRINT BOTTOM AND CROSS VARIABLE SCALE
   86 WRITE(6,7)
      WRITE(6,8)(YPR(IP),IP=1,6)
      WRITE(6,9)
      RETURN
      END
```

CHAPTER D2 SUBROUTINES (OVMOD)

```
      SUBROUTINE OVMOD(ME,AV,XF,P,IN,U,NE,X,XN,XD,XCN,KB,L)
C *********************************************************************
C SUBROUTINE CVMOD
C
C PURPOSE
C   TC COMPUTE ANY SELECTED CCMBINATION CF THE TOTAL LINEAR
C   AND ANGULAR ACCELERATIONS AT A GIVEN TIME FOR A GENERAL
C   OCEAN VEHICLE INCLUDING ALL DESIRED INPUTS AND,IF
C   SPECIFIED,A NOISY ENVIRONMENT
C     EQUATION-
C       ADX/CT=EE(XKIJ*XI*XJ)+XEFFECTORS+XGRAVITY+
C               XDISTURBANCES+XCCNSTANT+XSECCNDARY
C       WHERE- E IS SUMMATICN FOR 6 DEGREES OF FREEDOM OR
C               ANY SELECTED COMBINATICN CF 6 DEGREES
C
C LANGUAGE : FORTRAN IV
C
C SUBROUTINES REQUIRED
C   TANKS,GRAVT,ACALC,SHCAL,XDCAL,SIMQ,XDIST
C
C DESCRIPTION
C   CHAPTER D2,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C *********************************************************************
      INTEGER AV,XF
     0DIMENSION ME(1),AV(1),XF(1),P(1),IN(1),U(1),NE(1),
     1X(1),XN(1),XD(1),XCN(1),L(1)
      DIMENSION XGRAV(6),XPD(6),PG(8),XS(6),A(36),AN(36)
C     SETUP FOR TANKS AND GRAVITY CALCULATICNS
C     SET PG INITIALLY FROM P
      MT=ME(17)-1
      DO 5 KT=1,8
      PG(KT)=P(MT+KT)
    5 CONTINUE
C     TEST FOR TANKS CALCULATICNS
      IF(ME(1).NE.1)GO TO 1
      CALL TANKS(P,ME(10),U,NE(1),PG)
C     CALCULATE GRAVITY FORCES AND MCMENTS
    1 CALL GRAVT(ME(8),U,NE(10),PG,XGRAV,KB,L)
C     CALCULATE THE DERIVATIVE TRANSFORMATICN MATRIX - A
      MG=ME(18)+1
      G=P(MG)
      CALL ACALC(P,PG,G,A,KB,L)
      IKB=KB*KB
      DO 19 IA=1,IKB
      AN(IA)=A(IA)
   19 CONTINUE
C     BEGIN NOISY AND NOISELESS DERIVATIVE EVALUATIONS
      ISW=1
      IF(ME(6).EQ.1)GO TO 3
C     TEST FOR SHROUD CALCULATICN - NOISELESS
    2 IF(ME(3).NE.1)GO TO 6
     0CALL SHCAL(P,ME(12),X,U,NE(7),NE(2),NE(11),NE(12),
```

```
     1XS,KB,L)
C      CALCULATE NOISELESS FORCES AND MOMENTS
    6 DO 7 K=1,KB
      KX=L(K)
      CALL XDCAL(ME,AV,XF,P,PG,U,NE,X,XDC,KB,L,KX)
      IF(ME(3).NE.1)GO TO 9
      XPD(K)=XDC+XGRAV(KX)+XS(KX)
      GO TO 7
    9 XPD(K)=XDC+XGRAV(KX)
    7 CONTINUE
C      SETUP AND SOLVE SIMULTANEOUS EQUATIONS FOR
C      NOISELESS DERIVATIVES
      IF(KB.EQ.1)GO TO 20
      CALL SIMQ(A,XPD,KB,KSIM)
C      TEST FOR VALID SOLUTION
      IF(KSIM.EQ.0)GO TO 8
   16 WRITE(6,18)
   18 FORMAT(1X,'SIMQ IN OVMOD GIVEN SINGULAR A-MATRIX')
      WRITE(6,15)(PG(J),J=1,8)
   15 FORMAT(1X,'PG=',8E15.7)
      DO 10 K=1,KB
      XD(K)=0.
      XDN(K)=0.
   10 CONTINUE
      RETURN
   20 IF(A(1).EQ.0.)GO TO 16
      XPD(1)=XPD(1)/A(1)
C      PLACE CALCULATED NOISELESS DERIVATIVES IN XD
    8 DO 11 K=1,KB
      XD(K)=XPD(K)
   11 CONTINUE
      GO TO 4
C      CALCULATE NOISY FORCES AND MOMENTS
    3 IF(ME(9).EQ.1)ISW=2
C      TEST FOR SHROUD CALCULATION - NOISY
      IF(ME(3).NE.1)GO TO 12
     0CALL SHCAL(P,ME(12),XN,U,NE(7),NE(2),NE(11),NE(12),
     1XS,KB,L)
C      CALCULATE NOISY FORCES AND MOMENTS
   12 DO 13 K=1,KB
      KX=L(K)
      CALL XDCAL(ME,AV,XF,P,PG,U,NE,XN,XDC,KB,L,KX)
      CALL XDIST(IN,P,ME(15),XNO,KX,6)
      IF(ME(3).NE.1)GO TO 14
      XPD(K)=XDC+XGRAV(KX)+XS(KX)+XNO
      GO TO 13
   14 XPD(K)=XDC+XGRAV(KX)+XNO
   13 CONTINUE
C      SETUP AND SOLVE SIMULTANEOUS EQUATIONS FOR
C      NOISY DERIVATIVES
      IF(KB.EQ.1)GO TO 21
      CALL SIMQ(AN,XPD,KB,KSIM)
C      TEST FOR VALID SOLUTION
```

```
      IF(KSIM.EQ.1)GO TO 16
      GO TO 22
   21 IF(AN(1).EQ.0.)GO TO 16
      XPD(1)=XPD(1)/AN(1)
C     PLACE CALCULATED NOISY DERIVATIVES IN XDN
   22 DO 17 K=1,KB
      XDN(K)=XPD(K)
   17 CONTINUE
      GO TO (4,2),ISW
    4 RETURN
      END
```

```
      SUBROUTINE ACALC(P,PG,G,A,KB,L)
C ****************************************************************
C SUBROUTINE ACALC
C
C PURPOSE
C   TO CALCULATE THE OCEAN VEHICLE A-MATRIX CONTAINING
C   ADDED MASS AND INERTIA TERMS
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   CHAPTER D2,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ****************************************************************
      DIMENSION P(1),PG(1),A(1),L(1)
      J=1
      VM=PG(1)/G
      DO 38 K=1,KB
      KM=L(K)
      DO 39 I=1,KB
      IM=L(I)
      N=IM+6*(KM-1)
     0GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,
     119,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,
     236),N
    1 AC=VM-P(1)
      GO TO 37
    2 AC=0.
      GO TO 37
    3 AC=0.
      GO TO 37
    4 AC=0.
      GO TO 37
    5 AC=VM*PG(4)
      GO TO 37
    6 AC=-VM*PG(3)
      GO TO 37
    7 AC=0.
      GO TO 37
    8 AC=VM-P(8)
      GO TO 37
    9 AC=0.
      GO TO 37
   10 AC=-VM*PG(4)-P(20)
      GO TO 37
   11 AC=0.
      GO TO 37
   12 AC=VM*PG(2)-P(32)
      GO TO 37
   13 AC=0.
      GO TO 37
   14 AC=0.
      GO TO 37
```

```
15 AC=VM-P(15)
   GO TO 37
16 AC=VM*PG(3)
   GO TO 37
17 AC=-VM*PG(2)-P(27)
   GO TO 37
18 AC=0.
   GO TO 37
19 AC=0.
   GO TO 37
20 AC=-VM*PG(4)-P(10)
   GO TO 37
21 AC=VM*PG(3)
   GO TO 37
22 AC=P(37)-P(22)
   GO TO 37
23 AC=0.
   GO TO 37
24 AC=0.
   GO TO 37
25 AC=VM*PG(4)
   GO TO 37
26 AC=0.
   GO TO 37
27 AC=-VM*PG(2)-P(17)
   GO TO 37
28 AC=0.
   GO TO 37
29 AC=P(39)-P(29)
   GO TO 37
30 AC=0.
   GO TO 37
31 AC=-VM*PG(3)
   GO TO 37
32 AC=VM*PG(2)-P(12)
   GO TO 37
33 AC=0.
   GO TO 37
34 AC=0.
   GO TO 37
35 AC=0.
   GO TO 37
36 AC=P(42)-P(36)
37 A(J)=AC
   J=J+1
39 CONTINUE
38 CONTINUE
   RETURN
   END
```

```
      SUBROUTINE XDCAL(ME,AV,XF,P,PG,U,N,X,XD,KB,L,K)
C ***********************************************************************
C SUBROUTINE XDCAL
C
C PURPOSE
C   TO COMBINE COEFFICIENT AND EFFECTOR FORCES WITH ANY
C   SELECTED GROUP OF EFFECTORS AND ANY SELECTED GROUP OF 6
C   DEGREES OF FREEDOM , SUBROUTINES ARE SPECIFIC TO THE
C   DSRV , ONLY SECOND DEGREE COEFFICIENTS USED
C
C SUBROUTINES REQUIRED
C   XFUNS,SECAL,PRCAL,THCAL,COCAL
C
C DESCRIPTION
C   SEE SUBROUTINE DVMOD
C   CHAPTER D2,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***********************************************************************
      INTEGER AV,XF
      DIMENSION ME(1),AV(1),XF(1),P(1),PG(1),U(1),N(1),X(1)
      DIMENSION L(1),PC(8),NT(8)
C     SET PARAMETERS FOR XFUNS
      MC=ME(18)
      PC(1)=PG(1)/P(MC+1)
      PC(2)=PG(2)
      PC(3)=PG(3)
      PC(4)=PG(4)
      PC(5)=P(37)
      PC(6)=P(39)
      PC(7)=P(42)
      PC(8)=P(43)
C     BEGIN PRIMARY TERM CALCULATIONS
      XD=0.
      DO 1 I=1,KB
      IM=L(I)
      DO 2 J=I,KB
C     CALCULATE PRIMARY TERM INDICES
      JM=L(J)
      IJK=IM+(JM*(JM-1))/2+21*(K-1)
      KIJ=IJK+6
C     TEST FOR TERM TO BE SKIPPED
      IF(XF(IJK).EQ.0)GO TO 2
      CALL XFUNS(PC,X,KIJ,XC,XF)
C     TEST FOR ABSOLUTE VALUE TERMS AND CALCULATE
      IF(AV(IJK))8,10,9
    8 XD=XD+XC*ABS(X(IM))*X(JM)
      GO TO 2
    9 XD=XD+XC*X(IM)*ABS(X(JM))
      GO TO 2
   10 XD=XD+XC*X(IM)*X(JM)
    2 CONTINUE
    1 CONTINUE
C     SECONDARY TERM CALCULATIONS
```

```
      IF(ME(2).NE.1)GO TO 3
      CALL SECAL(P,ME(11),X,XS,K,XSO,XST)
      XD=XD+XS
C     PROPELLOR CALCULATIONS
    3 IF(ME(4).NE.1)GO TO 4
      CALL PRCAL(P,ME(13),X,U,N(8),N(3),XP,K)
      XD=XD+XP
C     THRUSTER CALCULATIONS
    4 IF(ME(5).NE.1)GO TO 5
      NT(1)=N(4)
      NT(2)=N(9)
      DO 7 I=3,8
      NT(I)=N(I+10)
    7 CONTINUE
      CALL THCAL(P,ME(14),U,NT,X,XT,K)
      XD=XD+XT
C     CONSTANT TERMS CALCULATIONS
    5 IF(ME(7).NE.1)GO TO 6
      CALL COCAL(P,ME(16),X,XCO,K)
      XD=XD+XCO
    6 RETURN
      END
```

```
      SUBROUTINE XFUNS(P,X,K,XC,I)
C ***************************************************************
C SUBROUTINE XFUNS
C
C PURPOSE
C   TO COMPUTE THE FUNCTIONAL VALUES OF THE SECOND DEGREE
C   COEFFICIENTS OF THE DSRV
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   SEE SUBROUTINE XDCAL
C   CHAPTER D2,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***************************************************************
      DIMENSION P(1),X(1),I(1)
      ISW=I(K-6)
C     INITIAL CONDITIONAL GO TO STATEMENT, BRANCH TO
C     PROPER FUNCTION
     0GO TO (13,14,15,17,19,21,30,31,35,37,41,42,50,52,53,
     157,61,62,71,72,74,79,83,92,95,97,101,103,114,118,119,
     2121,122,127,128,129,130),ISW
   13 XC=X(K)-P(1)
      RETURN
   14 XC=P(1)*P(3)
      RETURN
   15 XC=X(K)+P(1)*P(2)
      RETURN
   17 XC=X(K)+P(1)
      RETURN
   19 XC=X(K)-P(1)*P(4)
      RETURN
   21 XC=X(K)+P(1)*P(2)
      RETURN
   30 XC=X(K)+P(1)
      RETURN
   31 XC=P(1)*P(3)
      RETURN
   35 XC=X(K)-P(1)*P(2)
      RETURN
   37 IF(X(1).LT.0.)GO TO 131
      XC=X(K)*SIGN(1.,X(1))-P(1)
      RETURN
  131 XC=X(K+126)*SIGN(1.,X(1))-P(1)
      RETURN
   41 XC=-P(1)*P(4)
      RETURN
   42 XC=P(1)*P(3)
      RETURN
   50 XC=X(K)-P(1)
      RETURN
   52 XC=X(K)+P(1)*P(4)
```

```
      RETURN
   53 IF(X(1).LT.0.)GO TO 132
      XC=X(K)*SIGN(1.,X(1))+P(1)
      RETURN
  132 XC=X(K+126)*SIGN(1.,X(1))+P(1)
      RETURN
   57 XC=P(1)*P(4)
      RETURN
   61 XC=X(K)-P(1)*P(2)
      RETURN
   62 XC=-P(1)*P(3)
      RETURN
   71 XC=-P(1)*P(3)
      RETURN
   72 XC=X(K)-P(1)*P(4)
      RETURN
   74 XC=P(1)*P(3)
      RETURN
   79 IF(X(1).LT.0.)GO TO 133
      XC=X(K)*SIGN(1.,X(1))+P(1)*P(4)
      RETURN
  133 XC=X(K+126)*SIGN(1.,X(1))+P(1)*P(4)
      RETURN
   83 XC=X(K)+P(6)-P(7)
      RETURN
   92 XC=X(K)+P(1)*P(2)
      RETURN
   95 IF(X(1).LT.0.)GO TO 134
      XC=X(K)*SIGN(1.,X(1))-P(1)*P(2)
      RETURN
  134 XC=X(K+126)*SIGN(1.,X(1))-P(1)*P(2)
      RETURN
   97 XC=-P(1)*P(4)
      RETURN
  101 XC=X(K)+P(1)*P(4)
      RETURN
  103 XC=X(K)+P(7)-P(5)
      RETURN
  114 XC=X(K)+P(1)*P(2)
      RETURN
  118 XC=P(1)*P(3)
      RETURN
  119 XC=X(K)+P(5)-P(6)
      RETURN
  121 IF(X(1).LT.0.)GO TO 135
      XC=X(K)*SIGN(1.,X(1))-P(1)*P(2)
      RETURN
  135 XC=X(K+126)*SIGN(1.,X(1))-P(1)*P(2)
      RETURN
  122 XC=-P(1)*P(3)
      RETURN
  127 XC=X(K)
      RETURN
```

```
123 IF(X(1).LT.0.)GO TO 136
    XC=X(K)
    RETURN
136 XC=X(K+126)
    RETURN
129 IF(X(3).LT.0.)GO TO 137
    XC=X(K)
    RETURN
137 XC=0.
    RETURN
130 IF(X(1))139,138,138
133 IF((X(2)-P( 8)*X(1)).GT.0.)GO TO 137
    XC=X(K)
    RETURN
139 IF((X(2)-P( 8)*X(1)).LT.0.)GO TO 137
    XC=X(K)
    RETURN
    END
```

```
      SUBROUTINE SECAL(P,M,X,XS,K,XSO,XST)
C ***********************************************************************
C SUBROUTINE SECAL
C
C PURPOSE
C   TO CALCULATE THE SECONDARY DRAG FORCES AND MOMENTS FOR
C   THE DSRV
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   CHAPTER D2,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***********************************************************************
      DIMENSION P(1),X(1)
C     INITIAL SECONDARY DRAG REGION DETERMINATION
      IF((K.EQ.1).OR.(K.EQ.4))GO TO 1
      KM=8-K
      IF((K.EQ.2).OR.(K.EQ.6))ISW=1
      IF((K.EQ.3).OR.(K.EQ.5))ISW=2
      GO TO (2,3),ISW
C     K=2 OR K=6 REGION DETERMINATION
    2 TA=X(2)+P(M+30)*X(6)
      TB=X(2)+P(M+29)*X(6)
      XC=X(30)
      IF(X(6))17,16,16
   16 IF((TA.GE.0.).OR.(TB.LE.0.))GO TO 4
      ISW=2
      GO TO 5
   17 IF((TA.LE.0.).OR.(TB.GE.0.))GO TO 4
      ISW=2
      GO TO 5
    4 ISW=1
    5 GO TO (8,9),ISW
    1 XS=0.
      XSO=0.
      XST=0.
      RETURN
C     K=3 OR K=5 REGION DETERMINATION
    3 TA=X(3)-P(M+30)*X(5)
      TB=X(3)-P(M+29)*X(5)
      XC=X(54)
      IF(X(5))19,18,18
   18 IF((TA.LE.0.).OR.(TB.GE.0.))GO TO 6
      ISW=2
      GO TO 7
   19 IF((TA.GE.0.).OR.(TB.LE.0.))GO TO 6
      ISW=2
      GO TO 7
    6 ISW=1
    7 GO TO (10,11),ISW
C     BEGIN SELECTED SECONDARY DRAG CALCULATIONS
```

```
    8 IF(K.EQ.2)GO TO 14
C      K=5 OR K=6 REGION ONE
   12 N=M+19+(K-5)*7
     0XS=ABS(X(KM))*(P(N)*X(KM)+P(N-1)*X(K))+
     1P(N+1)*SIGN(1.,X(KM))*X(K)*X(K)
      XSO=XS
      XST=XS
      XS=XS*XC
      RETURN
    9 IF(K.EQ.2)GO TO 15
C      K=5 OR K=6 REGION TWO
   13 N=M+14+(K-5)*7
     0XS=P(N)*X(K)*ABS(X(K))+P(N+3)*X(KM)*ABS(X(K))+
     1SIGN(1.,X(K))*X(K)*X(K)*(P(N+2)*((X(KM)/X(K))**2 )+
     2P(N+1))
      XSO=XS
      XST=XS
      XS=XS*XC
      RETURN
   10 IF(K.EQ.5)GO TO 12
C      K=2 OR K=3 REGION ONE
   14 N=M+(K-2)*7
     0XS=ABS(X(K))*(P(N)*X(K)+P(N+1)*X(KM))+
     1P(N+2)*SIGN(1.,X(K))*X(KM)*X(KM)
      XSO=XS
      XST=XS
      XS=XS*XC
      RETURN
   11 IF(K.EQ.5)GO TO 13
C      K=2 OR K=3 REGION TWO
   15 N=M+3+(K-2)*7
     0XS=SIGN(1.,X(KM))*X(K)*X(K)*(P(N)*(X(K)/X(KM))+P(N+1))
     1+P(N+2)*X(K)*ABS(X(KM))+P(N+3)*X(KM)*ABS(X(KM))
      XSO=XS
      XST=XS
      XS=XS*XC
      RETURN
      END
```

CHAPTER D3 SUBROUTINES (EFFECTORS)

```
      SUBROUTINE TANKS(P,M,U,NT,PG)
C ********************************************************************
C SUBROUTINE TANKS
C
C PURPOSE
C   TO COMPUTE THE VEHICLE WEIGHT AND CENTER OF GRAVITY OF
C   A GENERAL OCEAN VEHICLE BASED UPON ITS TANK LOADINGS
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   CHAPTER D3,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ********************************************************************
      DIMENSION P(1),U(1),PG(1)
      DW=0.
      XDW=0.
      YDW=0.
      ZDW=0.
      J=M
      DO 1 I=1,NT
      D=U(I)-P(J)
      DW=DW+D
      K=J+NT
      XDW=XDW+D*P(K)
      K=K+NT
      YDW=YDW+D*P(K)
      K=K+NT
      ZDW=ZDW+D*P(K)
      J=J+1
    1 CONTINUE
      WT=PG(1)+DW
      PG(2)=(PG(2)*PG(1)+XDW)/WT
      PG(3)=(PG(3)*PG(1)+YDW)/WT
      PG(4)=(PG(4)*PG(1)+ZDW)/WT
      PG(1)=WT
      RETURN
      END
```

```
      SUBROUTINE GRAVT(M,A,N,P,X,K,L)
C ******************************************************************
C SUBROUTINE GRAVT
C
C PURPOSE
C   TO COMPUTE THE GRAVITY FORCES AND MOMENTS ON A GENERAL
C   OCEAN VEHICLE BASED UPON ITS WEIGHT,BUOYANCY,CENTERS OF
C   GRAVITY AND BUOYANCY,AND VEHICLE ANGLES
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   CHAPTER D3,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ******************************************************************
      LOGICAL AM
      DIMENSION A(1),P(1),X(1),L(1)
      WMB=P(1)-P(5)
      SPH=SIN(A(N))
      CPH=COS(A(N))
      STH=SIN(A(N+1))
      CTH=COS(A(N+1))
      AM=M.EQ.1
      DO 10 I=1,K
      J=L(I)
      IF(AM.AND.(J.GT.3))J=J+3
      GO TO (1,2,3,4,5,6,7,8,9),J
C     FORCE EQUATIONS FOR BOTH ZERO AND NON-ZERO C.B.
    1 X(J)=-WMB*STH
      GO TO 10
    2 X(J)=WMB*CTH*SPH
      GO TO 10
    3 X(J)=WMB*CTH*CPH
      GO TO 10
C     MOMENT EQUATIONS FOR ZERO CENTER OF BUOYANCY
    4 X(J)=(P(3)*CPH-P(4)*SPH)*P(1)*CTH
      GO TO 10
    5 X(J)=-P(1)*(P(2)*CTH*CPH+P(4)*STH)
      GO TO 10
    6 X(J)=P(1)*(P(2)*CTH*SPH+P(3)*STH)
      GO TO 10
C     MOMENT EQUATIONS FOR NON-ZERO CENTER OF BUOYANCY
    7 X(J-3)=CTH*((P(3)*P(1)-P(7)*P(5))*CPH-(P(4)*P(1)-P(8)*
     1P(5))*SPH)
      GO TO 10
    8 X(J-3)=-(P(2)*P(1)-P(6)*P(5))*CTH*CPH-(P(4)*P(1)-P(8)*
     1P(5))*STH
      GO TO 10
    9 X(J-3)=(P(2)*P(1)-P(6)*P(5))*CTH*SPH+(P(3)*P(1)-P(7)*
     1P(5))*STH
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SHCAL(P,M,X,U,N,NS,NL,ND,XS,KB,L)
C ***********************************************************************
C SUBROUTINE SHCAL
C
C PURPOSE
C   TO CALCULATE THE FORCES AND MOMENTS ON THE DSRV CAUSED
C   BY SHROUD DEFLECTION ANGLES
C
C SUBROUTINES REQUIRED
C   UTCOM,ASCOM,RGCOM,SIMQ
C
C DESCRIPTION
C   CHAPTER D3,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***********************************************************************
      DIMENSION P(1),X(1),U(1),XS(1),L(1)
      DIMENSION A(18),UT(3),FU(3)
      IF((KB.LT.2).AND.(L(1).EQ.4))GO TO 1
      XSHR=P(M+47)
      CALL UTCOM(U,N,XSHR,A)
      RAD=57.2957795
C     CALCULATE U2S(1)
      VSQ=0.
      DO 2 I=1,3
      UT(I)=0.
      DO 3 K=1,KB
      J=L(K)
      MN=I+3*J-3
      UT(I)=UT(I)+A(MN)*X(J)
    3 CONTINUE
      VSQ=VSQ+UT(I)*UT(I)
    2 CONTINUE
C     CALCULATE SHROUD ANGLE ALPHA-S
      EPS=P(M+50)
      CALL ASCOM(UT,AS,IND,EPS,VNS)
      IF(AS.LT.0.)AS=AS+3.14159265
      AS=AS*RAD
C     TEST IND AND INCREMENT A SHROUD RUN PARAMETER
      IF(IND.EQ.1)P(M+51)=P(M+51)+1.
C     COMPUTE LIFT AND DRAG COEFFICIENTS
      CALL RGCOM(M,NL,P,IND,AS,CL)
      MN=M+3*NL+1
      CALL RGCOM(MN,ND,P,IND,AS,CD)
C     COMPUTE VLIFT AND VDRAG
      B=0.5*P(M+49)*VSQ*P(M+48)
      VLIFT=B*CL
      VDRAG=B*CD
C     COMPUTE VNF AND FU(1)
      AS=AS/RAD
      CAS=COS(AS)
      SAS=SIN(AS)
      VNF=VLIFT*CAS+VDRAG*SAS
      FU(1)=VLIFT*SAS-VDRAG*CAS
```

```
C     TEST VNS FROM ASCCM
      IF(VNS.EQ.0.)VNS=EPS
      FU(2)=-UT(2)*VNF/VNS
      FU(3)=-UT(3)*VNF/VNS
C     SOLVE SIMULTANEOUS EQUATIONS FOR SHROUD FORCES
      XS(1)=FU(1)
      XS(2)=FU(2)
      XS(3)=FU(3)
      CALL SIMQ(A,XS,3,IND)
C     TEST SIMQ INDICATOR
      IF(IND.EQ.0)GO TO 4
      WRITE(6,5)
    5 FORMAT(1X,'SIMQ IN SHCAL GIVEN SINGULAR SYSTEM')
      DO 6 K=1,KB
      KM=L(K)
      XS(KM)=0.
    6 CONTINUE
      RETURN
C     SELECT AND CALCULATE MOMENTS
    4 DO 7 K=1,KB
      IF(L(K).LT.5)GO TO 7
      IF(L(K).EQ.5)XS(5)=XSHR*XS(3)
      IF(L(K).EQ.6)XS(6)=-XSHR*XS(2)
    7 CONTINUE
    1 XS(4)=0.
      RETURN
      END
```

```
      SUBROUTINE RGCOM(M,N,P,I,X,Y)
C ***********************************************************************
C SUBROUTINE RGCOM
C
C PURPOSE
C   TO COMPUTE A LINEAR VALUE WITHIN A RANGE GIVEN STRAIGHT
C   LINE SLOPES AND INTERCEPTS
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   SEE SUBROUTINES SHCAL AND THCAL
C   CHAPTER D3,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***********************************************************************
      DIMENSION P(1)
      DO 1 K=1,N
      IF(X.GT.P(M+K))GO TO 1
      I=K
      MV=M+N+K
      Y=P(MV)*X+P(MV+N)
      RETURN
    1 CONTINUE
      I=-1
      Y=0.
      RETURN
      END
```

```
      SUBROUTINE ASCOM(UT,A,I,E,V)
C ********************************************************************
C SUBROUTINE ASCOM
C
C PURPOSE
C   TO COMPUTE SHROUD ANGLE ALPHA-S
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   SEE SUBROUTINE SHCAL
C   CHAPTER D3,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ********************************************************************
      DIMENSION UT(1)
      I=0
C     TEST UT(1) FOR ZERO
      IF(UT(1).EQ.0.)GO TO 1
    2 V=SQRT(UT(2)*UT(2)+UT(3)*UT(3))
      A=ATAN(V/UT(1))
      RETURN
    1 UT(1)=E
      I=1
      GO TO 2
      END
```

```
      SUBROUTINE UTCOM(U,N,X,A)
C *****************************************************************
C SUBROUTINE UTCOM
C
C PURPOSE
C   TO COMPUTE THE DSRV SHROUD ANGLE TRANSFORMATION MATRIX
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   SEE SUBROUTINE SHCAL
C   CHAPTER D3,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C *****************************************************************
      DIMENSION U(1),A(1)
      CP=COS(U(N))
      SP=SIN(U(N))
      CY=COS(U(N+1))
      SY=SIN(U(N+1))
      A(1)=CY*CP
      A(2)=-SY*CP
      A(3)=SP
      A(4)=SY
      A(5)=CY
      A(6)=0.
      A(7)=-CY*SP
      A(8)=SY*SP
      A(9)=CP
      A(10)=0.
      A(11)=0.
      A(12)=0.
      A(13)=X*A(7)
      A(14)=X*A(8)
      A(15)=X*A(9)
      A(16)=-X*A(4)
      A(17)=-X*A(5)
      A(18)=-X*A(6)
      RETURN
      END
```

```
      SUBROUTINE PRCAL(P,M,X,U,N,NP,XP,K)
C ************************************************************************
C SUBROUTINE PRCAL
C
C PURPOSE
C   TO CALCULATE THE FORCES AND MOMENTS ON THE DSRV CAUSED
C   BY PROPELLOR ROTATION AND ROTATIONAL ACCELERATION
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   CHAPTER D3,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ************************************************************************
      DIMENSION P(1),X(1),U(1)
      LOGICAL A,B,C
      PM=P(M+36)
      VP=X(2)+PM*X(6)
      WP=X(3)-PM*X(5)
      GO TO (1,2,3,4,5,6),K
C     SURGE REGION DETERMINATION AND EQUATION SELECTION
    1 A=X(1).GE.0.
      B=U(N).GE.0.
      C=(U(N)-P(M+35)*X(1)).GE.0.
      PV=P(M+32)*U(N)*SQRT(VP*VP+WP*WP)
      IF(A.AND.C)ISW=1
      IF(A.AND..NOT.C)ISW=2
      IF(.NOT.A.AND.B)ISW=3
      IF(.NOT.(A.OR.B))ISW=4
      GO TO (7,8,9,10),ISW
C     PROPELLOR SURGE EQUATIONS
    70XP=P(M)*U(N)*ABS(U(N))+P(M+1)*X(1)*U(N)+
     1P(M+2)*X(1)*X(1)+PV
      RETURN
    80XP=P(M+3)*U(N)*ABS(U(N))+P(M+4)*X(1)*U(N)+
     1P(M+5)*X(1)*X(1)+PV
      RETURN
    90XP=P(M+6)*U(N)*ABS(U(N))+P(M+7)*X(1)*U(N)+
     1P(M+8)*X(1)*X(1)+PV
      RETURN
   100XP=P(M+9)*U(N)*ABS(U(N))+P(M+10)*X(1)*U(N)+
     1P(M+11)*X(1)*X(1)+PV
      RETURN
C     SWAY REGION DETERMINATION AND EQUATION SELECTION
    2 IF(U(N).LT.0.)GO TO 11
C     PROPELLOR SWAY EQUATIONS
      XP=P(M+12)*U(N)*VP
      RETURN
   11 XP=P(M+13)*U(N)*VP
      RETURN
C     HEAVE REGION DETERMINATION AND EQUATION SELECTION
    3 IF(U(N).LT.0.)GO TO 12
```

```
      XP=P(M+14)*U(N)*WP
      RETURN
   12 XP=P(M+15)*U(N)*WP
      RETURN
C     RCLL REGION DETERMINATICN AND EQUATION SELECTION
    4 A=X(1).GE.0.
      B=U(N).GE.0.
      C=(U(N)-P(M+35)*X(1)).GE.0.
      PV=P(M+33)*U(N)*SQRT(VP*VP+WP*WP)
      PD=P(M+34)*U(N+1)
      IF(A.AND.C)ISW=1
      IF(A.AND..NOT.C)ISW=2
      IF(.NOT.A.AND.B)ISW=3
      IF(.NOT.(A.OR.B))ISW=4
      GO TC (13,14,15,16),ISW
C     PROPELLCR ROLL EQUATICNS
  130XP=P(M+16)*U(N)*ABS(U(N))+P(M+17)*X(1)*U(N)+
     1P(M+18)*X(1)*X(1)+PV+PD
      RETURN
  140XP=P(M+19)*U(N)*ABS(U(N))+P(M+20)*X(1)*U(N)+
     1P(M+21)*X(1)*X(1)+PV+PD
      RETURN
  150XP=P(M+22)*U(N)*ABS(U(N))+P(M+23)*X(1)*U(N)+
     1P(M+24)*X(1)*X(1)+PV+PD
      RETURN
  160XP=P(M+25)*U(N)*ABS(U(N))+P(M+26)*X(1)*U(N)+
     1P(M+27)*X(1)*X(1)+PV+PD
      RETURN
C     PITCH REGICN DETERMINATION AND EQUATICN SELECTICN
    5 IF(U(N).LT.0.)GO TO 17
C     PROPELLOR PITCH EQUATICNS
      XP=P(M+28)*U(N)*WP
      RETURN
   17 XP=P(M+29)*U(N)*WP
      RETURN
C     YAW REGION DETERMINATION ANC EQUATION SELECTION
    6 IF(U(N).LT.0.)GO TO 18
C     PROPELLOR YAW EQUATIONS
      XP=P(M+30)*U(N)*VP
      RETURN
   18 XP=P(M+31)*U(N)*VP
      RETURN
      END
```

```
      SUBROUTINE THCAL(P,M,U,N,X,XT,K)
C ***********************************************************************
C SUBROUTINE THCAL
C
C PURPOSE
C   TO CALCULATE THE FORCES AND MOMENTS ON THE DSRV CAUSED
C   BY THRUSTER PROPELLOR ROTATIONS
C
C SUBROUTINES REQUIRED
C   RGCOM
C
C DESCRIPTION
C   CHAPTER D3,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***********************************************************************
C     STEADY STATE EQUATIONS USED FOR THRUSTER CALCULATIONS
      DIMENSION P(1),U(1),N(1),X(1)
      GO TO (1,2,3,4,5,6),K
C     THRUSTER SURGE EQUATIONS
    1 XT=0.
      I=N(2)-1
      NT=N(1)
      JS=4
      JR=-1
      MV=M+60
      DO 7 J=1,NT
      JS=JS+JR
      IJ=I+J
      JT=JS+2
      MV=MV+JR*10
      UI=ABS(U(IJ))
      IF(UI.EQ.0.)GO TO 16
      A=X(1)/UI
      PV=P(M+128)
      IF(A.GT.PV)A=PV
      PW=P(M+127)
      IF(A.LT.PW)A=PW
      CALL RGCOM(MV,N(JT),P,IR,A,TS)
      XT=XT+U(IJ)*U(IJ)*TS
   16 JR=-JR
    7 CONTINUE
      RETURN
C     THRUSTER SWAY EQUATIONS
    2 XT=0.
      I=N(2)-1
      ISW=0
      JT=3
      MV=M
      PV=P(M+128)
      PW=P(M+127)
   12 I=I+1
      UI=ABS(U(I))
      IF(UI.EQ.0.)GO TO 17
```

```
      A=X(1)/UI
      IF(A.GT.PV)A=PV
      IF(A.LT.PW)A=PW
      CALL RGCOM(MV,N(JT),P,IR,A,TS)
      XT=XT+U(I)*ABS(U(I))*TS
   17 MV=MV+25
      JT=JT+1
      ISW=ISW+1
      GO TO (12,13),ISW
   13 RETURN
C     THRUSTER HEAVE EQUATIONS
    3 XT=0.
      I=N(2)+1
      ISW=0
      JT=3
      MV=M
      PV=P(M+128)
      PW=P(M+127)
   14 I=I+1
      UI=ABS(U(I))
      IF(UI.EQ.0.)GO TO 18
      A=X(1)/UI
      IF(A.GT.PV)A=PV
      IF(A.LT.PW)A=PW
      CALL RGCOM(MV,N(JT),P,IR,A,TS)
      XT=XT+U(I)*ABS(U(I))*TS
   18 MV=MV+25
      JT=JT+1
      ISW=ISW+1
      GO TO (14,15),ISW
   15 RETURN
C     THRUSTER ROLL EQUATIONS
    4 XT=0.
      RETURN
C     THRUSTER PITCH EQUATIONS
    5 XT=0.
      I=N(2)
      JV=M+122
      XT=XT+P(JV)*U(I)*ABS(U(I))
      I=I+1
      XT=XT+P(JV)*U(I)*ABS(U(I))
      ISW=0
      MV=M+76
      JT=7
      JV=M+120
      PV=P(M+128)
      PW=P(M+127)
    8 I=I+1
      UI=ABS(U(I))
      IF(UI.EQ.0.)GO TO 19
      A=X(1)/UI
      IF(A.GT.PV)A=PV
      IF(A.LT.PW)A=PW
```

```
      CALL RGCOM(MV,N(JT),P,IR,A,TS)
      XT=XT+P(JV)*U(I)*ABS(U(I))*TS
   19 MV=MV+22
      JV=JV+1
      JT=JT+1
      ISW=ISW+1
      GO TO (8,9),ISW
    9 RETURN
C     THRUSTER YAW EQUATIONS
    6 XT=0.
      I=N(2)-1
      ISW=0
      MV=M+76
      JT=7
      JV=M+123
      PV=P(M+128)
      PW=P(M+127)
   10 I=I+1
      UI=ABS(U(I))
      IF(UI.EQ.0.)GO TO 20
      A=X(1)/UI
      IF(A.GT.PV)A=PV
      IF(A.LT.PW)A=PW
      CALL RGCCM(MV,N(JT),P,IR,A,TS)
      XT=XT+P(JV)*U(I)*ABS(U(I))*TS
   20 MV=MV+22
      JV=JV+1
      JT=JT+1
      ISW=ISW+1
      GO TO (10,11),ISW
   11 I=I+1
      XT=XT+P(JV)*U(I)*ABS(U(I))
      I=I+1
      XT=XT+P(JV)*U(I)*ABS(U(I))
      RETURN
      END
```

```
      SUBROUTINE COCAL(P,M,X,XC,K)
C ****************************************************************
C SUBROUTINE COCAL
C
C PURPOSE
C   TO SELECT THE CONSTANT FORCES AND MOMENTS TO BE
C   APPLIED TO THE DSRV
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   CHAPTER D3,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ****************************************************************
      DIMENSION P(1),X(1)
      MP=M+K-1
      GO TO (1,2,3,4,5,6),K
    1 XC=P(MP)
      RETURN
    2 XC=P(MP)
      RETURN
    3 XC=P(MP)
      RETURN
    4 XC=P(MP)
      RETURN
    5 XC=P(MP)
      RETURN
    6 XC=P(MP)
      RETURN
      END
```

```
      SUBROUTINE XDIST(I,P,M,X,K,N)
C ******************************************************************************
C SUBROUTINE XDIST
C
C PURPOSE
C   TO COMPUTE A VECTOR OF GAUSSIAN RANDOM NUMBERS
C
C SUBROUTINES REQUIRED
C   GAUSS(IBM/SSP)
C
C DESCRIPTION
C   CHAPTER D3,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ******************************************************************************
      DIMENSION I(1),P(1)
      IX=I(K)
      MV=M+K-1
      AM=P(MV)
      MV=MV+N
      S=P(MV)
      CALL GAUSS(IX,S,AM,X)
      RETURN                          I(K) = IX
      END
```

CHAPTER D5 SUBROUTINES (OVDER, GRADIENT)

```
      SUBROUTINE OVDER(ME,U,AV,XF,KF,LF,KB,L,X,KP,LP,P,F,NF,NE)
C ***********************************************************************
C SUBROUTINE OVDER
C
C PURPOSE
C   TO COMPUTE SELECTED STATE DERIVATIVES WITH RESPECT TO
C   SECOND-DEGREE COEFFICIENTS AND PARAMETERS FOR A
C   GENERAL OCEAN VEHICLE
C
C SUBROUTINES REQUIRED
C   TANKS,ACALC,A3INV,MINV,XDDER,SEDER,PRDER,THDER,SHDER,
C   XFDER,XPDER
C
C DESCRIPTION
C   CHAPTER D5,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***********************************************************************
      INTEGER AV,XF
      DIMENSION ME(1),U(1),NE(1),AV(1),XF(1),LF(1),L(1),X(1)
      DIMENSION LP(1),P(1),F(1)
      DIMENSION A(36),ATH(9),NT(14),MM(6),PG(8),PC(8),XD(36)
      DIMENSION XS(36),XFD(756),XPD(756)
      MT=ME(17)-1
      DO 5 KT=1,8
      PG(KT)=P(MT+KT)
    5 CONTINUE
      IF(ME(1).NE.1)GO TO 1
      CALL TANKS(P,ME(10),U,NE(1),PG)
    1 MG=ME(18)+1
      G=P(MG)
      CALL ACALC(P,PG,G,A,KB,L)
C     BEGIN INVERSION OF A
      IF(KB.NE.1)GO TO 2
      IF(A(1).EQ.0.)GO TO 3
      A(1)=1./A(1)
      GO TO 8
    2 IF(KB.NE.2)GO TO 4
      DET=A(1)*A(4)-A(2)*A(3)
      IF(DET.EQ.0.)GO TO 3
      DET=1./DET
      AH=A(1)
      A(1)=A(4)*DET
      A(2)=-A(2)*DET
      A(3)=-A(3)*DET
      A(4)=AH*DET
      GO TO 8
    4 IF(KB.NE.3)GO TO 6
      DO 7 K=1,9
      ATH(K)=A(K)
    7 CONTINUE
      CALL A3INV(ATH,A)
      GO TO 8
    6 CALL MINV(A,KB,DET,NT,MM)
```

```
      IF(DET.EQ.0.)GO TO 3
      GO TO 8
    3 IKB=KB*KB
      DO 9 K=1,IKB
      A(K)=0.
    9 CONTINUE
      WRITE(6,10)
   10 FORMAT(1X,'SINGULAR DETERMINANT IN OVDER, SETTING F=0.')
      GO TO 11
    8 MC=ME(18)
      PC(1)=PG(1)/P(MC+1)
      PC(2)=PG(2)
      PC(3)=PG(3)
      PC(4)=PG(4)
      PC(5)=P(37)
      PC(6)=P(39)
      PC(7)=P(42)
      PC(8)=P(43)
      NT(1)=NE(4)
      NT(2)=NE(9)
      DO 34 I=3,8
      NT(I)=NE(I+10)
      NT(I+6)=NE(I+19)
   34 CONTINUE
      DO 12 KM=1,KB
      K=L(KM)
      DO 13 KN=1,KB
      M=L(KN)
      NF=M+6*(K-1)
      DX=0.
      CALL XDDER(AV,XF,P,PC,X,DX,K,M,KB,L)
      IF(ME(2).NE.1)GO TO 14
      CALL SEDER(P,ME(11),X,DXS,NF)
      DX=DX+DXS
   14 IF(ME(4).NE.1)GO TO 15
      CALL PRDER(P,ME(13),X,U,NE(8),NE(3),DXP,NF)
      DX=DX+DXP
   15 IF(ME(5).NE.1)GO TO 33
      CALL THDER(P,ME(14),U,NT,X,DXT,NF)
      DX=DX+DXT
   33 XD(NF)=DX
   13 CONTINUE
   12 CONTINUE
      IF(ME(3).NE.1)GO TO 16
     0CALL SHDER(P,ME(12),X,U,NE(7),NE(2),NE(11),NE(12),
     1NE(20),NE(21),XS,KB,L)
      NV=1
      DO 17 KM=1,KB
      K=L(KM)
      DO 18 KN=1,KB
      M=L(KN)
      NF=M+6*(K-1)
      F(NV)=XD(NF)+XS(NF)
```

```
      NV=NV+1
   18 CONTINUE
   17 CONTINUE
   16 IF(ME(3).EQ.1)GO TO 19
      NV=1
      DO 20 KM=1,KB
      K=L(KM)
      DO 21 KN=1,KB
      M=L(KN)
      NF=M+6*(K-1)
      F(NV)=XD(NF)
      NV=NV+1
   21 CONTINUE
   20 CONTINUE
   19 IF(KF.EQ.0)GO TO 24
      CALL XFDER(AV,XF,P,PC,X,XFD,KF,LF,KB,L,ME(11))
      ND=0
      NL=1
      NB=NV-1
      DO 23 KM=1,KB
      M=L(KM)
      DO 35 JM=1,KB
      J=L(JM)
      DO 36 IM=1,JM
      I=L(IM)
      IJM=I+(J*(J-1))/2+21*(M-1)
   38 IF(NL.GT.KF)GO TO 37
      IF(LF(NL).GT.IJM)GO TO 36
      IF(LF(NL).EQ.IJM)GO TO 39
      NL=NL+1
      GO TO 38
   39 ND=ND+1
      DO 22 KN=1,KB
      K=L(KN)
      IJK=K+6*(IJM-1)
      NK=NB+KN+KB*(ND-1)
      F(NK)=XFD(IJK)
   22 CONTINUE
      NL=NL+1
   36 CONTINUE
   35 CONTINUE
   23 CONTINUE
   37 NV=NV+KB*ND
   24 IF(KP.EQ.0)GO TO 25
      CALL XPDER(P,ME,U,NE,X,XPD,KP,LP,KB,L,NT)
      DO 26 KM=1,KP
      K=LP(KM)
      DO 27 KN=1,KB
      M=L(KN)
      NF=M+6*(K-1)
      F(NV)=XPD(NF)
      NV=NV+1
   27 CONTINUE
```

```
   26 CONTINUE
C     BEGIN DERIVATIVE TRANSFORMATION
   25 IR=0
      IK=0
      NF=KB+KF+KP
      DO 29 K=1,NF
      DO 28 KM=1,KB
      IK=IK+1
      PG(KM)=F(IK)
   28 CONTINUE
      DO 30 M=1,KB
      IR=IR+1
      JI=M-KB
      IB=0
      F(IR)=0.
      DO 31 I=1,KB
      JI=JI+KB
      IB=IB+1
      F(IR)=F(IR)+A(JI)*PG(IB)
   31 CONTINUE
   30 CONTINUE
   29 CONTINUE
      RETURN
   11 NF=KB+KF+KP
      IKB=KB*NF
      DO 32 K=1,IKB
      F(K)=0.
   32 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE XDDER(AV,XF,P,PC,X,DX,K,M,KB,L)
C ************************************************************************
C SUBROUTINE XDDER
C
C PURPOSE
C   TO COMPUTE THE STATE DERIVATIVES OF THE SECOND-DEGREE
C   COEFFICIENT-FUNCTIONS OF AN OCEAN VEHICLE
C
C SUBROUTINES REQUIRED
C   XFUNS
C
C DESCRIPTION
C   CHAPTER D5,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ************************************************************************
      INTEGER AV,XF
      DIMENSION AV(1),XF(1),P(1),PC(1),X(1),L(1)
      DX=0.
      DO 1 IM=1,KB
      I=L(IM)
      DO 2 JM=IM,KB
      J=L(JM)
      IJK=I+(J*(J-1))/2+21*(K-1)
      IF(XF(IJK).EQ.0)GO TO 2
      IF((I.NE.M).AND.(J.NE.M))GO TO 2
      KIJ=IJK+6
      CALL XFUNS(PC,X,KIJ,XC,XF)
      IF(I.EQ.J)GO TO 3
      IF(I.EQ.M)GO TO 4
      IF(AV(IJK))5,6,7
    5 DX=DX+ABS(X(I))*XC
      GO TO 2
    6 DX=DX+X(I)*XC
      GO TO 2
    7 DX=DX+X(I)*SIGN(1.,X(J))*XC
      GO TO 2
    4 IF(AV(IJK))10,11,12
   10 DX=DX+X(J)*SIGN(1.,X(I))*XC
      GO TO 2
   11 DX=DX+X(J)*XC
      GO TO 2
   12 DX=DX+ABS(X(J))*XC
       GO TO 2
    3 IF(AV(IJK))8,9,8
    8 DX=DX+2.0*ABS(X(I))*XC
       GO TO 2
    9 DX=DX+2.0*X(I)*XC
    2 CONTINUE
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SEDER(P,M,X,DXS,K)
C ************************************************************************
C SUBROUTINE SEDER
C
C PURPOSE
C   TO COMPUTE THE STATE DERIVATIVES OF THE SECCNDARY DRAG
C   TERMS FOR THE DSRV
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   CHAPTER D6,M,N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ************************************************************************
      DIMENSION P(1),X(1)
     OGO TO (1,1,1,1,1,1,1,2,1,1,1,8,1,1,4,1,6,1,1,1,1,1,1,
     11,1,1,5,1,7,1,1,3,1,1,1,9),K
    1 DXS=0.
      RETURN
    2 TA=X(2)+P(M+30)*X(6)
      TB=X(2)+P(M+29)*X(6)
      XC=X(30)
      IF(X(6))27,26,26
   26 IF((TA.GE.0.).OR.(TB.LE.0.))GO TO 28
      ISW=2
      GO TO 29
   27 IF((TA.LE.0.).OR.(TB.GE.0.))GO TO 28
      ISW=2
      GO TO 29
   28 ISW=1
   29 GO TO (10,11),ISW
    3 TA=X(2)+P(M+30)*X(6)
      TB=X(2)+P(M+29)*X(6)
      XC=X(30)
      IF(X(6))31,30,30
   30 IF((TA.GE.0.).OR.(TB.LE.0.))GO TO 32
      ISW=2
      GO TO 33
   31 IF((TA.LE.0.).OR.(TB.GE.0.))GO TO 32
      ISW=2
      GO TO 33
   32 ISW=1
   33 GO TO (12,13),ISW
    4 TA=X(3)-P(M+30)*X(5)
      TB=X(3)-P(M+29)*X(5)
      XC=X(54)
      IF(X(5))35,34,34
   34 IF((TA.LE.0.).OR.(TB.GE.0.))GO TO 36
      ISW=2
      GO TO 37
   35 IF((TA.GE.0.).OR.(TB.LE.0.))GO TO 36
      ISW=2
```

```
      GO TO 37
   36 ISW=1
   37 GO TO (14,15),ISW
    5 TA=X(3)-P(M+30)*X(5)
      TB=X(3)-P(M+29)*X(5)
      XC=X(54)
      IF(X(5))39,38,38
   38 IF((TA.LE.0.).OR.(TB.GE.0.))GO TO 40
      ISW=2
      GO TO 41
   39 IF((TA.GE.0.).OR.(TB.LE.0.))GO TO 40
      ISW=2
      GO TO 41
   40 ISW=1
   41 GO TO (16,17),ISW
    6 TA=X(3)-P(M+30)*X(5)
      TB=X(3)-P(M+29)*X(5)
      XC=X(54)
      IF(X(5))43,42,42
   42 IF((TA.LE.0.).OR.(TB.GE.0.))GO TO 44
      ISW=2
      GO TO 45
   43 IF((TA.GE.0.).OR.(TB.LE.0.))GO TO 44
      ISW=2
      GO TO 45
   44 ISW=1
   45 GO TO (18,19),ISW
    7 TA=X(3)-P(M+30)*X(5)
      TB=X(3)-P(M+29)*X(5)
      XC=X(54)
      IF(X(5))47,46,46
   46 IF((TA.LE.0.).OR.(TB.GE.0.))GO TO 48
      ISW=2
      GO TO 49
   47 IF((TA.GE.0.).OR.(TB.LE.0.))GO TO 48
      ISW=2
      GO TO 49
   48 ISW=1
   49 GO TO (20,21),ISW
    8 TA=X(2)+P(M+30)*X(6)
      TB=X(2)+P(M+29)*X(6)
      XC=X(30)
      IF(X(6))51,50,50
   50 IF((TA.GE.0.).OR.(TB.LE.0.))GO TO 52
      ISW=2
      GO TO 53
   51 IF((TA.LE.0.).OR.(TB.GE.0.))GO TO 52
      ISW=2
      GO TO 53
   52 ISW=1
   53 GO TO (22,23),ISW
    9 TA=X(2)+P(M+30)*X(6)
      TB=X(2)+P(M+29)*X(6)
```

```
      XC=X(30)
      IF(X(6))55,54,54
   54 IF((TA.GE.0.).OR.(TB.LE.0.))GO TO 56
      ISW=2
      GO TO 57
   55 IF((TA.LE.0.).OR.(TB.GE.0.))GO TO 56
      ISW=2
      GO TO 57
   56 ISW=1
   57 GO TO (24,25),ISW
   10 DXS=P(M)*2.0*ABS(X(2))+P(M+1)*X(6)*SIGN(1.,X(2))
      DXS=DXS*XC
      RETURN
   110DXS=P(M+3)*SIGN(1.,X(6))*3.0*X(2)*X(2)/X(6)+P(M+4)*
     1SIGN(1.,X(6))*2.0*X(2)+P(M+5)*ABS(X(6))
      DXS=DXS*XC
      RETURN
   12 DXS=P(M+1)*ABS(X(2))+P(M+2)*SIGN(1.,X(2))*2.0*X(6)
      DXS=DXS*XC
      RETURN
   130DXS=P(M+3)*SIGN(1.,X(6))*(X(2)/X(6))*(X(2)/X(6))*
     1(-X(2))+P(M+5)*X(2)*SIGN(1.,X(6))+P(M+6)*2.0*ABS(X(6))
      DXS=DXS*XC
      RETURN
   14 DXS=P(M+7)*2.0*ABS(X(3))+P(M+8)*X(5)*SIGN(1.,X(3))
      DXS=DXS*XC
      RETURN
   150DXS=P(M+10)*SIGN(1.,X(5))*3.0*X(3)*X(3)/X(5)+P(M+11)*
     1SIGN(1.,X(5))*2.0*X(3)+P(M+12)*ABS(X(5))
      DXS=DXS*XC
      RETURN
   16 DXS=P(M+ 8)*ABS(X(3))+P(M+9)*SIGN(1.,X(3))*2.0*X(5)
      DXS=DXS*XC
      RETURN
   170DXS=P(M+10)*SIGN(1.,X(5))*(X(3)/X(5))*(X(3)/X(5))*(-
     1X(3))+P(M+12)*X(3)*SIGN(1.,X(5))+P(M+13)*2.0*ABS(X(5))
      DXS=DXS*XC
      RETURN
   18 DXS=P(M+18)*SIGN(1.,X(3))*X(5)+P(M+19)*2.0*ABS(X(3))
      DXS=DXS*XC
      RETURN
   190DXS=P(M+15)*SIGN(1.,X(5))*2.0*X(3)+P(M+16)*SIGN(1.,
     1X(5))*4.0*(X(3)**3.)/(X(5)*X(5))+P(M+17)*ABS(X(5))
      DXS=DXS*XC
      RETURN
   20 DXS=P(M+18)*ABS(X(3))+P(M+20)*SIGN(1.,X(3))*2.0*X(5)
      DXS=DXS*XC
      RETURN
   210DXS=P(M+14)*2.0*ABS(X(5))+P(M+16)*SIGN(1.,X(5))*2.0*((
     1X(3)/X(5))**3.)*(-X(3))+P(M+17)*SIGN(1.,X(5))*X(3)
      DXS=DXS*XC
      RETURN
   22 DXS=P(M+25)*SIGN(1.,X(2))*X(6)+P(M+26)*2.0*ABS(X(2))
```

```
      DXS=DXS*XC
      RETURN
  230 DXS=P(M+22)*SIGN(1.,X(6))*2.0*X(2)+P(M+23)*SIGN(1.,
     1X(6))*4.0*((X(2)/X(6))**2 )*X(2)+P(M+24)*ABS(X(6))
      DXS=DXS*XC
      RETURN
   24 DXS=P(M+25)*ABS(X(2))+P(M+27)*SIGN(1.,X(2))*2.0*X(6)
      DXS=DXS*XC
      RETURN
  250 DXS=P(M+21)*2.0*ABS(X(6))+P(M+23)*SIGN(1.,X(6))*2.0*((
     1X(2)/X(6))**3.)*(-X(2))+P(M+24)*SIGN(1.,X(6))*X(2)
      DXS=DXS*XC
      RETURN
      END
```

```
      SUBROUTINE XFDER(AV,XF,P,PC,X,XFD,KF,LF,KB,L,MS)
C ***********************************************************************
C SUBROUTINE XFDER
C
C PURPOSE
C   TO COMPUTE THE DERIVATIVES WITH RES. TO SECOND-DEGREE
C   COEFFICIENT FUNCTIONS FOR AN OCEAN VEHICLE
C
C SUBROUTINES REQUIRED
C   XDFNS,SECAL
C
C DESCRIPTION
C   SEE SUBROUTINE CVDER
C   CHAPTER D5,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***********************************************************************
      INTEGER AV,XF
      DIMENSION AV(1),XF(1),P(1),PC(1),X(1),XFD(1),LF(1),L(1)
      NL=1
      DO 4 KN=1,KB
      K=L(KN)
      DO 1 KM=1,KB
      M=L(KM)
      DO 2 JM=1,KB
      J=L(JM)
      DO 3 IM=1,JM
      I=L(IM)
      IJM=I+(J*(J-1))/2+21*(M-1)
   11 IF(NL.GT.KF)GO TO 8
      IF(LF(NL).GT.IJM)GO TO 3
      IF(LF(NL).EQ.IJM)GO TO 12
      NL=NL+1
      GO TO 11
   12 IF(K.NE.M)GO TO 10
      IF(XF(IJM).EQ.0)GO TO 10
      NL=NL+1
      KIJ=IJM+6
      CALL XDFNS(PC,X,KIJ,XF,DX,IND)
      KIJ=IJM
      IJM=K+6*(IJM-1)
      IF(IND.EQ.0)GO TO 9
      IF(AV(KIJ))5,6,7
    5 XFD(IJM)=DX*ABS(X(I))*X(J)
      GO TO 3
    6 XFD(IJM)=DX*X(I)*X(J)
      GO TO 3
    7 XFD(IJM)=DX*X(I)*ABS(X(J))
      GO TO 3
   10 IJM=K+6*(IJM-1)
      NYV=24
      IF(LF(NL).EQ.NYV)GO TO 13
      NZW=48
      IF(LF(NL).EQ.NZW)GO TO 14
```

```
      NL=NL+1
    9 XFD(IJM)=0.
      GO TO 3
   13 CALL SECAL(P,MS,X,XS,K,XSO,XST)
      XFD(IJM)=XSO
      GO TO 3
   14 CALL SECAL(P,MS,X,XS,K,XSO,XST)
      XFD(IJM)=XST
    3 CONTINUE
    2 CONTINUE
    1 CONTINUE
    3 NL=1
    4 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE XDFNS(P,X,K,I,D,N)
C ***********************************************************************
C SUBROUTINE XDFNS
C
C PURPOSE
C   TO SELECT AND SUPPLY THE DERIVATIVES WITH RES. TO DSRV
C   SECOND-DEGREE COEFFICIENTS FOR SUBROUTINE XFDER
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   SEE SUBROUTINE XFDER
C   CHAPTER D5,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***********************************************************************
      DIMENSION P(1),X(1),I(1)
      ISW=I(K-6)
     0GO TO (2,1,2,2,2,2,2,1,2,3,1,1,2,2,3,1,2,1,1,2,1,3,2,
     12,3,1,2,2,2,1,2,3,1,2,2,4,5),ISW
    1 D=0.
      N=0
      RETURN
    2 D=1.
      N=1
      RETURN
    3 D=SIGN(1.,X(1))
      N=1
      RETURN
    4 IF(X(3).LT.0.)GO TO 6
      N=1
      D=1.
      RETURN
    6 D=0.
      N=0
      RETURN
    5 IF(X(1).LT.0.)GO TO 7
      IF((X(2)-P(8)*X(1)).GT.0.)GO TO 6
      N=1
      D=1.
      RETURN
    7 IF((X(2)-P(3)*X(1)).LT.0.)GO TO 6
      N=1
      D=1.
      RETURN
      END
```

```
      SUBROUTINE XPDER(P,ME,U,NE,X,XPD,KP,LP,KB,L,NT)
C ************************************************************************
C SUBROUTINE XPDER
C
C PURPOSE
C   TO COMPUTE THE DERIVATIVES WITH RES. TO VEHICLE AND
C   EFFECTOR PARAMETERS FOR AN OCEAN VEHICLE
C
C SUBROUTINES REQUIRED
C   SEDEP,PRDEP,THDEP
C
C DESCRIPTION
C   SEE SUBROUTINE CVDER
C   CHAPTER D5,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ************************************************************************
      DIMENSION P(1),ME(1),U(1),NE(1),X(1),XPD(1),LP(1),L(1)
      DIMENSICN NT(1)
      KME=9
      KMP=9
      DO 1 JM=1,KP
      J=LP(JM)
      KJ=KME
      IS=1
      DO 3 KM=1,KMP
      KJ=KJ+1
      IF(J.LT.ME(KJ))GO TO 4
      IS=IS+1
    3 CONTINUE
    4 DO 2 IM=1,KB
      I=L(IM)
      N=I+6*(J-1)
      GO TO (5,6,7,8,9,10,11,12,13,14),IS
    5 XPD(N)=0.
      GO TO 2
    6 XPD(N)=0.
      GO TO 2
    7 CALL SEDEP(P,ME(11),X,PX,I,J)
      XPD(N)=PX
      GO TO 2
    8 XPD(N)=0.
      GO TO 2
    9 CALL PRDEP(P,ME(13),X,U,NE(8),NE(3),XP,I,J)
      XPD(N)=XP
      GO TO 2
   10 CALL THDEP(P,ME(14),U,NT,X,XT,I,J)
      XPD(N)=XT
      GO TO 2
   11 XPD(N)=0.
      GO TO 2
   12 XPD(N)=1.
      GO TO 2
   13 XPD(N)=0.
```

```
      GO TO 2
   14 XPD(N)=0.
    2 CONTINUE
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SEDEP(P,M,X,XS,K,J)
C ****************************************************************
C SUBROUTINE SEDEP
C
C PURPOSE
C   TO COMPUTE THE DERIVATIVES WITH RES. TO DSRV SECONDARY
C   DRAG PARAMETERS
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   CHAPTER 06,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ****************************************************************
      DIMENSION P(1),X(1)
C     INITIAL SECONDARY DRAG REGION DETERMINATION
      IF((K.EQ.1).OR.(K.EQ.4))GO TO 1
      KM=8-K
      IF((K.EQ.2).OR.(K.EQ.6))ISW=1
      IF((K.EQ.3).OR.(K.EQ.5))ISW=2
      GO TO (2,3),ISW
C     K=2 OR K=6 REGION DETERMINATION
    2 TA=X(2)+P(M+30)*X(6)
      TB=X(2)+P(M+29)*X(6)
      XC=X(30)
      IF(X(6))17,16,16
   16 IF((TA.GE.0.).OR.(TB.LE.0.))GO TO 4
      ISW=2
      GO TO 5
   17 IF((TA.LE.0.).OR.(TB.GE.0.))GO TO 4
      ISW=2
      GO TO 5
    4 ISW=1
    5 GO TO (8,9),ISW
    1 XS=0.
      RETURN
C     K=3 OR K=5 REGION DETERMINATION
    3 TA=X(3)-P(M+30)*X(5)
      TB=X(3)-P(M+29)*X(5)
      XC=X(54)
      IF(X(5))19,18,18
   18 IF((TA.LE.0.).OR.(TB.GE.0.))GO TO 6
      ISW=2
      GO TO 7
   19 IF((TA.GE.0.).OR.(TB.LE.0.))GO TO 6
      ISW=2
      GO TO 7
    6 ISW=1
    7 GO TO (10,11),ISW
C     BEGIN SELECTED SECONDARY DRAG CALCULATIONS
    8 IF(K.EQ.2)GO TO 14
C     K=5 OR K=6 REGION ONE
```

```
   12 N=M+19+(K-5)*7
      IF(J.NE.N)GO TO 20
      XS=ABS(X(KM))*X(KM)*XC
      RETURN
   20 N=N-1
      IF(J.NE.N)GO TO 21
      XS=ABS(X(KM))*X(K)*XC
      RETURN
   21 N=N+2
      IF(J.NE.N)GO TO 1
      XS=SIGN(1.,X(KM))*X(K)*X(K)*XC
      RETURN
    9 IF(K.EQ.2)GO TO 15
C     K=5 OR K=6 REGION TWO
   13 N=M+14+(K-5)*7
      IF(J.NE.N)GC TO 22
      XS=X(K)*ABS(X(K))*XC
      RETURN
   22 N=N+1
      IF(J.NE.N)GO TO 23
      XS=SIGN(1.,X(K))*X(K)*X(K)*XC
      RETURN
   23 N=N+1
      IF(J.NE.N)GO TO 24
      XS=SIGN(1.,X(K))*X(K)*X(K)*((X(KM)/X(K))**2.)*XC
      RETURN
   24 N=N+1
      IF(J.NE.N)GO TO 1
      XS=X(KM)*ABS(X(K))*XC
      RETURN
   10 IF(K.EQ.5)GO TO 12
C     K=2 OR K=3 REGION CNE
   14 N=M+(K-2)*7
      IF(J.NE.N)GO TO 25
      XS=X(K)*ABS(X(K))*XC
      RETURN
   25 N=N+1
      IF(J.NE.N)GO TO 26
      XS=X(KM)*ABS(X(K))*XC
      RETURN
   26 N=N+1
      IF(J.NE.N)GO TO 1
      XS=SIGN(1.,X(K))*X(KM)*X(KM)*XC
      RETURN
   11 IF(K.EQ.5)GO TO 13
C     K=2 OR K=3 REGION TWO
   15 N=M+3+(K-2)*7
      IF(J.NE.N)GO TO 27
      XS=SIGN(1.,X(KM))*X(K)*X(K)*(X(K)/X(KM))*XC
      RETURN
   27 N=N+1
      IF(J.NE.N)GO TO 28
      XS=SIGN(1.,X(KM))*X(K)*X(K)*XC
```

```
      RETURN
   28 N=N+1
      IF(J.NE.N)GO TO 29
      XS=ABS(X(KM))*X(K)*XC
      RETURN
   29 N=N+1
      IF(J.NE.N)GO TO 1
      XS=ABS(X(KM))*X(KM)*XC
      RETURN
      END
```

```
      SUBROUTINE A3INV(A,AI)
C
C     ******************************************************************
C
C        SUBROUTINE A3INV
C
C        PURPOSE
C           INVERT A GENERAL, NONSINGULAR 3X3 MATRIX
C
C        USAGE
C           CALL A3INV(A,AI)
C
C        DESCRIPTION OF PARAMETERS
C           A  - GENERAL,NONSINGULAR 3X3 MATRIX IN
C                VECTOR FORM
C           AI - INVERSE OF A
C
C        REMARKS
C           (1)- THE ZERO VALUE FOR THE DETERMINANT HAS
C                BEEN ARBITRARILY SET AT 1.0E-40
C           (2)- AI WILL BE SET ZERO FOR SINGULAR MATRICES
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        AUTHOR
C           MICHAEL N. HAYES,RM.5-333,MIT X6807
C
C     ******************************************************************
C
      DIMENSION A(1),AI(1)
    3 FORMAT(1H1,'MATRIX A IS SINGULAR, DET(A)=',E15.7,
     1'    SETTING AI=0.0')
C        COMPUTE THE DETERMINANT
     0DET=A(1)*(A(5)*A(9)-A(6)*A(8))-A(2)*(A(4)*A(9)-
     1A(6)*A(7))+A(3)*(A(4)*A(8)-A(5)*A(7))
C        TEST DETERMINANT FOR ZERO
      IF(ABS(DET)-1.0E-40)1,2,2
    1 WRITE(6,3)DET
      DO 4 I=1,9
      AI(I)=0.0
    4 CONTINUE
      RETURN
C        MATRIX NONSINGULAR, COMPUTE INVERSE
    2 DET=1./DET
      AI(1)= (A(5)*A(9)-A(6)*A(8))*DET
      AI(2)=-(A(2)*A(9)-A(3)*A(8))*DET
      AI(3)= (A(2)*A(6)-A(3)*A(5))*DET
      AI(4)=-(A(4)*A(9)-A(6)*A(7))*DET
      AI(5)= (A(1)*A(9)-A(3)*A(7))*DET
      AI(6)=-(A(1)*A(6)-A(3)*A(4))*DET
      AI(7)= (A(4)*A(8)-A(5)*A(7))*DET
      AI(8)=-(A(1)*A(8)-A(2)*A(7))*DET
```

```
AI(9)= (A(1)*A(5)-A(2)*A(4))*DET
RETURN
END
```

CHAPTER D6 SUBROUTINES (EFFECTOR GRADIENT)

```
      SUBROUTINE SHDER(P,M,X,U,N,NS,NL,ND,NDL,NDD,XSD,KB,L)
C ****************************************************************
C SUBROUTINE SHDER
C
C PURPOSE
C   TO COMPUTE THE STATE DERIVATIVES OF THE SHROUD FORCES
C   AND MOMENTS FOR THE DSRV
C
C SUBROUTINES REQUIRED
C   UTCOM,ASDER,RGCOM,A3INV,GMPRD
C
C DESCRIPTION
C   CHAPTER D6,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ****************************************************************
      DIMENSION P(1),X(1),U(1),XSD(1),L(1)
      DIMENSION A(18),ASD(18),AI(9),UT(3),FU(3)
      DIMENSION DAS(6),DFUU(6),DFUV(6),DFUW(6)
      DIMENSION DVNS(6),DWNS(6),DVTS(6),DWTS(6),DUTS(6)
      IF((KB.LT.2).AND.(L(1).EQ.4))GO TO 1
      XSHR=P(M+47)
      CALL UTCOM(U,N,XSHR,A)
      RAD=57.2957795
C     CALCULATE U2S(I)
      VSQ=0.
      DO 2 I=1,3
      UT(I)=0.
      DO 3 K=1,KB
      J=L(K)
      MN=I+3*J-3
      UT(I)=UT(I)+A(MN)*X(J)
    3 CONTINUE
      VSQ=VSQ+UT(I)*UT(I)
    2 CONTINUE
C     CALCULATE SHROUD ANGLE ALPHA-S
      EPS=P(M+50)
      V=SQRT(VSQ)
     OCALL ASDER(UT,AS,I,EPS,A,DAS,VNSQ,WNSQ,VVNS,WVNS,VNS,
     1DVNS,DUTS,DVTS,DWTS)
      IF(AS.LT.0.)AS=AS+3.14159265
      AS=AS*RAD
C     COMPUTE LIFT AND DRAG COEFFICIENTS
C     AND DERIVATIVES
      CALL RGCOM(M,NL,P,IND,AS,CL)
      MN=M+3*NL+1
      CALL RGCOM(MN,ND,P,IND,AS,CD)
      MN=MN+3*ND+6
      CALL RGCOM(MN,NDL,P,IND,AS,DCL)
      MN=MN+3*NDL+1
      CALL RGCOM(MN,NDD,P,IND,AS,DCD)
C     COMPUTE VLIFT AND VDRAG
      BV=0.5*P(M+49)*P(M+48)
      B=BV*VSQ
```

```
      VLIFT=B*CL
      VDRAG=B*CD
C     COMPUTE VNF AND FU(1)
      AS=AS/RAD
      CAS=COS(AS)
      SAS=SIN(AS)
      VNF=VLIFT*CAS+VDRAG*SAS
      FU(1)=VLIFT*SAS-VDRAG*CAS
C     BEGIN COMPUTATION OF DFUU,DFUV,DFUW
      DO 6 K=1,6
      AA=B*DCL*DAS(K)
      AB=B*DCD*DAS(K)
      AC=2.0*(UT(1)*DUTS(K)+UT(2)*DVTS(K)+UT(3)*DWTS(K))
      DVL=AA+BV*CL*AC
      DVD=AB+BV*CD*AC
      DNF=DVL*CAS+DVD*SAS-FU(1)*DAS(K)
      DFUU(K)=DVL*SAS-DVD*CAS+VNF*DAS(K)
      DVVNS=DVTS(K)/VNS-VNSQ*DVNS(K)
      DWVNS=DWTS(K)/VNS-WNSQ*DVNS(K)
      DFUV(K)=-DNF*VVNS-VNF*DVVNS
      DFUW(K)=-DNF*WVNS-VNF*DWVNS
    6 CONTINUE
C     CALCULATE INVERSE OF TRANSFORMATION MATRIX
      CALL A3INV(A,AI)
C     PLACE SHROUD DERIVATIVES INTO A-MATRIX
      J=1
      DO 12 K=1,6
      A(J)=DFUU(K)
      A(J+1)=DFUV(K)
      A(J+2)=DFUW(K)
      J=J+3
   12 CONTINUE
C     TRANSFORM DERIVATIVES TO VEHICLE COORDINATES
      CALL GMPRD(AI,A,ASD,3,3,6)
C     SELECT AND FILL APPROPRIATE POSITIONS IN XSD(36)
      DO 13 I=1,KB
      IM=L(I)
      DO 14 J=1,KB
      JM=L(J)
      IJA=IM+3*(JM-1)
      IJX=IM+6*(JM-1)
      IF(IM.GT.3)GO TO 15
      XSD(IJX)=ASD(IJA)
      GO TO 14
   15 IF(IM.EQ.4)GO TO 16
      IF(IM.EQ.5)GO TO 17
      IF(IM.EQ.6)GO TO 18
   16 XSD(IJX)=0.
      GO TO 14
   17 IJW=3*JM
      XSD(IJX)=XSHR*ASD(IJW)
      GO TO 14
   18 IJW=3*JM-1
```

```
      XSD(IJX)=-XSHR*ASD(IJW)
   14 CONTINUE
   13 CONTINUE
      RETURN
    1 XSD(22)=0.
      RETURN
      END
```

```
      OSUBROUTINE ASDER(UT,AS,I,E,AM,DA,VQ,WQ,VS,WS,VN,DVN,
C ********************************************************************
C SUBROUTINE ASDER
C
C PURPOSE
C   TO COMPUTE THE DERIVATIVE OF DSRV SHROUD ANGLE ALPHA-S
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   SEE SUBROUTINE SHDER
C   CHAPTER D6,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ********************************************************************
     1DUT,DVT,DWT)
      DIMENSION UT(1),AM(1),DA(1),DVN(1),DVT(1),DWT(1),DUT(1)
      I=0
      VN=SQRT(UT(2)*UT(2)+UT(3)*UT(3))
      IF(VN.EQ.0.)VN=1.0E-08*E
      VS=UT(2)/VN
      VQ=VS/VN
      WS=UT(3)/VN
      WQ=WS/VN
      IF(UT(1).EQ.0.)UT(1)=E
      VUT=VN/UT(1)
      VUTS=VUT/UT(1)
      AS=ATAN(VUT)
      SVN=SQRT(VN)
      DO 1 I=1,6
      DUT(I)=AM(3*I-2)
      DVT(I)=AM(3*I-1)
      DWT(I)=AM(3*I)
      DVN(I)=(UT(2)*DVT(I)+UT(3)*DWT(I))/SVN
      DA(I)=(DVN(I)/UT(1)-VUTS*DUT(I))/(1.0+VUT*VUT)
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PRDER(P,M,X,U,N,NP,XP,K)
C ******************************************************************************
C SUBROUTINE PRDER
C
C PURPOSE
C   TO COMPUTE THE STATE DERIVATIVES OF THE PROPELLOR FORCES
C   AND MOMENTS FOR THE DSRV
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   CHAPTER D6,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ******************************************************************************
      LOGICAL A,B,C
      DIMENSION P(1),X(1),U(1)
      EPS=1.0E-08
      PM=P(M+36)
      VP=X(2)+PM*X(6)
      WP=X(3)-PM*X(5)
     0GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,
     119,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,
     236),K
    1 A=X(1).GE.0.
      B=U(N).GE.0.
      C=(U(N)-P(M+35)*X(1)).GE.0.
      IF(A.AND.C)ISW=1
      IF(A.AND..NOT.C)ISW=2
      IF(.NOT.A.AND.B)ISW=3
      IF(.NOT.(A.OR.B))ISW=4
      MV=M+3*(ISW-1)+1
      MW=MV+1
      XP=P(MV)*U(N)+2.*P(MW)*X(1)
      RETURN
    2 XP=0.
      RETURN
    3 XP=0.
      RETURN
    4 A=X(1).GE.0.
      B=U(N).GE.0.
      C=(U(N)-P(M+35)*X(1)).GE.0.
      IF(A.AND.C)ISW=1
      IF(A.AND..NOT.C)ISW=2
      IF(.NOT.A.AND.B)ISW=3
      IF(.NOT.(A.OR.B))ISW=4
      MV=M+3*(ISW-1)+17
      MW=MV+1
      XP=P(MV)*U(N)+2.*P(MW)*X(1)
      RETURN
    5 XP=0.
      RETURN
    6 XP=0.
```

```
      RETURN
    7 XP=VP*P(M+32)*U(N)
      VW=SQRT(VP*VP+WP*WP)
      IF(VW.EQ.0.)VW=EPS
      XP=XP/VW
      RETURN
    8 ISW=12
      A=X(1).GE.0.
      IF(.NOT.A)ISW=ISW+1
      XP=P(M+ISW)*U(N)
      RETURN
    9 XP=0.
      RETURN
   10 XP=P(M+33)*U(N)*VP
      VW=SQRT(VP*VP+WP*WP)
      IF(VW.EQ.0.)VW=EPS
      XP=XP/VW
      RETURN
   11 XP=0.
      RETURN
   12 ISW=30
      B=U(N).GE.0.
      IF(.NOT.B)ISW=ISW+1
      XP=P(M+ISW)*U(N)
      RETURN
   13 XP=WP*P(M+32)*U(N)
      VW=SQRT(VP*VP+WP*WP)
      IF(VW.EQ.0.)VW=EPS
      XP=XP/VW
      RETURN
   14 XP=0.
      RETURN
   15 ISW=14
      A=X(1).GE.0.
      IF(.NOT.A)ISW=ISW+1
      XP=P(M+ISW)*U(N)
      RETURN
   16 XP=P(M+33)*U(N)*WP
      VW=SQRT(VP*VP+WP*WP)
      IF(VW.EQ.0.)VW=EPS
      XP=XP/VW
      RETURN
   17 ISW=28
      B=U(N).GE.0.
      IF(.NOT.B)ISW=ISW+1
      XP=P(M+ISW)*U(N)
      RETURN
   18 XP=0.
      RETURN
   19 XP=0.
      RETURN
   20 XP=0.
      RETURN
```

```
21 XP=0.
   RETURN
22 XP=0.
   RETURN
23 XP=0.
   RETURN
24 XP=0.
   RETURN
25 XP=-P(M+36)*WP*P(M+32)*U(N)
   VW=SQRT(VP*VP+WP*WP)
   IF(VW.EQ.0.)VW=EPS
   XP=XP/VW
   RETURN
26 XP=0.
   RETURN
27 ISW=14
   A=X(1).GE.0.
   IF(.NOT.A)ISW=ISW+1
   XP=-P(M+ISW)*U(N)*P(M+36)
   RETURN
28 XP=-P(M+33)*U(N)*WP*P(M+36)
   VW=SQRT(VP*VP+WP*WP)
   IF(VW.EQ.0.)VW=EPS
   XP=XP/VW
   RETURN
29 ISW=28
   B=U(N).GE.0.
   IF(.NOT.B)ISW=ISW+1
   XP=-P(M+ISW)*U(N)*P(M+36)
   RETURN
30 XP=0.
   RETURN
31 XP= P(M+36)*VP*P(M+32)*U(N)
   VW=SQRT(VP*VP+WP*WP)
   IF(VW.EQ.0.)VW=EPS
   XP=XP/VW
   RETURN
32 ISW=12
   A=X(1).GE.0.
   IF(.NOT.A)ISW=ISW+1
   XP=P(M+ISW)*U(N)*P(M+36)
   RETURN
33 XP=0.
   RETURN
34 XP= P(M+33)*U(N)*VP*P(M+36)
   VW=SQRT(VP*VP+WP*WP)
   IF(VW.EQ.0.)VW=EPS
   XP=XP/VW
   RETURN
35 XP=0.
   RETURN
36 ISW=30
   B=U(N).GE.0.
```

```
IF(.NOT.B)ISW=ISW+1
XP= P(M+ISW)*U(N)*P(M+36)
RETURN
END
```

```
      SUBROUTINE THDER(P,M,U,N,X,XT,K)
C ******************************************************************
C SUBROUTINE THDER
C
C PURPOSE
C   TO COMPUTE THE STATE DERIVATIVES OF THE THRUSTER FORCES
C   AND MOMENTS FOR THE DSRV
C
C SUBROUTINES REQUIRED
C   RGCOM
C
C DESCRIPTION
C   CHAPTER D6,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ******************************************************************
      DIMENSION P(1),U(1),N(1),X(1)
      IF(K.GT.6)GO TO 21
      GO TO (1,2,3,4,5,6),K
   21 XT=0.
      RETURN
    1 XT=0.
      I=N(2)-1
      NT=N(1)
      JS=10
      JR=-1
      MV=M+207
      DO 7 J=1,NT
      JS=JS+JR
      IJ=I+J
      JT=JS+2
      MV=MV+JR*16
      UI=ABS(U(IJ))
      IF(UI.EQ.0.)GO TO 16
      A=X(1)/UI
      PV=P(M+128)
      IF(A.GT.PV)A=PV
      PW=P(M+127)
      IF(A.LT.PW)A=PW
      CALL RGCOM(MV,N(JT),P,IR,A,TS)
      XT=XT+ABS(U(IJ))*TS
   16 JR=-JR
    7 CONTINUE
      RETURN
    2 XT=0.
      I=N(2)-1
      ISW=0
      JT=9
      MV=M+129
      PV=P(M+128)
      PW=P(M+127)
   12 I=I+1
      UI=ABS(U(I))
      IF(UI.EQ.0.)GO TO 17
```

```
      SUBROUTINE THDER(P,M,U,N,X,XT,K)
C ****************************************************************
C SUBROUTINE THDER
C
C PURPOSE
C   TO COMPUTE THE STATE DERIVATIVES OF THE THRUSTER FORCES
C   AND MOMENTS FOR THE DSRV
C
C SUBROUTINES REQUIRED
C   RGCOM
C
C DESCRIPTION
C   CHAPTER D6,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ****************************************************************
      DIMENSION P(1),U(1),N(1),X(1)
      IF(K.GT.6)GO TO 21
      GO TO (1,2,3,4,5,6),K
   21 XT=0.
      RETURN
    1 XT=0.
      I=N(2)-1
      NT=N(1)
      JS=10
      JR=-1
      MV=M+207
      DO 7 J=1,NT
      JS=JS+JR
      IJ=I+J
      JT=JS+2
      MV=MV+JR*16
      UI=ABS(U(IJ))
      IF(UI.EQ.0.)GO TO 16
      A=X(1)/UI
      PV=P(M+128)
      IF(A.GT.PV)A=PV
      PW=P(M+127)
      IF(A.LT.PW)A=PW
      CALL RGCOM(MV,N(JT),P,IR,A,TS)
      XT=XT+ABS(U(IJ))*TS
   16 JR=-JR
    7 CONTINUE
      RETURN
    2 XT=0.
      I=N(2)-1
      ISW=0
      JT=9
      MV=M+129
      PV=P(M+128)
      PW=P(M+127)
   12 I=I+1
      UI=ABS(U(I))
      IF(UI.EQ.0.)GO TO 17
```

```
      SUBROUTINE THDER(P,M,U,N,X,XT,K)
C ****************************************************************
C SUBROUTINE THDER
C
C PURPOSE
C   TO COMPUTE THE STATE DERIVATIVES OF THE THRUSTER FORCES
C   AND MOMENTS FOR THE DSRV
C
C SUBROUTINES REQUIRED
C   RGCOM
C
C DESCRIPTION
C   CHAPTER D6,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ****************************************************************
      DIMENSION P(1),U(1),N(1),X(1)
      IF(K.GT.6)GO TO 21
      GO TO (1,2,3,4,5,6),K
   21 XT=0.
      RETURN
    1 XT=0.
      I=N(2)-1
      NT=N(1)
      JS=10
      JR=-1
      MV=M+207
      DO 7 J=1,NT
      JS=JS+JR
      IJ=I+J
      JT=JS+2
      MV=MV+JR*16
      UI=ABS(U(IJ))
      IF(UI.EQ.0.)GO TO 16
      A=X(1)/UI
      PV=P(M+128)
      IF(A.GT.PV)A=PV
      PW=P(M+127)
      IF(A.LT.PW)A=PW
      CALL RGCOM(MV,N(JT),P,IR,A,TS)
      XT=XT+ABS(U(IJ))*TS
   16 JR=-JR
    7 CONTINUE
      RETURN
    2 XT=0.
      I=N(2)-1
      ISW=0
      JT=9
      MV=M+129
      PV=P(M+128)
      PW=P(M+127)
   12 I=I+1
      UI=ABS(U(I))
      IF(UI.EQ.0.)GO TO 17
```

```
 9 RETURN
 6 XT=0.
   I=N(2)-1
   ISW=0
   JT=13
   MV=M+235
   JV=M+123
   PV=P(M+128)
   PW=P(M+127)
10 I=I+1
   UI=ABS(U(I))
   IF(UI.EQ.0.)GO TO 20
   A=X(I)/UI
   IF(A.GT.PV)A=PV
   IF(A.LT.PW)A=PW
   CALL RGCOM(MV,N(JT),P,IR,A,TS)
   XT=XT+P(JV)*U(I)*TS
20 MV=MV+31
   JV=JV+1
   JT=JT+1
   ISW=ISW+1
   GO TO (10,11),ISW
11 RETURN
   END
```

```
      SUBROUTINE PRDEP(P,M,X,U,N,NP,XP,K,J)
C ****************************************************************
C SUBROUTINE PRDEP
C
C PURPOSE
C   TO COMPUTE THE DERIVATIVES WITH RESPECT TO DSRV
C   PROPELLOR PARAMETERS
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   CHAPTER D6,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ****************************************************************
      DIMENSION P(1),X(1),U(1)
      LOGICAL A,B,C
      PM=P(M+36)
      VP=X(2)+PM*X(6)
      WP=X(3)-PM*X(5)
      GO TO (1,2,3,4,5,6),K
C     SURGE REGION DETERMINATION AND EQUATION SELECTION
    1 A=X(1).GE.0.
      B=U(N).GE.0.
      C=(U(N)-P(M+35)*X(1)).GE.0.
      IF(A.AND.C)ISW=1
      IF(A.AND..NOT.C)ISW=2
      IF(.NOT.A.AND.B)ISW=3
      IF(.NOT.(A.OR.B))ISW=4
C     PROPELLOR SURGE EQUATIONS
      NT=M+32
      IF(J.NE.NT)GO TO 21
      XP=U(N)*SQRT(VP*VP+WP*WP)
      RETURN
   21 GO TO (7,8,9,10),ISW
    7 NT=M
      IF(J.NE.NT)GO TO 19
      XP=U(N)*ABS(U(N))
      RETURN
   19 NT=M+1
      IF(J.NE.NT)GO TO 20
      XP=X(1)*U(N)
      RETURN
   20 NT=M+2
      IF(J.NE.NT)GO TO 22
      XP=X(1)*X(1)
      RETURN
   22 XP=0.
      RETURN
    8 NT=M+3
      IF(J.NE.NT)GO TO 23
      XP=U(N)*ABS(U(N))
      RETURN
```

```
   23 NT=M+4
      IF(J.NE.NT)GO TO 24
      XP=X(1)*U(N)
      RETURN
   24 NT=M+5
      IF(J.NE.NT)GO TO 22
      XP=X(1)*X(1)
      RETURN
    9 NT=M+6
      IF(J.NE.NT)GO TO 25
      XP=U(N)*ABS(U(N))
      RETURN
   25 NT=M+7
      IF(J.NE.NT)GO TO 26
      XP=X(1)*U(N)
      RETURN
   26 NT=M+8
      IF(J.NE.NT)GO TO 22
      XP=X(1)*X(1)
      RETURN
   10 NT=M+9
      IF(J.NE.NT)GO TO 27
      XP=U(N)*ABS(U(N))
      RETURN
   27 NT=M+10
      IF(J.NE.NT)GO TO 28
      XP=X(1)*U(N)
      RETURN
   28 NT=M+11
      IF(J.NE.NT)GO TO 22
      XP=X(1)*X(1)
      RETURN
C     SWAY REGION DETERMINATION AND EQUATION SELECTION
    2 IF(U(N).LT.0.)GO TO 11
C     PROPELLOR SWAY EQUATIONS
      NT=M+12
      IF(J.NE.NT)GO TO 22
      XP=U(N)*VP
      RETURN
   11 NT=M+13
      IF(J.NE.NT)GO TO 22
      XP=U(N)*VP
      RETURN
C     HEAVE REGION DETERMINATION AND EQUATION SELECTION
    3 IF(U(N).LT.0.)GO TO 12
      NT=M+14
      IF(J.NE.NT)GO TO 22
      XP=U(N)*WP
      RETURN
   12 NT=M+15
      IF(J.NE.NT)GO TO 22
      XP=U(N)*WP
      RETURN
```

```
    4 A=X(1).GE.0.
      B=U(N).GE.0.
      C=(U(N)-P(M+35)*X(1)).GE.0.
      NT=M+33
      IF(J.NE.NT)GO TO 29
      XP=U(N)*SQRT(VP*VP+WP*WP)
      RETURN
   29 NT=M+34
      IF(J.NE.NT)GO TO 30
      XP=U(N+1)
      RETURN
   30 IF(A.AND.C)ISW=1
      IF(A.AND..NOT.C)ISW=2
      IF(.NOT.A.AND.B)ISW=3
      IF(.NOT.(A.OR.B))ISW=4
      GO TO (13,14,15,16),ISW
C     PROPELLOR ROLL EQUATIONS
   13 NT=M+16
      IF(J.NE.NT)GO TO 31
      XP=U(N)*ABS(U(N))
      RETURN
   31 NT=M+17
      IF(J.NE.NT)GO TO 32
      XP=X(1)*U(N)
      RETURN
   32 NT=M+18
      IF(J.NE.NT)GO TO 22
      XP=X(1)*X(1)
      RETURN
   14 NT=M+19
      IF(J.NE.NT)GO TO 33
      XP=U(N)*ABS(U(N))
      RETURN
   33 NT=M+20
      IF(J.NE.NT)GO TO 34
      XP=X(1)*U(N)
      RETURN
   34 NT=M+21
      IF(J.NE.NT)GO TO 22
      XP=X(1)*X(1)
      RETURN
   15 NT=M+22
      IF(J.NE.NT)GO TO 35
      XP=U(N)*ABS(U(N))
      RETURN
   35 NT=M+23
      IF(J.NE.NT)GO TO 36
      XP=X(1)*U(N)
      RETURN
   36 NT=M+24
      IF(J.NE.NT)GO TO 22
      XP=X(1)*X(1)
      RETURN
```

```
   16 NT=M+25
      IF(J.NE.NT)GO TO 37
      XP=U(N)*ABS(U(N))
      RETURN
   37 NT=M+26
      IF(J.NE.NT)GO TO 38
      XP=X(1)*U(N)
      RETURN
   38 NT=M+27
      IF(J.NE.NT)GO TO 22
      XP=X(1)*X(1)
      RETURN
C     PITCH REGION DETERMINATION AND EQUATION SELECTION
    5 IF(U(N).LT.0.)GO TO 17
C     PROPELLOR PITCH EQUATIONS
      NT=M+28
      IF(J.NE.NT)GO TO 22
      XP=U(N)*WP
      RETURN
   17 NT=M+29
      IF(J.NE.NT)GO TO 22
      XP=U(N)*WP
      RETURN
C     YAW REGION DETERMINATION AND EQUATION SELECTION
    6 IF(U(N).LT.0.)GO TO 18
C     PROPELLOR YAW EQUATIONS
      NT=M+30
      IF(J.NE.NT)GO TO 22
      XP=U(N)*VP
      RETURN
   18 NT=M+31
      IF(J.NE.NT)GO TO 22
      XP=U(N)*VP
      RETURN
      END
```

```
      SUBROUTINE THDEP(P,M,U,N,X,XT,K,J)
C ***************************************************************************
C SUBROUTINE THDEP
C
C PURPOSE
C   TO COMPUTE THE DERIVATIVES WITH RESPECT TO DSRV THRUSTER
C   PARAMETERS
C
C SUBROUTINES REQUIRED
C   RPDER,RGCOM
C
C DESCRIPTION
C   CHAPTER D6,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***************************************************************************
      DIMENSION P(1),U(1),N(1),X(1)
      GO TO (1,2,3,4,5,6),K
C     THRUSTER SURGE EQUATIONS
    1 XT=0.
      I=N(2)-1
      NT=N(1)
      JS=4
      JR=-1
      MV=M+60
      DO 7 J=1,NT
      JS=JS+JR
      IJ=I+J
      JT=JS+2
      MV=MV+JR*10
      UI=ABS(U(IJ))
      IF(UI.EQ.0.)GO TO 16
      A=X(1)/UI
      PV=P(M+128)
      IF(A.GT.PV)A=PV
      PW=P(M+127)
      IF(A.LT.PW)A=PW
      CALL RPDER(MV,N(JT),P,IR,A,XT,J,L)
      XT=XT*U(IJ)*U(IJ)
      IF(L.NE.0)GO TO 13
   16 JR=-JR
    7 CONTINUE
      RETURN
C     THRUSTER SWAY EQUATIONS
    2 XT=0.
      I=N(2)-1
      ISW=0
      JT=3
      MV=M
      PV=P(M+128)
      PW=P(M+127)
   12 I=I+1
      UI=ABS(U(I))
      IF(UI.EQ.0.)GO TO 17
```

```
      A=X(1)/UI
      IF(A.GT.PV)A=PV
      IF(A.LT.PW)A=PW
      CALL RPDER(MV,N(JT),P,IR,A,XT,J,L)
      XT=XT*U(I)*ABS(U(I))
      IF(L.NE.0)GO TO 13
   17 MV=MV+25
      JT=JT+1
      ISW=ISW+1
      GO TO (12,13),ISW
   13 RETURN
C     THRUSTER HEAVE EQUATIONS
    3 XT=0.
      I=N(2)+1
      ISW=0
      JT=3
      MV=M
      PV=P(M+128)
      PW=P(M+127)
   14 I=I+1
      UI=ABS(U(I))
      IF(UI.EQ.0.)GO TO 18
      A=X(1)/UI
      IF(A.GT.PV)A=PV
      IF(A.LT.PW)A=PW
      CALL RPDER(MV,N(JT),P,IR,A,XT,J,L)
      XT=XT*U(I)*ABS(U(I))
      IF(L.NE.0)GO TO 13
   18 MV=MV+25
      JT=JT+1
      ISW=ISW+1
      GO TO (14,15),ISW
   15 RETURN
C     THRUSTER ROLL EQUATIONS
    4 XT=0.
      RETURN
C     THRUSTER PITCH EQUATIONS
    5 XT=0.
      I=N(2)
      JV=M+122
      IF(J.NE.JV)GO TO 21
      XT=U(I)*ABS(U(I))+U(I+1)*ABS(U(I+1))
      RETURN
   21 ISW=0
      I=I+1
      MV=M+76
      JT=7
      JV=M+120
      PV=P(M+128)
      PW=P(M+127)
    8 I=I+1
      UI=ABS(U(I))
      IF(UI.EQ.0.)GO TO 19
```

```
      A=X(1)/UI
      IF(A.GT.PV)A=PV
      IF(A.LT.PW)A=PW
      IF(J.NE.JV)GO TO 22
      CALL RGCOM(MV,N(JT),P,IR,A,TS)
      XT=TS*U(I)*ABS(U(I))
      RETURN
   22 CALL RPDER(MV,N(JT),P,IR,A,XT,J,L)
      XT=XT*U(I)*ABS(U(I))*P(JV)
      IF(L.NE.0)GO TO 13
   19 MV=MV+22
      JV=JV+1
      JT=JT+1
      ISW=ISW+1
      GO TO (8,9),ISW
    9 RETURN
C     THRUSTER YAW EQUATIONS
    6 XT=C.
      I=N(2)-1
      ISW=0
      MV=M+76
      JT=7
      JV=M+123
      PV=P(M+128)
      PW=P(M+127)
   10 I=I+1
      UI=ABS(U(I))
      IF(UI.EQ.0.)GO TO 20
      A=X(1)/UI
      IF(A.GT.PV)A=PV
      IF(A.LT.PW)A=PW
      IF(J.NE.JV)GO TO 23
      CALL RGCOM(MV,N(JT),P,IR,A,TS)
      XT=TS*U(I)*ABS(U(I))
      RETURN
   23 CALL RPDER(MV,N(JT),P,IR,A,XT,J,L)
      XT=XT*U(I)*ABS(U(I))*P(JV)
      IF(L.NE.0)GO TO 13
   20 MV=MV+22
      JV=JV+1
      JT=JT+1
      ISW=ISW+1
      GO TO (10,11),ISW
   11 I=I+1
      IF(J.NE.JV)GO TO 13
      XT=U(I)*ABS(U(I))+U(I+1)*ABS(U(I+1))
      RETURN
      END
```

```
      SUBROUTINE RPDER(M,N,P,I,X,Y,J,L)
C ***********************************************************************
C SUBROUTINE RPDER
C
C PURPOSE
C   TO COMPUTE THE PARAMETRIC DERIVATIVES OF A LINEAR VALUE
C   WITHIN A RANGE WITH RESPECT TO STRAIGHT LINE SLOPES
C   AND INTERCEPTS
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   CHAPTER D6,M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***********************************************************************
      DIMENSION P(1)
      L=0
      DO 1 K=1,N
      IF(X.GT.P(M+K))GO TO 1
      I=K
      MV=M+N+K
      IF(J.NE.NT)GO TO 2
      Y=X
      L=1
      RETURN
    2 NT=NT+N
      IF(J.NE.NT)GO TO 3
      Y=1.
      L=2
      RETURN
    3 Y=0.
      RETURN
    1 CONTINUE
      I=-1
      Y=0.
      RETURN
      END
```

APPENDIX A10

THE MEAN OF A SQUARED GAUSSIAN RANDOM VARIABLE

Given a normally distributed, zero mean random variable x as expressed by equation A10.1, and a general squared transformation as in equation A10.2, the resulting random variable y is shown to be of non-zero mean given in equation A10.3.

$$x = N\,(0,\ \sigma^2) \qquad \text{A10.1}$$

$$y = a\ x^2 \qquad \text{A10.2}$$

$$E[y] = a\sigma^2 \qquad \text{A10.3}$$

The probability density function for x is given by equation A10.4 and the associated probability density function for y is given by equation A10.5 (P-3, p. 130). The mean of the random variable y is expressed by equations A10.6 A10.7, and A10.8.

$$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{x^2}{2\sigma^2}} \qquad \text{A10.4}$$

$$f_y(y) = \frac{1}{\sigma\sqrt{2\pi a y}}\, e^{-\frac{y}{2a\sigma^2}}\, U(y) \qquad \text{A10.5}$$

$U(y)$ = step function in y

$$E\,[y] = \int_{-\infty}^{\infty} y\ f_y(y)\ dy \qquad \text{A10.6}$$

$$E\,[y] = \int_{0}^{\infty} \frac{\sqrt{y}}{\sigma\sqrt{2\pi a}}\, e^{-\frac{y}{2a\sigma^2}}\ dy \qquad \text{A10.7}$$

$$E[y] = K \int_0^\infty \sqrt{y} \quad e^{-by} \, dy \qquad \text{A10.8}$$

$$\text{Where } K = \frac{1}{\sigma\sqrt{2\pi a}}$$

$$b = \frac{1}{2a\sigma^2}$$

The integral in equation A10.8 may be transformed to the integral in equation A10.10 by using the separation of variables in equation A10.9. Equation A10.10 is a standard definite integral (S-28, Integral #531) whose result is given by equation A10.11.

$$\int u\,dv = \int v\,du - uv \Big| \qquad \text{A10.9}$$

$$\text{Where:} \quad u = \sqrt{y} \qquad dv = e^{-by}\,dy$$

$$du = \tfrac{1}{2}y^{-\frac{1}{2}} \qquad v = -\frac{1}{b}\,e^{-by}$$

$$E[y] = \frac{K}{2b} \int_0^\infty \frac{e^{-by}}{\sqrt{y}}\,dy \qquad \text{A10.10}$$

$$\int_0^\infty \frac{e^{-by}}{\sqrt{y}}\,dy = \sqrt{\frac{\pi}{b}} \qquad \text{A10.11}$$

With the substitution of equation A10.11 into A10.10 the result is directly given by equation A10.3. This then shows that the mean of the square of a zero mean gaussian random variable is not necessarily of zero mean. Many of the biases arising in identification processes for nonlinear stochastic systems may be partially or wholly attributed to this fact.

APPENDIX A11

A NONLINEAR OBSERVABILITY CRITERION APPLIED TO THE DSRV SINGLE DEGREE OF FREEDOM EQUATION

A nonlinear dynamic system given by equations A11.1 and A11.2 is observable if S, u(t), and y(t) determine x_o uniquely. A sufficiency criterion for checking nonlinear system observability is given by Schoenwandt (S-5) and is applied here to the DSRV single degree of freedom equation A11.3. The general nonlinear state estimation problem for equation A11.3 is given by equations A11.4 and A11.5.

$$\dot{\underline{x}} = \underline{f}\ (\underline{x}, \underline{u}, \underline{t}) \quad ; \quad t \quad [t_o, t_1] \qquad \text{A11.1}$$

$$\underline{y} = \underline{g}\ (\underline{x}, \underline{u}, \underline{t}) \quad ; \quad x_o \quad S \qquad \text{A11.2}$$

$$\dot{x} = ax^2 + bu^2 \quad ; \quad y = cx \qquad \text{A11.3}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2\ x_1^2 + x_3\ u^2 \\ 0 \\ 0 \end{bmatrix} \qquad \text{A11.4}$$

$$y = [c\ 0\ 0]\ [x_1\ x_2\ x_3]^T \qquad \text{A11.5}$$

Schoenwandt's criterion consists mainly of finding enough initial values of $\underline{y}$ and its derivatives to specify $\underline{x}_o$. The following chain (A11.6 - A11.15) of equations develops the observability criterion steps for c = 1 in Schoenwandt's terminology. Equations A11.6 through

$$u(t) = u_o \qquad \text{A11.6}$$

$$\dot{u}_o = 0 \qquad \text{A11.7}$$

$$y = x_1 \qquad \text{A11.8}$$

$$g_o = x_{10} \qquad \text{A11.9}$$

$$g_1 = \left.\frac{\partial g}{\partial t}\right|_{t_o} + \left.\frac{\partial g}{\partial x}\right|_{t_o} f(x_o, u_o, t_o) + \left.\frac{\partial g}{\partial u}\right|_{t_o} \dot{u}(t_o) \qquad \text{A11.10}$$

$$g_1 = x_{20}\, x_{10}^2 + x_{30}\, u_o^2 \qquad \text{A11.11}$$

$$g_2 = \left.\frac{\partial g_1}{\partial t}\right|_{t_o} + \left.\frac{\partial g_1}{\partial x}\right|_{t_o} f(x_o, u_o, t_o) + \left.\frac{\partial g_1}{\partial u}\right|_{t_o} \dot{u}(t_o) \qquad \text{A11.12}$$

$$g_2 = 2x_{10}\, x_{20}\, g_1 \qquad \text{A11.13}$$

$$g_3 = \left.\frac{\partial g_2}{\partial t}\right|_{t_o} + \left.\frac{\partial g_2}{\partial x}\right|_{t_o} f(x_o, u_o, t_o) + \left.\frac{\partial g_2}{\partial u}\right|_{t_o} \dot{u}(t_o) \qquad \text{A11.14}$$

$$g_3 = g_2^2 \,/\, g_1 + 2x_{20}\, g_1^2 \qquad \text{A11.15}$$

A11.15 can now be shown to determine $\underline{x}_o$ uniquely in terms of g_o, g_1, g_2, and g_3 as in equations A11.16 through A11.18.

$$x_{10} = g_o \quad \text{uniquely} \qquad \text{A11.16}$$

$$x_{20} = (g_3\, g_1 - g_2^2) \,/\, 2g_1^3 \quad \text{uniquely; } g_1 \neq 0 \qquad \text{A11.17}$$

$$x_{30} = g_1\, (1 - x_{20}\, g_1) \,/\, u_o^2 \quad \text{uniquely; } u_o \neq 0 \qquad \text{A11.18}$$

The initial state vector $\underline{x}_o$ is uniquely determined in equations A11.16 through A11.18 and therefore the nonlinear state estimation problem is observable and all of its states may be estimated. The condition $g_1 \neq 0$ imposes a limitation on the initial value of the input function given by equation A11.19. This equation simply says

that the system is not initially at its steady-state value. The

$$u_o^2 \neq - \frac{x_{20}}{x_{30}} x_{10}^2 \qquad \text{A11.19}$$

other condition on the input $u_o \neq 0$ says that there must be some initial non-zero input to the system for observability to be possible. This appendix applies to Chapter P2.3.

CHAPTER P3 MODEL REFERENCE SINGLE DEGREE OF FREEDOM PROGRAM AND SUBROUTINES

```
C     PROGRAM FOR NONLINEAR IDENTIFICATION COST FUNCTION
C     WITH NOISE
      REAL M
      DIMENSION VEC(47),XM(47),XV(94)
      DIMENSION CD(47),CN(51),CCST(2397)
      DIMENSION P(282),DD(5),GG(5)
      COMMON /PNOIS/AM,S
      COMMON /PND/D
      COMMON /PNG/G
      AM=0.
      S=1.
      NDD=2
      NGG=2
      M=4507.
      CDZ=-16.7
      CNZ=755.
      CDST=-22.2
      CNST=555.
      DELCD=0.25
      DELCN=10.
      AZ=CDZ/M
      BZ=CNZ/M
      C=1.
      AST=CDST/M
      BST=CNST/M
      DA=DELCD/M
      DB=DELCN/M
      N=47
      READ(5,20)(DD(I),I=1,5)
      READ(5,20)(GG(I),I=1,5)
   20 FORMAT(10F8.2)
      TI=0.
      XI=0.
      H=10.
      IXV=111
      IXW=11
      NPL=47
      NCN=51
      NCD=47
      ICN=NCN-1
      ICD=NCD-1
      NS=0
      NC=1
      CD(1)=CDST
      DO 5 I=1,ICD
      CD(I+1)=CD(I)+DELCD
    5 CONTINUE
      CN(1)=CNST
```

```
      DO 6 J=1,ICN
      CN(J+1)=CN(J)+DELCN
    6 CONTINUE
      DO 10 IDD=1,NDD
      D=CC(IDD)
      DC 11 IGG=1,NGG
      G=GG(IGG)
C     COMPUTE VEC
      CALL RK(H,TI,XI,K,N,AZ,BZ,C,VEC,IXW,IXV)
      DO 1 K=1,NPL
      XV(K)=TI+FLOAT(K)*H
      XV(K+NPL)=VEC(K)
    1 CONTINUE
      CALL PLOT(0,XV,NPL,2,0)
C     BEGIN A AND B PARAMETER VARIATION AND COST CALCULATION
      B=BST
      DC 3 J=1,NCN
      A=AST
      DO 2 I=1,NCD
      CALL RKINT(H,TI,XI,N,A,B,C,XM)
      NC=(J-1)*NCD+I
      COST(NC)=0.
      DO 4 JSQ=1,N
C     INTEGRATE THE SQUARE OF THE ERROR
      XDIF=VEC(JSQ)-XM(JSQ)
      COST(NC)=COST(NC)+XDIF*XDIF*H
    4 CONTINUE
      A=AST+DA*FLOAT(I-1)
    2 CONTINUE
      B=BST+DB*FLOAT(J-1)
    3 CONTINUE
      CALL CONTUR(NJ,CD,CN,COST,NCD,NCN,NS)
C     PLOT SETUP
      DO 7 I=1,NCD
      P(I)=CD(I)
      P(NCD+I)=COST(I)
      P(2*NCD+I)=COST(13*NCD+I)
      P(3*NCD+I)=COST(25*NCD+I)
      P(4*NCD+I)=COST(38*NCD+I)
      P(5*NCD+I)=COST(50*NCD+I)
    7 CONTINUE
      CALL PLOT(NO,P,NCD,6,NS)
      NO=NO+1
   11 CONTINUE
   10 CONTINUE
      STOP
      END
```

```
      REAL FUNCTION F(T,X,U,A,B,W)
C     NONLINEAR ABSOLUTE SQUARE LAW VEHICLE
      COMMON /PNG/G
      F=A*X*ABS(X)+B*U*ABS(U)+G*W
      RETURN
      END
```

```
      SUBROUTINE RK(H,TI,XI,K,N,A,B,C,VEC,IXW,IXV)
      DIMENSION VEC(1)
      COMMON /PND/D
      X=XI
      T=TI
      DO 2 II=1,N
      UV=U(T,X)
      WV=W(IXW)
      VV=V(IXV)
      X=X+H*F(T,X,UV,A,B,WV)
      T=T+H
      VEC(II)=X+D*VV
    2 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RKINT(H,TI,XI,N,A,B,C,VEC)
      DIMENSION VEC(1)
      X=XI
      T=TI
      DO 2 I=1,N
      UV=U(T,X)
      X=X+H*FUN(T,X,UV,A,B)
      T=T+H
      VEC(I)=X
    2 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION FUN(T,X,U,A,B)
C     NONLINEAR ABSOLUTE SQUARE LAW MODEL
      FUN=A*X*ABS(X)+B*U*ABS(U)
      RETURN
      END
```

```
      REAL FUNCTION V(IX)
C     N(0,1) DISTRIBUTION
      COMMON /PNOIS/AM,S
      CALL GAUSS(IX,S,AM,V)
      RETURN
      END


      REAL FUNCTION U(T,X)
C     STEP INPUT OF UNIT HEIGHT
      U=1.
      RETURN
      END


      REAL FUNCTION W(IX)
C     N(0,1) DISTRIBUTION
      COMMON /PNOIS/AM,S
      CALL GAUSS(IX,S,AM,W)
      RETURN
      END
```

Additional Subroutines:

GAUSS, RANDU	Appendix A4
CONTUR, PLOT	Appendix A5

APPENDIX A13

MODEL REFERENCE STUDIES OF A LINEAR SURGE EQUATION

Many of the studies conducted in Chapter P3 for the nonlinear DSRV surge equation were also conducted upon the linear equation A13.1. Many of the same inputs were used and most of the results are presented here in the same form as in Chapter P3. The inputs used in

$$4507\,\dot{x} = -100x + 755\,u \qquad \text{A13.1}$$

these studies are the step, staircase, simulated impulse and sinusoidal with 150 second-period. A listing of the applicable tables and figures is given in Table A13.1. The inputs correspond to those in Chapter P3.

Input	Contours	Noise Behavior
Step	Figure A13.1	Table A13.2
Staircase	Figure A13.2	Table A13.3
Impulse	Figure A13.3	Table A13.4
Sinusoidal-150	Figure A13.4	Table A13.5

Table A13.1 TABLES AND FIGURES FOR LINEAR MODEL REFERENCE STUDIES

Step Input	p_1^* (-100)	p_2^* (755)	C ($\underline{p}^*$)	C max
v=0, w=0	-100	755	0	33220
v=0, w=10%	-114	870	18	32500
v=10%, w=0	-98	750	306	33610
v-10%, w=10%	-102	850	299	33700

Table A13.2 NOISE CHARACTERISTICS, LINEAR SYSTEM, STEP INPUT

No.	%v	%w	p_1^* (-100)	p_2^* (755)	$C(p^*)$	C max
1	0	0	-100	755	0	6999
2	0	1	-100	755	0.3	6969
3	0	10	-106	775	19.7	7470
4	0	100	-112	1050	2684	8182
5	0	200	-150	900	6353	16890
6	1	0	-99	755	1.7	6912
7	1	1	-97	750	2.2	6973
8	1	10	-88	710	24	6338
9	1	100	-110	1050	3396	8645
10	1	200	-88	670	6423	14240
11	10	0	-92	770	281	7303
12	10	1	-92	770	246	7069
13	10	10	-104	805	324	7139
14	10	100	-116	555	1948	14340
15	10	200	-150	940	13370	27820
16	100	0	-80	555	23400	32090
17	100	1	-58	650	22570	26140
18	100	10	-150	700	29150	43040
19	100	100	-112	1055	30740	37670
20	100	200	-150	960	27150	42140
21	200	0	-120	900	102100	112200
22	200	1	-110	560	77750	90740
23	200	10	-150	560	74650	94440
24	200	100	-150	560	07700	118300
25	200	200	-58	580	64980	69930

Table A13.3 NOISE CHARACTERISTICS, LINEAR SYSTEM, STAIRCASE INPUT

Simulated Impulse	p_1^* (-100)	p_2^* (755)	C ($\underset{\sim}{p}$*)	C max
v=0, w=0	-100	755	0	10500
v=0, w=10%	-94	775	18	9807
v=10%, w=0	-104	785	293	11010
v=10%, w=10%	-89	720	264	10490

Table A13.4 NOISE CHARACTERISTICS, LINEAR SYSTEM, SIMULATED IMPULSE INPUT

Sinusoidal 150 sec.	p_1^* (-100)	p_2^* (755)	C ($\underset{\sim}{p}$*)	C max
v=0, w=0	-100	755	0	2488
v=0, w=10%	-96	755	30	2407
v=10%, w=0	-94	765	298	2529
v=10%, w=10%	-100	745	286	3082

Table A13.5 NOISE CHARACTERISTICS, LINEAR SYSTEM, SINUSOIDAL 150 INPUT

Additional studies were made using the linear vehicle in equation A13.1 and a nonlinear absolute square model to simulate the fitting of a nonlinear model to linear data. The contours of these runs using a sinusoidal 150 second-period input are shown in Figure A13.5 and show that the best nonlinear fit for equation A13.1 is given by equation A13.2. The noise characteristics of the nonlinear model for linear data are presented in Table A13.6.

$$4507\,\dot{x} = -19.7\,x\,|x| + 645\,u\,|u| \qquad \text{A13.2}$$

Lin. Sys.,Nonlinear Model	p_1^*	p_2^*	C ($\underline{p}^*$)	C max
v=0, w=0	-19.7	645	116	3533
v=0, w=10%	-19.4	640	173	3558
v=10%, w=0	-18.5	660	339	3423
v=10%, w=10%	-21.5	620	471	3488

Table A13.6 NOISE CHARACTERISTICS, LINEAR SYSTEM, NONLINEAR MODEL, SINUSOIDAL 150 INPUT

The contours and tables in this appendix show that the sinusoidal input function is the best of those inputs tested for identification using model reference contouring. In addition, the noise characteristics indicate that the 10% v and w noises generally permit about 5% parameter identifications.

```
                         CONTOUR     1
              *....* INCREMENT IS     0.5000000E 02
      0.5550000E 03          0.8050000E 03          0.1055000E 04
  0.56E 03  0.66E 03  0.76E 03  0.86E 03  0.96E 03  0.11E 04
          *....*....*....*....*....*....*....*....*....*....*
-0.150E 03*555554444444443333333333222222222222222222222222222*
-0.148E 03*555554444444443333333333322222222222222222222222222*
-0.146E 03*555554444444444333333333322222222222222222222222222*
-0.144E 03*555444444444433333333333222222222222222222222222222*
-0.142E 03*555444444444333333333333222222222222222222222222222*
-0.140E 03*555444444444333333333332222222222222222222222222222*
-0.138E 03*554444444444333333333333222222222222222222222222222*
-0.136E 03*444444444443333333333332222222222222222222222222222*
-0.134E 03*444444444433333333333222222222222222222222222222222*
-0.132E 03*444444444333333333333222222222222222222222222222222*
-0.130E 03*444444433333333333322222222222222222222222222222222*
-0.128E 03*444444333333333333222222222222222222222222222222222*
-0.126E 03*444443333333333332222222222222222222222222222222222*
-0.124E 03*444443333333333322222222222222222222222222222222222*
-0.122E 03*444433333333333322222222222222222222222222222222222*
-0.120E 03*444333333333333222222222222222222222222222222222222*
-0.118E 03*443333333333332222222222222222222222222222222222222*
-0.116E 03*333333333332222222222222222222222222112222222222222*
-0.114E 03*333333333322222222222222222222222211122222222222222*
-0.112E 03*333333333222222222222222222222221112222222222222222*
-0.110E 03*333333332222222222222222222222211122222222222222222*
-0.108E 03*333333322222222222222222222222111222222222222222223*
-0.106E 03*333332222222222222222222222211112222222222222222233*
-0.104E 03*333322222222222222222222221111222222222222222223333*
-0.102E 03*333222222222222222222222211112222222222222222333333*
-0.100E 03*332222222222222222222221112222222222222222223333333*
-0.980E 02*222222222222222222221111122222222222222222233333334*
-0.960E 02*222222222222222222211112222222222222222223333333444*
-0.940E 02*222222222222222222111122222222222222222333333344444*
-0.920E 02*222222222222222221112222222222222222233333333444445*
-0.900E 02*222222222222222111122222222222222233333333444444555*
-0.880E 02*222222222222222111222222222222223333333344444455555*
-0.860E 02*222222222222211222222222222222233333333444444555566*
-0.840E 02*222222222221122222222222222223333333444444555556666*
-0.820E 02*222222222222222222222222222233333333444445555566677*
-0.800E 02*222222222222222222222222223333333444445555666677778*
-0.780E 02*222222222222222222222222233333344444555566667777888*
-0.760E 02*222222222222222222222223333333444445555666677788999*
-0.740E 02*2222222222222222222223333333444445555666677788999AA*
-0.720E 02*22222222222222222222333333444445555666677788899AABB*
-0.700E 02*2222222222222222223333334444455556667778899AAABBCC*
-0.680E 02*222222222222222233333344444555566677788899AABBBCCDD*
-0.660E 02*22222222222222333333444444555666677788899AAABBCCDDEEF*
-0.640E 02*22222222222333333444445556666777889999AABBCCCDDEEFGG*
-0.620E 02*222222222233333444444555666777888899AAABBCCDDEEFFGHHJ*
-0.600E 02*2222222233333444444555666777888899AABBCCDDEEFFGGHJJKK*
-0.580E 02*222222333334444555566677788999AABBCCDDEFFGGHJJKKLMM*
          *....*....*....*....*....*....*....*....*....*....*
  0.56E 03  0.66E 03  0.76E 03  0.86E 03  0.96E 03  0.11E 04
```

Figure A13.1 LINEAR SYSTEM, NO NOISE, STEP FUNCTION CONTOURS

```
                              CONTOUR    1
                    *....* INCREMENT IS     0.50000000E 02
          0.55500000E 03          0.80500000E 03          0.10550000E 04
     0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
               *....*....*....*....*....*....*....*....*....*....*
-0.150E 03*55555555444444444333333333333322222222222222222222*
-0.148E 03*55555555444444444433333333333322222222222222222222*
-0.146E 03*55555554444444444333333333333322222222222222222222*
-0.144E 03*55555544444444443333333333333222222222222222222222*
-0.142E 03*55555444444444443333333333332222222222222222222222*
-0.140E 03*55555444444444433333333333322222222222222222222222*
-0.138E 03*55554444444443333333333332222222222222222222222222*
-0.136E 03*55544444444433333333333322222222222222222222222222*
-0.134E 03*55444444444333333333333222222222222222222222222222*
-0.132E 03*44444444433333333333322222222222222222222222222222*
-0.130E 03*44444444433333333332222222222222222222222222222222*
-0.128E 03*44444444333333333322222222222222222222222222222222*
-0.126E 03*44444443333333333322222222222222222222222222222222*
-0.124E 03*44444433333333333222222222222222222222222222222222*
-0.122E 03*44444333333333333222222222222222222222222222222222*
-0.120E 03*44443333333333332222222222222222222222222222222222*
-0.118E 03*44433333333333322222222222222222222222222222222222*
-0.116E 03*44333333333332222222222222222222222222222222222222*
-0.114E 03*33333333333222222222222222222222222222222222222222*
-0.112E 03*33333333322222222222222222222222222222222222222222*
-0.110E 03*33333333222222222222222222222222222222222222222223*
-0.108E 03*33333332222222222222222222222222222222222222222333*
-0.106E 03*33333322222222222222222222222222222222222222223333*
-0.104E 03*33333222222222222222222222112222222222222222333333*
-0.102E 03*33332222222222222222222221111222222222222233333333*
-0.100E 03*33322222222222222222222211112222222222222333333334*
-0.980E 02*33222222222222222222222111222222222222222333333344*
-0.960E 02*22222222222222222222221112222222222222223333334444*
-0.940E 02*22222222222222222222211222222222222222333333344444*
-0.920E 02*22222222222222222222122222222222222223333334444455*
-0.900E 02*22222222222222222222222222222222222333333444444555*
-0.880E 02*22222222222222222222222222222222223333333444445556*
-0.860E 02*22222222222222222222222222222222333333444445555566*
-0.840E 02*22222222222222222222222222222223333333444455555667*
-0.820E 02*22222222222222222222222222222333333344444555566677*
-0.800E 02*22222222222222222222222222233333334444455556667778*
-0.780E 02*22222222222222222222222223333333444445555666777889*
-0.760E 02*22222222222222222222222333333344444555566677778899*
-0.740E 02*22222222222222222222223333333444445555666777788899AA*
-0.720E 02*2222222222222222222223333334444455556667777888999AAABB*
-0.700E 02*222222222222222222233333344444455556666777888999AABBCC*
-0.680E 02*2222222222222222233333444444555566677788899AAABBCCDDD*
-0.660E 02*222222222222233333344444455556667778889999AABBBCCDDEEF*
-0.640E 02*2222222222333333344444555566677788899AABBCCCDEEFFGG*
-0.620E 02*2222222333333444445555666777888899AABBBCCDDEEFFGHHJ*
-0.600E 02*22222333333344444555566667778889999AABBCCCCDEEFFGGHJJKL*
-0.580E 02*2233333334444455556667778889 9AABBBCCDEEFFGGHJJKKLMM*
               *....*....*....*....*....*....*....*....*....*....*
     0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
```

Figure A13.2 LINEAR SYSTEM, NO NOISE, STAIRCASE FUNCTION CONTOURS

```
                          CONTOUR    1
                 *....* INCREMENT IS      0.5000000E 02
        0.5550000E 03            0.8050000E 03              0.1055000E 04
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
              *....*....*....*....*....*....*....*....*....*....*
-0.150E 03*66666665555554444444433333333333333333333333333333*
-0.148E 03*66666665555554444444433333333333333333333333333333*
-0.146E 03*66666655555554444444433333333333333333333333333333*
-0.144E 03*66666555555544444444433333333333333333333333333333*
-0.142E 03*66665555555444444444333333333333333333333333333333*
-0.140E 03*66665555554444444443333333333333322222222223333333*
-0.138E 03*66655555554444444433333333333322222222222222333333*
-0.136E 03*66555555544444444333333333332222222222222222333333*
-0.134E 03*66555555444444444333333333332222222222222222333333*
-0.132E 03*55555554444444433333333332222222222222222222333333*
-0.130E 03*55555444444443333333333222222222222222222222333333*
-0.128E 03*55555444444433333333322222222222222222222222333333*
-0.126E 03*55554444444333333333222222222222222222222222333333*
-0.124E 03*55544444443333333332222222222222222222222222333333*
-0.122E 03*55544444443333333322222222222222222222222222333333*
-0.120E 03*55444444433333332222222222222222222222222223333333*
-0.118E 03*44444444333333322222222222222222222222222223333333*
-0.116E 03*44444443333333222222222222222222222222222233333333*
-0.114E 03*44444433333333222222222222222222222222222233333334*
-0.112E 03*44444433333332222222222222222222222222222333333344*
-0.110E 03*44443333333322222222222222222222222222222333333444*
-0.108E 03*44433333333222222222222222222222222222223333333444*
-0.106E 03*44433333332222222222222222222222222222223333334444*
-0.104E 03*44333333322222222222222222222222222222233333344444*
-0.102E 03*33333333322222222222222222222222222222333333444445*
-0.100E 03*33333333222222222222222111222222222222223333334444455*
-0.980E 02*3333332222222222222222111222222222222233333344444455*
-0.960E 02*3333332222222222222221112222222222222333333444445555*
-0.940E 02*33333222222222222222222222222222222223333334444555566*
-0.920E 02*333322222222222222222222222222222223333333444455555666*
-0.900E 02*33332222222222222222222222222222233333344444555556667*
-0.880E 02*333222222222222222222222222222222333333444445555666677*
-0.860E 02*333222222222222222222222222222223333334444555556666777*
-0.840E 02*33322222222222222222222222222223333333444445555666777788*
-0.820E 02*3322222222222222222222222222222333333444445555666667778899*
-0.800E 02*33222222222222222222222222223333333444445555666677788899*
-0.780E 02*3322222222222222222222222223333333444445555666677788899999A*
-0.760E 02*332222222222222222222222333333334444455556666077778899999AAB*
-0.740E 02*33222222222222222222223333333444445555566667777889999AABBB*
-0.720E 02*33222222222222222222233333333444445555566667777888899AAABBCC*
-0.700E 02*332222222222222333333333444444555556666777788899AAABBCCCDD*
-0.680E 02*3332222222222333333333344444455555666677778888899AAABBCCDDFE*
-0.660E 02*33333333333333333333334444445555566666777888899AABBCCDDEEFG*
-0.640E 02*3333333333333333333333444445555566666777888899AAABBCCCDEEFGGH*
-0.620E 02*333333333333333333344444445555566667777888899AAABBCCCCEFFGGHJJ*
-0.600E 02*3333333333333444444444555556666777888899AABBBCCDFEFFGGHJJKL*
-0.580E 02*333333344444444555556666677778889999AABBCCDDEEFFGHHJKKLMM*
              *....*....*....*....*....*....*....*....*....*....*
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
```

Figure A13.3 LINEAR SYSTEM, NO NOISE, SIMULATED IMPULSE FUNCTION CONTOURS

```
                                 CONTOUR    1
                        *....* INCREMENT IS     0.5000000E 02
        0.5550000E 03                0.8050000E 03               0.1055000E 04
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
            *....*....*....*....*....*....*....*....*....*....*
-0.150E 03*BBAAA998877776666555555554444444445555555566666777788*
-0.148E 03*BBAAA998887777666655555555444444444555555566666777788*
-0.146E 03*BBAA9998877776666655555544444444444445555556666677778*
-0.144E 03*AAA9998887776666555554444444444444444555555666777778*
-0.142E 03*AAA998887776666555554444444444444444444555556666667778*
-0.140E 03*AA99988777666655555444444444444444444455555666667778*
-0.138E 03*AA9988777666655555444444443333334444444445555556667778*
-0.136E 03*99988877766655555444444433333333334444444455555666667778*
-0.134E 03*999887776665555544444443333333333333444444455555666667778*
-0.132E 03*9988877666555544444433333333333333334444445555566667778*
-0.130E 03*998877766655544444333333333333333333444444555566667778*
-0.128E 03*8887776665554444433333333333333333333334444445556667778*
-0.126E 03*8887766655544444333333333333333333333344444555666677788*
-0.124E 03*88777666555444433333332222223333333334444555666677788*
-0.122E 03*8877666555444433333322222222223333334444555666677788*
-0.120E 03*77776655544443333332222222222223333334444555666677788*
-0.118E 03*777666555444433333222222222222223333344445556677778 8*
-0.116E 03*7776655544443333322222222222222233333444455566777888*
-0.114E 03*7766655544433333222222222222222223333344445556677788 9*
-0.112E 03*776655544444333322222222222222222223333444445566677788 9*
-0.110E 03*7766555444333332222222222222222222333344445555667778 89*
-0.108E 03*66665544443333222222222222222222233333444555667788 99*
-0.106E 03*6665554443333222222222222222222223333344455566677888 99*
-0.104E 03*66655544433333222222222222222222223333444555666777889A*
-0.102E 03*666554443333322222222222222222222233334445556667788899A*
-0.100E 03*66555444333332222222221222222222223333444555666777889AA*
-0.980E 02*66555444333322222222221222222222223333444555667778899AA*
-0.960E 02*66554443333322222222221222222222223333444555667778899AB*
-0.940E 02*66554443333222222222222222222222233334445556677788 99AAB*
-0.920E 02*55554443333222222222222222222222233333444555667778 89AABB*
-0.900E 02*555544433332222222222222222222223333444555667778899AABC*
-0.880E 02*555544433333222222222222222222223333344455566677889AABBC*
-0.860E 02*555544433333222222222222222222223333344455566778899ABBCD*
-0.840E 02*55554443333322222222222222222223333444555667788 99AABCCD*
-0.820E 02*5555444333322222222222222222233333444455566677889AABBCDD*
-0.800E 02*5555444333332222222222222222333334445556677888 99ABBCDDE*
-0.780E 02*555544433333222222222222223333333444555667788899AABCCDEE*
-0.760E 02*55554443333332222222222223333334445556677778889AABBCDEFF*
-0.740E 02*55554444333333322222333333334444555566677888 99ABBCDDEFG*
-0.720E 02*5555444443333333333333333333444445555666777888 99ABBCCDEFFG*
-0.700E 02*555554444433333333333333333444445556666778899AABCCDEFFGH*
-0.680E 02*66555444444333333333333333444444555666778899AABCCDEEFGHJ*
-0.660E 02*6655554444443333333344444445555566677888 99AABCCDEEFGHHJ*
-0.640E 02*6655554444444444444444444445555666777888 99AABCCDEEFGGHJK*
-0.620E 02*66655554444444444444444445555566667778899AABCCDDEFGGHJKL*
-0.600E 02*66665555544444444444555555666777888 99AABCCDDEFGGHJKLM*
-0.580E 02*66666555555554444555555555666777888899AABCCDEEFGGHJKLMM*
            *....*....*....*....*....*....*....*....*....*....*
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
```

Figure A13.4 LINEAR SYSTEM, NO NOISE, SINUSOIDAL 150 SECOND INPUT CONTOURS

```
                              CONTOUR    1
                    *....* INCREMENT IS      0.5000000E 02
        0.5550000E 03              0.8050000E 03              0.1055000E 04
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
              *....*....*....*....*....*....*....*....*....*....*
-0.222E 02*22222222222222222222223333344445556667778899AABBC*
-0.219E 02*22222222222222222222223333344445556667778899AABBC*
-0.217E 02*222222222222222222222233333444455566677788999AABBCC*
-0.214E 02*22222222222222222222223333344445556667778889AAABBCC*
-0.212E 02*222222222222222222222233333444455566677788999AAABBCC*
-0.209E 02*222222222222222222222233333444455566677788999AAABBCC*
-0.207E 02*22222222222222222222223333344445556667778889AABBBCC*
-0.204E 02*22222222222222222222223333344445556667778899AABBCCC*
-0.202E 02*22222222212222222222233333444455566677788999AABBCCD*
-0.199E 02*222222221122222222222333334444555666777888999AABBCCD*
-0.197E 02*222222221112222222222333334444555566677888999AAABBCCD*
-0.194E 02*22222222112222222222233333444455556667788999AAABBCCD*
-0.192E 02*222222222122222222222333334444555666777888999AABBCCCD*
-0.189E 02*2222222222222222222233333444455566677788999AABBCCDD*
-0.187E 02*2222222222222222222233333444455566677788999AABBCCDD*
-0.184E 02*22222222222222222222233333444455566677788999AAABBCCDD*
-0.182E 02*22222222222222222222233333444455566677788899AABBBCCCD*
-0.179E 02*2222222222222222222233333444455566677788899AABBCCDDE*
-0.177E 02*2222222222222222222233333444455566677788999AABBCCDDE*
-0.174E 02*222222222222222222223333344445556667778899AABBCCDDE*
-0.172E 02*2222222222222222222233333444455566677788999AABBCCDDEE*
-0.169E 02*2222222222222222222233333444455566677788999AABBCCDDEE*
-0.167E 02*2222222222222222222233333444455566677788999AABBCCDDEE*
-0.164E 02*22222222222222222223333344445556667778899AAABBCCDEEF*
-0.162E 02*22222222222222222223333344445556667778899AABBCCDDEEF*
-0.159E 02*22222222222222222223333344445556667778899AABBCCDDEEF*
-0.157E 02*2222222222222222223333334444555666777889999AABBCCDEEFF*
-0.154E 02*22222222222222222233333444455566677788999AABBCCDDEEFF*
-0.152E 02*222222222222222222333333444455566677788999AABBCCDDEEFG*
-0.149E 02*3322222222222222233333344445555666777889999AABBCCDEEFFG*
-0.147E 02*332222222222222223333344445555666777888999AABBCCDDEEFFGG*
-0.144E 02*33322222222222222333333444455566677788999AABBCCDDEFFGG*
-0.142E 02*333222222222222333333444455556677788999AABBCCDDEEFFGH*
-0.139E 02*33332222222222333333344445555666777888999AABBCCDDEFFGGH*
-0.137E 02*333332222222233333333444455555666777888999AABBCCDEEFFGHH*
-0.134E 02*33333322222333333333444455566677788999AABBCCDDEEFGHH*
-0.132E 02*333333333333333333344444555566677788899AABBCCDDEFFGGHJ*
-0.129E 02*333333333333333333344444555666777888999AABCCDDEEFFGGHHJ*
-0.127E 02*3333333333333333334444555566677788999AABBCCDEEFFGGHJJ*
-0.124E 02*333333333333333334444455566677788999AABBCDDEEFFGHHJK*
-0.122E 02*333333333333333344444555566677788999AABBCCDDEFFGGHJJK*
-0.119E 02*33333333333333344444555666777889999AABBCDDEEFFGHHJKK*
-0.117E 02*4433333333333344444455556667778899AABBCCDDEFFGGHJJKL*
-0.114E 02*44433333333344444445555666777888999AABBCDDEEFFGHHJKLL*
-0.112E 02*44444333334444444455555666777888999AABBCCDDEFFGGHJJKLM*
-0.109E 02*44444444444444444555556667778899999AABBCDDEEFFGGHHJKLLM*
-0.107E 02*444444444444444445555566677788899AABBCCDDEFFGHHJKKLMM*
              *....*....*....*....*....*....*....*....*....*....*
   0.56E 03   0.66E 03   0.76E 03   0.86E 03   0.96E 03   0.11E 04
```

Figure A13.5 LINEAR SYSTEM, NONLINEAR MODEL, SINUSOIDAL 150, NO NOISE CONTOURS

CHAPTER P4 EKF SINGLE DEGREE OF FREEDOM PROGRAM AND SUBROUTINES

```
C       PROGRAM FOR KALMAN FILTER STUDIES IN A SINGLE DEGREE
C       OF FREEDOM WITH ONE STATE AND TWO PARAMETERS
        REAL M
        DIMENSION ZS(188),US(188),TS(188),XV(141)
        DIMENSION A(9),B(3),XHT(3),XBAR(3),HZ(3)
        DIMENSION EHT(9),EBAR(9),EGAIN(3)
        DIMENSION XPL(376),APL(376),BPL(376)
        DIMENSION EPX(376),EPA(376),EPB(376),EPL(1316)
        COMMON /RKI/D
        COMMON /REF/G
        COMMON /UFUN/HV,TP,TPP,TO
        COMMON /NOISE/AM,S
        COMMON /STOP /DX,DA,DB,DEX,DEA,DEB,TSC
        NDN=1
        IXV=1
        IXW=11111
        D=0.07
        DO 5 KDNO=1,NDN
        G=0.0017
        DO 6 KGNO=1,NDN
        HV=0.5
        TP=100.
        TPP=200.
        TO=0.
        AM=0.
        S=1.
        M=4507.0
        EM=M
        DX=1.
        DA=1./EM
        DB=1./EM
        DEX=1.
        DEA=1./(EM*EM)
        DEB=1./(EM*EM)
        CDZ=-10.7
        CNZ=755.0
        AZ=CDZ/M
        BZ=CNZ/M
        TI=0.
        XI=0.
        C=1.
        N=188
        H=2.5
        TSC=1.
        SC=0.
C       GENERATE INITIAL VEHICLE DATA WITH NOISE
        CALL RKNL(H,TI,XI,N,AZ,BZ,C,ZS,US,TS,IXW,IXV)
C       PLOT INITIAL DATA
        USC=0.
        NPL=47
        KS=4
        DO 1 K=1,NPL
        L=KS*K
```

```
      XV(K)=TS(L)/TSC
      XV(NPL+K)=US(L)+USC
      XV(2*NPL+K)=ZS(L)+SC
    1 CONTINUE
      CALL PLOT(0,XV,NPL,3,0)
C     INITIAL SETUP
C     SET KALMAN FILTER INCREMENT,STARTING INDEX,
C     AND STOPPING INDEX
      KST=1
      KFINC=1
      KB=KST+KFINC
      KFIN=188
C     SET INITIAL STARTING STATE ESTIMATE
      XST=1.
      AST=-11.2/M
      BST=943./M
C     SET XHAT(0)
      XHT(1)=XST
      XHT(2)=AST
      XHT(3)=BST
C     SET EXAGERATED NOISE PARAMETERS
      PW=1.
      QW=1.
C     SET NOISE PARAMETERS,Q FOR VEHICLE NOISE W,
C     AND P FOR MEASUREMENT NOISE V
      P=PW*D*D                Note:  P = R used in Chapter P4
      Q=QW*G*G
C     INITIALIZE A,B,AND EHAT MATRICES
      DO 2 K=1,9
      A(K)=0.
      EHT(K)=0.
    2 CONTINUE
      B(2)=0.
      B(3)=0.
C     SET INITIAL ERROR COVARIANCES
      EHT(1)=0.5
      EHT(5)=20./(M*M)
      EHT(9)=3500./(M*M)
C     DEFINE H MATRIX
      HZ(1)=C
      HZ(2)=0.
      HZ(3)=0.
C     PLACE INITIAL VALUES IN OUTPUT
      NP=3
      DO 3 K=1,NP
      USV=US(1)
      UZ=U(TI,XI)
      US(1)=UZ
      DT=H
      CALL PROP(DT,US,XHT,EHT,A,Q,XBAR,EBAR,1)
      CALL GAIN(EBAR,HZ,P,EGAIN)
      CALL UPDAT(ZS,HZ,EBAR,XBAR,EGAIN,XHT,EHT,1)
     0CALL STORE(XHT,EHT,TS,XPL,APL,BPL,EPL,EPX,EPA,EPB,1,
```

```
     1KFIN)
      US(1)=USV
C     BEGIN ITERATIONS FOR FILTERING
      DO 4 II=KB,KFIN
      NV=II-1
      DT=TS(II)-TS(NV)
C     PROPAGATE THE STATE AND ERROR COVARIANCE MATRIX USING
C     THE NONLINEAR MODEL AND AN EULER INTEGRATION
C     METHOD FOR INCREMENT OF TIME DT
      CALL PROP(DT,US,XHT,EHT,A,Q,XBAR,EBAR,II)
C     COMPUTE THE KALMAN GAIN MATRIX
      CALL GAIN(EBAR,HZ,P,EGAIN)
C     UPDATE THE STATE AND ERROR COVARIANCE MATRIX BASED
C     UPON MOST RECENT MEASUREMENT
      CALL UPDAT(ZS,HZ,EBAR,XBAR,EGAIN,XHT,EHT,II)
C     STORE THESE VALUES OF XHAT AND EHAT FOR LATER USE
C     AND FOR PLOTTING
     0CALL STORE(XHT,EHT,TS,XPL,APL,BPL,EPL,EPX,EPA,EPB,II,
     1KFIN)
    4 CONTINUE
C     PLOT OUTPUT FUNCTIONS
      CALL OUTP(XPL,APL,BPL,EPL,EPX,EPA,EPB,KFIN)
    3 CONTINUE
      G=0.017
    6 CONTINUE
      D=0.7
    5 CONTINUE
      STOP
      END
```

```
      SUBROUTINE RKNL(H,TI,XI,N,AZ,BZ,C,ZS,US,TS,IXW,IXV)
      DIMENSION ZS(1),US(1),TS(1)
      COMMON /RKI/D
      X=XI
      T=TI
      A=AZ
      B=BZ
      UV=U(T,X)
      WV=W(IXW)
      DO 2 II=1,N
      T1=H*F(T,X,UV,A,B,WV)
      X=X+T1
      T=T+H
      TS(II)=T
      UV=U(T,X)
      US(II)=UV
      VV=V(IXV)
      Z=C*X+D*VV
      ZS(II)=Z
    2 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION F(T,X,U,A,B,W)
C     NONLINEAR ABSOLUTE SQUARE LAW VEHICLE
      COMMON /REF/G
      F=A*X*ABS(X)+B*U*ABS(U)+G*W
      RETURN
      END
```

```
      REAL FUNCTION U(T,X)
C     STAIRCASE WITH 2 STEPS
      COMMON /UFUN/H,TP,TPP,T0
      IF(T-TP)1,2,2
    1 U=H
      RETURN
    2 IF(T-TPP)3,4,4
    3 U=2.*H
      RETURN
    4 U=0.
      RETURN
      END
```

```
      SUBROUTINE PROP(H,US,XHT,EHT,A,Q,XBAR,EBAR,I)
      DIMENSION US(1),XHT(1),EHT(1),A(1),XBAR(1),EBAR(1)
      DIMENSION E1(9),E2(9),E3(9),E4(9),EJ(9)
      X=XHT(1)
      DO 3 J=1,9
      EJ(J)=EHT(J)
    3 CONTINUE
      UV=US(I)
      T1=H*FUN1(XHT,UV)
      CALL EFUN(A,EHT,Q,E1,XHT,UV)
      XBAR(1)=X+T1
      XBAR(2)=XHT(2)
      XBAR(2)=XHT(3)
      DO 5 J=1,9
      EBAR(J)=EJ(J)+E1(J)*H
    5 CONTINUE
      RETURN
      END
```

```
 SUBROUTINE EFUN(A,E,Q,EB,X,U)
 DIMENSION A(1),E(1),EB(1),X(1)
 A(1)=2.*X(2)*ABS(X(1))
 A(4)=X(1)*ABS(X(1))
 A(7)=U*ABS(U)
 EB(1)=2.*(A(1)*E(1)+A(4)*E(2)+A(7)*E(3))+Q
0EB(2)=E(2)*(A(5)+A(1))+A(2)*E(1)+A(8)*E(3)+
1A(4)*E(5)+A(7)*E(6)
0EB(3)=E(3)*(A(1)+A(9))+A(3)*E(1)+A(6)*E(2)+
1A(4)*E(6)+A(7)*E(9)
 EB(4)=EB(2)
 EB(5)=2.*(A(2)*E(2)+A(5)*E(5)+A(8)*E(6))
0EB(6)=E(6)*(A(5)+A(9))+A(3)*E(2)+A(6)*E(5)+
1A(2)*E(3)+A(8)*E(9)
 EB(7)=EB(3)
 EB(8)=EB(6)
 EB(9)=2.*(A(3)*E(3)+A(6)*E(6)+A(9)*E(9))
 RETURN
 END
```

```
REAL FUNCTION FUN1(X,U)
DIMENSION X(1)
FUN1=X(2)*X(1)*ABS(X(1))+X(3)*U*ABS(U)
RETURN
END
```

```
SUBROUTINE GAIN(E,H,P,EG)
DIMENSION E(1),H(1),EG(1)
C=H(1)*H(1)*E(1)+P
EG(1)=H(1)*E(1)/C
EG(2)=H(1)*E(2)/C
EG(3)=H(1)*E(3)/C
RETURN
END
```

```
      SUBROUTINE UPDAT(Z,H,EB,XB,EG,XH,EH,I)
      DIMENSION Z(1),H(1),EB(1),XB(1),EG(1),XH(1),EH(1)
      DO 1 J=1,3
      XH(J)=XB(J)+EG(J)*(Z(I)-H(1)*XB(1))
    1 CONTINUE
      EH(1)=EB(1)*(1.-EG(1)*H(1))
      EH(2)=EB(2)-EG(2)*H(1)*EB(1)
      EH(3)=EB(3)-EG(3)*H(1)*EB(1)
      EH(4)=EH(2)
      EH(5)=EB(5)-EG(2)*H(1)*EB(2)
      EH(6)=EB(6)-EG(3)*H(1)*EB(2)
      EH(7)=EH(3)
      EH(8)=EH(6)
      EH(9)=EB(9)-EG(3)*H(1)*EB(3)
      RETURN
      END
```

```
      SUBROUTINE  STORE(XH,EH,T,XP,AP,BP,EP,EX,EA,EB,I,K)
     0DIMENSION XH(1),EH(1),T(1),XP(1),AP(1),BP(1),EP(1),
     1EX(1),EA(1),EB(1)
      COMMON  /STOR /DX,DA,DB,DEX,DEA,DEB,C
      D=T(I)/C
      XP(I)=D
      AP(I)=D
      BP(I)=D
      EP(I)=D
      EX(I)=D
      EA(I)=D
      EB(I)=D
      N=I+K
      XP(N)=XH(1)/DX
      AP(N)=XH(2)/DA
      BP(N)=XH(3)/DB
      EX(N)=SQRT(ABS(EH(1)/DEX))
      EA(N)=SQRT(ABS(EH(5)/DEA))
      EB(N)=SQRT(ABS(EH(9)/DEB))
      RETURN
      END
```

```
      SUBROUTINE OUTP(XP,AP,BP,EP,EX,EA,EB,K)
      DIMENSION XP(1),AP(1),BP(1),EP(1),EX(1),EA(1),EB(1)
      KV=K
      K=47
      L=4
      DO 1 I=1,K
      IL=I*L
      XP(I)=XP(IL)
      AP(I)=AP(IL)
      BP(I)=BP(IL)
      EX(I)=EX(IL)
      EA(I)=EA(IL)
      EB(I)=EB(IL)
    1 CONTINUE
      DO 2 I=1,K
      IK=I+K
      KP=KV+I*L
      XP(IK)=XP(KP)
      AP(IK)=AP(KP)
      BP(IK)=BP(KP)
      EX(IK)=EX(KP)
      EA(IK)=EA(KP)
      EB(IK)=EB(KP)
    2 CONTINUE
      NS=0
      N=1
      M=2
      CALL PLOT(N,XP,K,M,NS)
      N=N+1
      CALL PLOT(N,AP,K,M,NS)
      N=N+1
      CALL PLOT(N,BP,K,M,NS)
      N=N+1
      CALL PLOT(N,EX,K,M,NS)
      N=N+1
      CALL PLOT(N,EA,K,M,NS)
      N=N+1
      CALL PLOT(N,EB,K,M,NS)
      K=KV
      KP=2*K
      WRITE(6,10) AP(KP),EA(KP),BP(KP),EB(KP)
   10 FORMAT(1H0,2X,'A=',2X,E14.7,' + OR - ',E14.7,
     15X,'B=',2X,E14.7,' + OR - ',E14.7)
      RETURN
      END
```

<u>Additional Subroutines:</u>

GAUSS, RANDU - Appendix A4

V, W - Appendix A12

PLOT - Appendix A5

PUTTING A "WHAMMY" ON NEGATIVE E IN EXTENDED KALMAN FILTERING

The E matrix in the propagation equation A15.1 for an extended Kalman filter should never become negative for a positive definite Q

$$\dot{E} = FE + EF^T + Q \tag{A15.1}$$

matrix. However, in many of the nonlinear ocean vehicle simulations the E matrix approaches zero very rapidly, and either computer noise or system noise may cause it to become slightly negative, especially for large integrator time steps. Equation A15.1 usually becomes completely unstable for negative E; and unless some provisions are made to correct the situation, the entire Kalman filter run will be program interrupted out of the computer due to exponential overflows.

The following Fortran statement A15.2 has been used by the author on several occasions and has saved quite a few Kalman filter runs which would have otherwise "blown up." The use of this simple, fast,

```
IF (E.LT. 0.) E = -E    ;  E ≐ element of E matrix A15.2
```

and definite correction technique is almost devoid of theoretical justification and may be likened to a man beating on the top of his TV set with his fist to get a better picture. Sometimes it works very well.

APPENDIX A16

SUBROUTINES PROP AND RKNL WITH RUNGE-KUTTA INTEGRATION

```
      SUBROUTINE PROP(H,US,XHT,EHT,A,Q,XBAR,EBAR,I)
      DIMENSION US(1),XHT(1),EHT(1),A(1),XBAR(1),EBAR(1)
      DIMENSION E1(9),E2(9),E3(9),E4(9),EJ(9)
      I=I-1
      X=XHT(1)
      DO 3 J=1,9
      EJ(J)=EHT(J)
    3 CONTINUE
      UV=US(I)
      T1=H*FUN1(XHT,UV)
      CALL EFUN(A,EHT,Q,E1,XHT,UV)
      XHT(1)=X+T1/2.
      UV=(US(I+1)+US(I))/2.
      T2=H*FUN1(XHT,UV)
      DO 1 J=1,9
      EHT(J)=EJ(J)+E1(J)/2.
    1 CONTINUE
      CALL EFUN(A,EHT,Q,E2,XHT,UV)
      XHT(1)=X+T2/2.
      T3=H*FUN1(XHT,UV)
      DO 2 J=1,9
      EHT(J)=EJ(J)+E2(J)/2.
    2 CONTINUE
      CALL EFUN(A,EHT,Q,E3,XHT,UV)
      XHT(1)=X+T3
      UV=US(I+1)
      T4=H*FUN1(XHT,UV)
      DO 4 J=1,9
      EHT(J)=EJ(J)+E3(J)
    4 CONTINUE
      CALL EFUN(A,EHT,Q,E4,XHT,UV)
      XBAR(1)=X+(T1+2.*T2+2.*T3+T4)/6.
      XBAR(2)=XHT(2)
      XBAR(3)=XHT(3)
      DO 5 J=1,9
      EBAR(J)=(E1(J)+2.*E2(J)+2.*E3(J)+E4(J))*(H/6.)+EJ(J)
    5 CONTINUE
      I=I+1
      RETURN
      END
```

```
      SUBROUTINE RKNL(H,TI,XI,N,AZ,BZ,C,ZS,US,TS,IXW,IXV)
      DIMENSION ZS(1),US(1),TS(1)
      COMMON /RKI/D
      H2=H/2.
      X=XI
      T=TI
      A=AZ
      B=BZ
      DO 2 II=1,N
      UV=U(T,X)
      WV=W(IXW)
      T1=H*F(T,X,UV,A,B,WV)
      TN=T+H2
      XN=X+T1/2.
      UV=U(TN,XN)
      WV=W(IXW)
      T2=H*F(TN,XN,UV,A,B,WV)
      XO=X+T2/2.
      UV=U(TN,XO)
      WV=W(IXW)
      T3=H*F(TN,XO,UV,A,B,WV)
      TO=T+H
      XP=X+T3
      UV=U(TO,XP)
      US(II)=UV
      WV=W(IXW)
      T4=H*F(TO,XP,UV,A,B,WV)
      X=X+(T1+2.*T2+2.*T3+T4)/6.
      T=T+H
      TS(II)=T
      VV=V(IXV)
      Z=C*X+D*VV
      ZS(II)=Z
    2 CONTINUE
      RETURN
      END
```

SUBROUTINES USED IN 6 * 6 MODEL REFERENCE CONTOURING

```
      SUBROUTINE SEATP(T,P,X,XN)
C ************************************************************************
C SUBROUTINE SEATP
C
C PURPOSE
C   TO COMPUTE SEA TRIAL TRAJECTORIES FOR A GENERAL OCEAN
C   VEHICLE IN ANY SELECTED COMBINATION OF 6 DEGREES OF
C   FREEDOM USING AN EULER INTEGRATOR
C
C SUBROUTINES REQUIRED
C   UINPT,OVMOD,XDIST
C
C DESCRIPTION
C   SECTION 6 (C),M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ************************************************************************
      DIMENSION T(47),U(846),Z(282),ZN(282)
      DIMENSION P(1),X(1),XN(1)
      DIMENSION Q(6),R(6),TST(180),UST(180),US(18)
      DIMENSION ME(20),AV(126),XF(126),IN(6),NE(27),L(6)
      DIMENSION XZ(6),XNZ(6),XD(6),XDN(6),INV(6)
      DIMENSION LF(126),LP(126),UI(18),RV(12)
      INTEGER AV,XF
      COMMON /OV/ME,AV,XF,IN,NE,KB,L,KF,LF,KP,LP
      COMMON /SNOIS/Q,R,INV,RV
      COMMON /STEPS/TST,UST,US
      COMMON /SINTG/TZ,DT,T
      COMMON /SUOUT/U
      COMMON /SZOUT/Z,ZN
      COMMON /TEST/EPS,EMAX
      COMMON /INTGR/NDT,KU,NST,KS.IONE,ITHRE,KUA
      CALL UINPT(TST,UST,US,DT,TZ,T,U)
      DO 3 KM=1,KB
      K=L(KM)
      XZ(K)=X(K)
      XNZ(K)=XN(K)
    3 CONTINUE
      DO 14 J=1,KS
      XD(J)=0.
      XDN(J)=0.
   14 CONTINUE
      DO 1 J=1,NDT
      JU=J
      DO 5 JA=1,KU
      UI(JA)=U(JU)
      JU=JU+NDT
    5 CONTINUE
      CALL OVMOD(ME,AV,XF,P,IN,UI,NE,X,XN,XD,XDN,KB,L)
      IF(ME(19).NE.IONE)GO TO 7
      DO 8 KM=1,KB
      K=L(KM)
      IF(K.LE.ITHRE)GO TO 8
      KUI=KUA+K-ITHRE-1
```

```
      UI(KUI)=UI(KUI)+X(K)*DT
    8 CONTINUE
      IF(J.EQ.NDT)GO TO 7
      JU=J+1+(KUA-1)*NDT
      DO 9 JA=KUA,KU
      U(JU)=UI(JA)
      JU=JU+NDT
    9 CONTINUE
    7 DO 2 KM=1,KB
      KZ=J+NDT*(KM-1)
      K=L(KM)
      IF(ME(6).EQ.1)GO TO 12
   13 X(K)=X(K)+XD(KM)*DT
      Z(KZ)=X(K)
      IF(ABS(Z(KZ)).GT.EMAX)GO TO 6
      GO TO 2
   12 XN(K)=XN(K)+XDN(KM)*DT
      CALL XDIST(INV,RV,1,VN,K,KS)
      ZN(KZ)=XN(K)+VN
      IF(ABS(ZN(KZ)).GT.EMAX)GO TO 6
      IF(ME(9).EQ.1)GO TO 13
    2 CONTINUE
    1 CONTINUE
      I=1
      GO TO 10
    6 I=0
   10 DO 11 KM=1,KB
      K=L(KM)
      X(K)=XZ(K)
      XN(K)=XNZ(K)
   11 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE UINPT(TS,US,UZ,DT,TZ,T,U)
C ***********************************************************************
C SUBROUTINE UINPT
C
C PURPOSE
C   TO GENERATE A TIME VECTOR AND A SET OF COMBINED-STEP-
C   FUNCTION INPUTS FOR A GENERAL OCEAN VEHICLE
C
C SUBROUTINES REQUIRED
C   NONE
C DESCRIPTION
C   SECTION 6 (C),M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***********************************************************************
      DIMENSION TS(1),US(1),T(1),U(1),UZ(1)
      COMMON /INTGR/NDT,KU,NST,KS,IONE,ITHRE,KUA
      T(1)=TZ+DT
      DO 5 IT=2,NDT
      T(IT)=T(IT-1)+DT
    5 CONTINUE
      NS=NST/KU
      JU=0
      DO 1 IU=1,KU
      IST=NS*(IU-1)
      DO 2 ID=1,NDT
      JU=JU+1
      U(JU)=UZ(IU)
      JS=IST
      DO 3 IS=1,NS
      JS=JS+1
      IF(TS(JS))3,6,6
    6 IF(TS(JS)-T(ID))4,4,3
    4 U(JU)=U(JU)+US(JS)
    3 CONTINUE
    2 CONTINUE
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MOREF(I,P,X)
C ***********************************************************************
C SUBROUTINE MOREF
C
C PURPOSE
C   TO COMPUTE TRAJECTORIES FOR A GENERAL OCEAN VEHICLE IN
C   ANY COMBINATION OF 6 DEGREES OF FREEDOM WITH VARIABLE
C   COEFFICIENTS AND PARAMETERS FOR USE IN MODEL REFERENCE
C   IDENTIFICATION , USING AN EULER INTEGRATOR
C
C SUBROUTINES REQUIRED
C   OVMOD
C
C DESCRIPTION
C   SECTION 6 (C),M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***********************************************************************
      DIMENSION P(1),X(1)
      DIMENSION T(47),U(846),Z(282),ZN(282)
      DIMENSION ME(20),AV(126),XF(126),IN(6),NE(27),L(6)
      DIMENSION LF(126),LP(126)
      DIMENSION XZ(6),XNZ(6),XD(6),XDN(6)
      DIMENSION UI(18),TST(180),UST(180),US(18)
      INTEGER AV,XF
      COMMON /OV/ME,AV,XF,IN,NE,KB,L,KF,LF,KP,LP
      COMMON /STEPS/TST,UST,US
      COMMON /SINTG/TZ,DT,T
      COMMON /SUOUT/U
      COMMON /SZOUT/Z,ZN
      COMMON /TEST/EPS,EMAX
      COMMON /INTGR/NDT,KU,NST,KS,IONE,ITHRE,KUA
      DO 3 KM=1,KB
      K=L(KM)
      XZ(K)=X(K)
      XNZ(K)=0.
    3 CONTINUE
      DO 1 J=1,NDT
      JU=J
      DO 5 JA=1,KU
      UI(JA)=U(JU)
      JU=JU+NDT
    5 CONTINUE
      CALL OVMOD(ME,AV,XF,P,IN,UI,NE,X,XNZ,XD,XDN,KB,L)
      IF(ME(19).NE.IONE)GO TO 7
      DO 8 KM=1,KB
      K=L(KM)
      IF(K.LE.ITHRE)GO TO 8
      KUI=KUA+K-ITHRE-1
      UI(KUI)=UI(KUI)+X(K)*DT
    8 CONTINUE
      IF(J.EQ.NDT)GO TO 7
      JU=J+1+(KUA-1)*NDT
      DO 9 JA=KUA,KU
```

```
      U(JU)=UI(JA)
      JU=JU+NDT
    9 CONTINUE
    7 DO 2 KM=1,KB
      KZ=J+NDT*(KM-1)
      K=L(KM)
      X(K)=X(K)+XD(KM)*DT
      Z(KZ)=Y(K)
      IF(ABS(Z(KZ)).GT.EMAX)GO TO 6
    2 CONTINUE
    1 CONTINUE
      I=1
      GO TO 10
    6 I=0
   10 DO 11 KM=1,KB
      K=L(KM)
      X(K)=XZ(K)
   11 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE  STOUT(T,U,Z,ZN,IU,N,NP)
C  ****************************************************************
C  SUBROUTINE  STOUT
C
C  PURPOSE
C    TO GENERATE AND OUTPUT PLOTS OF SEA TRIAL NOISY AND
C    NOISELESS TRAJECTORIES AND PLOTS OF VEHICLE INPUTS
C
C  SUBROUTINES REQUIRED  .
C    PLOT
C
C  DESCRIPTION
C    SECTION 6 (C),M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C  ****************************************************************
      INTEGER AV,XF
      DIMENSION T(1),U(1),Z(1),ZN(1),ZPL(376)
      DIMENSION ME(20),AV(126),XF(126),IN(6),NE(27),L(6)
      DIMENSION LF(126),LP(126)
      COMMON /OV/ME,AV,XF,IN,NE,KB,L,KF,LF,KP,LP
   30 FORMAT(1H ,10X,'NOISELESS U,V,OR W',5X,'L(KM)=',I3)
   31 FORMAT(1H ,10X,'NOISELESS P,Q,OR R',5X,'L(KM)=',I3)
   32 FORMAT(1H ,10X,'NOISY     U,V,OR W',5X,'L(KM)=',I3)
   33 FORMAT(1H ,10X,'NOISY     P,Q,OR R',5X,'L(KM)=',I3)
   340FORMAT(1H ,10X,'TANKS 1-RES,2-STBD,3-PORT,4-FWDM,',
     1'5-AFTM,6-FWDW,7-AFTW')
   350FORMAT(1H ,10X,'SHROUD 1-DELTA P,2-DELTA Y ** +DP=',
     1'-Z,-M * +DY=+Y,-N  **')
   36 FORMAT(1H ,10X,'PROPELLOR 1-RPS,2-RPS/SEC ;  +RPS=+X')
   370FORMAT(1H ,10X,'THRUSTERS 1-YFWD,2-YAFT,3-ZFWD,4-ZAF',
     1'T   +U=+Y,+M,-Z,-N')
   38 FORMAT(1H ,10X,'ANGLES    1-PHI,2-THETA,3-PSI')
      KW=6
C     PLACE T(N) INTO ZPL(NMAX*N)
      NPSA=NP
      DO 1 K=1,N
      ZPL(K)=T(K)
    1 CONTINUE
      IF(ME(9).EQ.0)GO TO 2
      NC=2
      ND=2
      DO 3 KM=1,KB
      K=L(KM)
      JZ=N*(KM-1)
      IF(K.GT.3)GO TO 4
      J=N
      DO 5 KN=1,N
      JZ=JZ+1
      J=J+1
      ZPL(J)=Z(JZ)
    5 CONTINUE
      CALL PLOT(NP,ZPL,N,NC,0)
      WRITE(KW,30)K
```

```
      NP=NP+1
      GO TO 3
    4 J=N
      DO 6 KN=1,N
      JZ=JZ+1
      J=J+1
      ZPL(J)=Z(JZ)
    6 CONTINUE
      NP=NP+1
      CALL PLOT(NP,ZPL,N,ND,0)
      WRITE(KW,31)K
    3 CONTINUE
    2 IF(ME(6).EQ.0)GO TO 7
      NC=2
      ND=2
      DO 8 KM=1,KB
      K=L(KM)
      JZ=N*(KM-1)
      IF(K.GT.3)GO TO 9
      J=N
      DO 10 KN=1,N
      JZ=JZ+1
      J=J+1
      ZPL(J)=ZN(JZ)
   10 CONTINUE
      NP=NP+1
      CALL PLOT(NP,ZPL,N,NC,0)
      WRITE(KW,32)K
      GO TO 8
    9 J=N
      DO 11 KN=1,N
      JZ=JZ+1
      J=J+1
      ZPL(J)=ZN(JZ)
   11 CONTINUE
      NP=NP+1
      CALL PLOT(NP,ZPL,N,ND,0)
      WRITE(KW,32)K
    8 CONTINUE
    7 IF(IU.EQ.0)GO TO 12
      IF(ME(1).EQ.0)GO TO 13
      J=N
      NU=N*(NE(6)-1)
      NO=NE(1)
      DO 14 JT=1,NO
      DO 15 KN=1,N
      J=J+1
      NU=NU+1
      ZPL(J)=U(NU)
   15 CONTINUE
   14 CONTINUE
      NO=NO+1
      NP=NP+1
```

```
      CALL PLOT(NP,ZPL,N,NO,0)
      WRITE(KW,34)
   13 IF(ME(3).EQ.0)GO TO 16
      J=N
      NU=N*(NE(7)-1)
      NO=NE(2)
      DO 17 JT=1,NO
      DO 18 KN=1,N
      J=J+1
      NU=NU+1
      ZPL(J)=U(NU)
   18 CONTINUE
   17 CONTINUE
      NO=NO+1
      NP=NP+1
      CALL PLOT(NP,ZPL,N,NO,0)
      WRITE(KW,35)
   16 IF(ME(4).EQ.0)GO TO 19
      J=N
      NU=N*(NE(8)-1)
      NO=NE(3)
      DO 20 JT=1,NO
      DO 21 KN=1,N
      J=J+1
      NU=NU+1
      ZPL(J)=U(NU)
   21 CONTINUE
   20 CONTINUE
      NO=NO+1
      NP=NP+1
      CALL PLOT(NP,ZPL,N,NO,0)
      WRITE(KW,36)
   19 IF(ME(5).EQ.0)GO TO 22
      J=N
      NU=N*(NE(9)-1)
      NO=NE(4)
      DO 23 JT=1,NO
      DO 24 KN=1,N
      J=J+1
      NU=NU+1
      ZPL(J)=U(NU)
   24 CONTINUE
   23 CONTINUE
      NO=NO+1
      NP=NP+1
      CALL PLOT(NP,ZPL,N,NO,0)
      WRITE(KW,37)
   22 IF(ME(19).EQ.0)GO TO 25
      WRITE(6,26)
   26 FORMAT(5X,'ANGLES ARE INTEGRATED ANGULAR VELOCITIES')
   25 IF(ME(20).EQ.0)GO TO 12
      J=N
      NU=N*(NE(10)-1)
```

```
      NO=NE(5)
      DO 28 JT=1,NO
      DO 29 KN=1,N
      J=J+1
      NU=NU+1
      ZPL(J)=U(NU)
   29 CONTINUE
   28 CONTINUE
      NO=NO+1
      NP=NP+1
      CALL PLOT(NP,ZPL,N,NO,0)
      WRITE(KW,38)
   12 NP=NPSA
      RETURN
      END
```

Additional Subroutines

OVMOD - Appendix A6

PLOT - Appendix A5

XDIST - Appendix A7

SUBROUTINES USED IN 6*6 EXTENDED KALMAN FILTERING

```
      SUBROUTINE EHDOT(X,U,P,E,XD,ED)
C *******************************************************************
C SUBROUTINE EHDOT
C
C PURPOSE
C   TO COMPUTE THE DERIVATIVE OF THE ERROR COVARIANCE MATRIX
C   FOR A GENERAL OCEAN VEHICLE KALMAN FILTER
C     SOLVES THE EQUATIONS
C       EHDOT - ED=F*E+E*FTRANS+Q
C       F     - OVDER
C       XD    - OVMOD
C
C SUBROUTINES REQUIRED
C   OVMOD,CVDER
C
C DESCRIPTION
C   SECTION 6 (C),M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C *******************************************************************
      DIMENSION X(1),U(1),P(1),E(1),XD(1),ED(1)
      DIMENSION ME(20),AV(126),XF(126),IN(6),NE(27),L(6)
      DIMENSION LF(126),LP(126)
      DIMENSION Q(6),XN(6),XND(6)
      DIMENSION F(1200),R(6),INV(6),RV(12)
      INTEGER AV,XF
      COMMON /CV/ME,AV,XF,IN,NE,KB,L,KF,LF,KP,LP
      COMMON /SNOIS/Q,R,INV,RV
C     CALCULATE XD AND F
      CALL OVMOD(ME,AV,XF,P,IN,U,NE,X,XN,XD,XND,KB,L)
      CALL CVDER(ME,U,AV,XF,KF,LF,KB,L,X,KP,LP,P,F,NF,NE)
C     CALCULATE THE 1-ST KB*KB TERMS OF ED
      NV=1
      N=1
      DO 1 J=1,KB
      DO 2 I=1,KB
      ED(NV)=0.
      DO 3 K=1,NF
      MF=I+KB*(K-1)
      MFT=K+NF*(J-1)
      ED(NV)=ED(NV)+F(MF)*E(MFT)+E(MF)*F(MFT)
    3 CONTINUE
      IF(I.NE.J)GO TO 4
      ED(NV)=ED(NV)+Q(N)
      N=N+1
    4 NV=NV+1
    2 CONTINUE
    1 CONTINUE
C     CALCULATE THE REMAINING KB*(NF-KB) TERMS OF ED
      NR=NF-KB
      DO 5 J=1,NR
      DO 6 I=1,KB
      ED(NV)=0.
      DO 7 K=1,NF
```

```
      MF=I+KB*(K-1)
      MFT=K+NF*(J-1)
      ED(NV)=ED(NV)+F(MF)*E(MFT)+E(MF)*F(MFT)
    7 CCNTINUE
      NV=NV+1
    6 CONTINUE
    5 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE KGAIN(E,KB,NF,EG)
C ***********************************************************************
C SUBROUTINE KGAIN
C
C PURPOSE
C   TO COMPUTE THE GAIN MATRIX FOR A KALMAN FILTER
C     SOLVES THE EQUATION
C       GAIN  - EG=E*(E+R)INV
C
C SUBROUTINES REQUIRED
C   MINV (IBM/SSP)
C
C DESCRIPTION
C   SECTION 6 (C),M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***********************************************************************
      DIMENSION E(1),EG(1)
      DIMENSION R(6),EI(36),LV(6),MV(6)
      DIMENSION Q(6),INV(6),RV(12)
      COMMON /SNOIS/Q,R,INV,RV
C     FORM MATRIX INVERSE OF (HEHT+R)
      NV=1
      N=1
      DO 1 J=1,KB
      DO 4 I=1,KB
      EI(NV)=E(NV)
      IF(I.EQ.J)GO TO 5
      GO TO 6
    5 EI(NV)=EI(NV)+R(N)
      N=N+1
    6 NV=NV+1
    4 CONTINUE
    1 CONTINUE
      CALL MINV(EI,KB,D,LV,MV)
      IF(D.EQ.0.)GO TO 8
C     FORM GAIN MATRIX EHT(HEHT+R)INV
      NV=1
      DO 2 J=1,KB
      DO 3 I=1,NF
      EG(NV)=0.
      DO 7 K=1,KB
      IK=I+NF*(K-1)
      IE=K+KB*(J-1)
      EG(NV)=EG(NV)+E(IK)*EI(IE)
    7 CONTINUE
      NV=NV+1
    3 CONTINUE
    2 CONTINUE
      RETURN
    8 WRITE(6,9)
    9 FORMAT(1X,'SINGULAR MATRIX IN KGAIN, ',
     1'SETTING EI=1./DIAG(EI) ')
      EPS=1.0E-40
```

```
      IKB=KB*KB
      DO 10 K=1,IKB
      EI(K)=0.
   10 CONTINUE
      DO 11 K=1,KB
      IK=K+KB*(K-1)
      EIV=E(IK)+R(K)
      IF(EIV.EQ.0.)EIV=EPS
      EI(K)=1./EIV
   11 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE UPDAT(Z,X,E,EG,XH,EH,KB,NF)
C ***********************************************************************
C SUBROUTINE UPDAT
C
C PURPOSE
C   TO COMPUTE THE UPDATED STATE VECTOR AND ERROR
C   COVARIANCE MATRIX FOR A KALMAN FILTER
C     SOLVES THE EQUATIONS
C       STATE - XH(UPDATED)=XH+EG*(Z-XH)
C       COV.  - EH(UPDATED)=EH-EG*EH
C
C SUBROUTINES REQUIRED
C   NONE
C
C DESCRIPTION
C   SECTION 6 (C),M.N.HAYES THESIS,MIT,1971,NAME DEPARTMENT
C
C ***********************************************************************
      DIMENSION Z(1),X(1),E(1),EG(1),XH(1),EH(1)
      DIMENSION XZ(6)
C     FORM Z-H*X
      DO 1 K=1,KB
      XZ(K)=Z(K)-X(K)
    1 CONTINUE
C     UPDATE THE NF STATES, XH
      DO 3 I=1,NF
      XH(I)=0.
      DO 2 J=1,KB
      KF=I+NF*(J-1)
      XH(I)=XH(I)+EG(KF)*XZ(J)
    2 CONTINUE
    3 CONTINUE
C     UPDATE THE (NF*(NF+1))/2 ERROR COVARIANCES, EH
      NV=1
      DO 4 J=1,NF
      DO 5 I=1,J
      IF(I.GT.KB)GO TO 7
      KI=I+KB*(J-1)
      EH(NV)=E(KI)
      GO TO 8
    7 EH(NV)=0.
    8 DO 6 K=1,KB
      KE=I+NF*(K-1)
      KI=K+KB*(J-1)
      EH(NV)=EH(NV)-EG(KE)*E(KI)
    6 CONTINUE
      NV=NV+1
    5 CONTINUE
    4 CONTINUE
      RETURN
      END
```

APPENDIX A19

MAIN 6 * 6 IDENTIFICATION PROGRAM STATEMENTS IN FORTRAN IV (MAIN)

```
C ****************************************************************
C MAIN
C
C MODEL REFERENCE AND KALMAN FILTER PROGRAM FOR THE DSRV
C
C PURPOSE
C   TO STUDY IDENTIFIABILITY OF THE SECOND DEGREE AND EFFECTOR
C   COEFFICIENTS OF THE DSRV FROM INTERNALLY GENERATED FULL
C   SCALE SEA TRIAL DATA
C
C LANGUAGE : FORTRAN IV
C
C SUBROUTINES REQUIRED
C   SEATR,STOUT,MOREF,CONTUR,PLOT
C
C COMPILED CORE REQUIREMENTS
C   IBM 360/65 250K BYTES
C
C AUTHOR
C   MICHAEL N. HAYES ,RM.5-333,MIT,EXT. 6807
C
C ****************************************************************
C
C DECLARATION STATEMENTS
      DIMENSION ME(20),AV(126),XF(126),IN(6),NE(27),L(6),M(6)
      DIMENSION LF(126),LP(126),Q(6),PG(8),U(18),R(6),P(587)
      DIMENSION PS(587),X(258),XS(258),XN(258),XH(258),PH(587)
      DIMENSION EH(20100),E(1200),F(1200),IOUT(10)
      DIMENSION IBL(5),IBLT(10),JBLT(10),ABLT(10)
      DIMENSION TST(180),UST(180),ON(6),RN(6),US(18)
      DIMENSION IXWR(6),JXWR(6),IXW(8),JXW(8),KXW(8)
      DIMENSION LXW(8),LFW(126)
      DIMENSION MGP(14),NGP(14),LFP(587)
      DIMENSION CD(47),CN(51),COST(2397)
      DIMENSION IMRF(4),INV(6),RV(12),IMRT(4)
      DIMENSION XINC(258),PINC(587)
      DIMENSION T(47),JD(846),Z(282),ZN(282)
      DIMENSION XSTR(258),PSTR(587)
      INTEGER AV,XF
      INTEGER IBL/'B','L','O','C','K'/
      INTEGER IXWR/'X','Y','Z','K','M','N'/
      INTEGER JXWR/'U','V','W','P','Q','R'/
      INTEGER IMRF/'P','A','S','S'/
      COMMON /OV/ME,AV,XF,IN,NE,KB,L,KF,LF,KP,LP
      COMMON /SNOIS/Q,R,INV,RV
      COMMON /STEPS/TST,UST,U
      COMMON /SINTG/TZ,DT,T
      COMMON /SUOUT/UD
      COMMON /SZOUT/Z,ZN
      COMMON /TEST/EPS,EMAX
      COMMON /INTGR/NDT,KU,NST,KS,IONE,ITHRE,KUA
      KR=5
      KW=6
```

```
C
C ****************************************************************
C INITIAL PROGRAM DATA INPUT
     0READ(KR,1)KS,KME,KAV,KNE,MP,KMV,KMP,KPF,KX,KPG,KU,
     1NHMAX,NIND,NIN,NST,NOUT,NBLT,KNP,NMRT
      READ(KR,1)MREF,KFIL,IONE,NCN,NCD,NMR,IABO,ITHRE,KUA
      READ(KR,204)ISIXT,ISEVT,KEH,KEF,EPS,EMAX
  204 FORMAT(4I5,2E10.2)
      READ(KR,121)NGPI,(MGP(J),J=1,NGPI)
      READ(KR,122)(NGP(J),J=1,NGPI)
  121 FORMAT(I4,14I4)
  122 FORMAT(14I4)
      READ(KR,2)(M(J),J=1,KS)
      READ(KR,1)(ME(J),J=1,KME)
      DO 6 J=1,KS
      IN(J)=J*1111
      INV(J)=J*1001
    6 CONTINUE
    1 FORMAT(20I4)
    2 FORMAT(40I2)
    3 FORMAT(10F8.2)
      READ(KR,2)(AV(J),J=1,KAV)
      READ(KR,2)(XF(J),J=1,KAV)
      READ(KR,2)(NE(J),J=1,KNE)
      DO 4 I=1,KMP
      MV=ME(I+KMV)-1
      READ(KR,3)(PS(J),J=MP,MV)
      MP=ME(I+KMV)
    4 CONTINUE
      READ(KR,3)(PS(J),J=MP,KMP)
      READ(KR,3)(XS(J),J=1,KX)
      READ(KR,3)(PG(J),J=1,KPG)
      READ(KR,3)(JS(J),J=1,KU)
      READ(KR,3)(XINC(J),J=1,KX)
      READ(KR,3)(PINC(J),J=1,KNP)
      READ(KR,39)(J,XH(J),JJ=1,KX)
      READ(KR,39)(J,PH(J),JJ=1,KNP)
C
C ****************************************************************
C INITIAL STRUCTURE SELECTOR SETUP
      DO 200 J=1,KS
      L(J)=0
  200 CONTINUE
      KB=0
      DO 5 I=1,KS
      KB=KB+M(I)
      IF(M(I).EQ.1)L(KB)=I
    5 CONTINUE
C
C ****************************************************************
C SET ALL QUANTITIES TO INITIAL VALUES OR TO ZERO
      DO 7 J=1,KX
      X(J)=XS(J)
```

```
      XN(J)=XS(J)
      XSTR(J)=XH(J)
    7 CONTINUE
      DO 181 J=1,282
      Z(J)=0.
      ZN(J)=0.
  181 CONTINUE
      DO 182 J=1,2397
      COST(J)=0.
  182 CONTINUE
      DO 183 J=1,846
      UD(J)=0.
  183 CONTINUE
      DO 184 J=1,NOUT
      IOUT(J)=0
  184 CONTINUE
      DO 201 J=1,NIND
      IBLT(J)=0
      JBLT(J)=0
      ABLT(J)=0.
  201 CONTINUE
      DO 202 J=1,51
      CN(J)=0.
  202 CONTINUE
      DO 203 J=1,47
      CD(J)=0.
      T(J)=0.
  203 CONTINUE
      DO 8 J=1,KNP
      P(J)=PS(J)
      PSTR(J)=PH(J)
    8 CONTINUE
      KF=0
      KP=0
      DO 9 J=1,KAV
      LF(J)=0
      LP(J)=0
    9 CONTINUE
      DO 13 J=1,KEH
      FH(J)=0.
   13 CONTINUE
      DO 14 J=1,KEF
      E(J)=0.
      F(J)=0.
   14 CONTINUE
      DO 60 J=1,KS
      Q(J)=0.
      R(J)=0.
      QN(J)=0.
      RN(J)=0.
   60 CONTINUE
      DO 61 J=1,NST
      TST(J)=-1.0
```

```
      UST(J)=0.
   61 CONTINUE
      DO 80 J=1,KU
      U(J)=US(J)
   80 CONTINUE
C
C ****************************************************************************
C WRITE INITIAL DATA FOR REFERENCE
      WRITE(KW,91)
      WRITE(KW,253)
  253 FORMAT(1H ,1X,119('*'))
      WRITE(KW,254)
  254 FORMAT(1H ,54X,'INITIAL DATA',54X)
      WRITE(KW,92)
      WRITE(KW,255)KS,KME,KAV,KNE,MP,KMV,KMP,KPF,KX,KPG,KU
  2550FORMAT(1H ,1X,'  KS =',I4,'  KME=',I4,'  KAV=',I4,
     1'  KNE=',I4,'  MP =',I4,'  KMV=',I4,'  KMP=',I4,
     2'  KPF=',I4,'  KX =',I4,'  KPG=',I4,'  KU =',I4)
     0WRITE(KW,269)NIN,NST,KNP,NIND,NOUT,NBLT,NMRT,IONE,
     1IABO,KEF
  2690FORMAT(1H ,1X,' NIN =',I4,' NST =',I4,' KNP =',I4,
     1' NIND=',I4,' NOUT=',I4,' NBLT=',I4,' NMRT=',I4,
     2' IONE=',I4,' IABO=',I4,' KEF =',I4)
      WRITE(KW,256)NHMAX,ISIXT,ISEVT,ITHRE
  2560FORMAT(1H ,1X,' NHMAX =',I4,' ISIXT =',I4,' ISEVT =',
     1I4,' ITHRE =',I4)
      WRITE(KW,257)MREF,KFIL,NCD,NCN,NMR,KEH,NGPI
  2570FORMAT(1H ,1X,'  MREF=',I5,'  KFIL=',I5,'  NCD =',I5,
     1'  NCN =',I5,'  NMR =',I5,'  KEH =',I5,'  NGPI=',I5)
      WRITE(KW,258)(J,MGP(J),J=1,NGPI)
  258 FORMAT(1H ,1X,'MGP(J)=',4X,14(1X,I2,'=',I3))
      WRITE(KW,259)(J,NGP(J),J=1,NGPI)
  259 FORMAT(1H ,1X,'NGP(J)=',4X,14(1X,I2,'=',I3))
      WRITE(KW,260)(J,M(J),J=1,KS)
  260 FORMAT(1H ,1X,'M(J) -',6(2X,I2,'=',I2))
      WRITE(KW,261)(J,ME(J),J=1,10)
      WRITE(KW,261)(J,ME(J),J=11,KME)
  261 FORMAT(1H ,1X,'ME(J)-',10(2X,I2,'=',I4))
      WRITE(KW,262)(J,IN(J),J=1,KS)
  262 FORMAT(1H ,1X,'IN(J)-',6(2X,I2,'=',I7))
      WRITE(KW,263)(J,INV(J),J=1,KS)
  263 FORMAT(1H ,1X,'INV(J)',6(2X,I2,'=',I7))
      WRITE(KW,264)(J,NE(J),J=1,14)
  264 FORMAT(1H ,1X,'NE(J)-',14(2X,I2,'=',I2))
      WRITE(KW,264)(J,NE(J),J=15,KNE)
      JO=1
      JT=14
      DO 265 I=1,8
      WRITE(KW,266)(J,AV(J),J=JO,JT)
      JO=JT+1
      JT=JT+14
  265 CONTINUE
      WRITE(KW,266)(J,AV(J),J=JO,KAV)
```

```
266 FORMAT(1H ,1X,'AV(J)-',14(2X,I3,'=',I2))
    J0=1
    JT=14
    DO 267 I=1,8
    WRITE(KW,268)(J,XF(J),J=J0,JT)
    J0=JT+1
    JT=JT+14
267 CONTINUE
    WRITE(KW,268)(J,XF(J),J=J0,KAV)
268 FORMAT(1H ,1X,'XF(J)-',14(2X,I3,'=',I2))
    WRITE(KW,91)
    WRITE(KW,253)
    WRITE(KW,254)
    WRITE(KW,92)
    J0=1
    JT=5
    DO 270 I=1,51
    WRITE(KW,271)(J,XS(J),J=J0,JT)
    J0=JT+1
    JT=JT+5
270 CONTINUE
    WRITE(KW,271)(J,XS(J),J=J0,KX)
271 FORMAT(1H ,1X,'XS(J) -',5(3X,I3,'=',F13.4))
    WRITE(KW,91)
    WRITE(KW,253)
    WRITE(KW,254)
    WRITE(KW,92)
    J0=1
    JT=5
    DO 272 I=1,51
    WRITE(KW,273)(J,XINC(J),J=J0,JT)
    J0=JT+1
    JT=JT+5
272 CONTINUE
    WRITE(KW,273)(J,XINC(J),J=J0,KX)
273 FORMAT(1H ,1X,'XINC(J)',5(3X,I3,'=',F13.4))
    ISW=1
    J0=1
    JT=5
274 WRITE(KW,91)
    WRITE(KW,253)
    WRITE(KW,254)
    WRITE(KW,92)
    DO 275 I=1,51
    WRITE(KW,276)(J,PS(J),J=J0,JT)
    J0=JT+1
    JT=JT+5
275 CONTINUE
    GO TO (277,278),ISW
277 ISW=2
    GO TO 274
278 WRITE(KW,91)
    WRITE(KW,253)
```

```
      WRITE(KW,254)
      WRITE(KW,92)
      DO 279 I=1,15
      WRITE(KW,276)(J,PS(J),J=JO,JT)
      JO=JT+1
      JT=JT+5
  279 CONTINUE
      WRITE(KW,276)(J,PS(J),J=JO,KNP)
  276 FORMAT(1H ,1X,'PS(J) -',5(3X,I3,'=',F13.4))
      ISW=1
      JO=1
      JT=5
  280 WRITE(KW,91)
      WRITE(KW,253)
      WRITE(KW,254)
      WRITE(KW,92)
      DO 281 I=1,51
      WRITE(KW,282)(J,PINC(J),J=JO,JT)
      JO=JT+1
      JT=JT+5
  281 CONTINUE
      GO TO (283,284),ISW
  283 ISW=2
      GO TO 280
  284 WRITE(KW,91)
      WRITE(KW,253)
      WRITE(KW,254)
      WRITE(KW,92)
      DO 285 I=1,15
      WRITE(KW,282)(J,PINC(J),J=JO,JT)
      JO=JT+1
      JT=JT+5
  285 CONTINUE
      WRITE(KW,282)(J,PINC(J),J=JO,KNP)
  282 FORMAT(1H ,1X,'PINC(J)',5(3X,I3,'=',F13.4))
      WRITE(KW,92)
      WRITE(KW,286)(J,PG(J),J=1,5)
      WRITE(KW,286)(J,PG(J),J=6,KPG)
  286 FORMAT(1H ,1X,'PG(J) -',5(3X,I3,'=',F13.4))
      WRITE(KW,92)
      WRITE(KW,287)(J,US(J),J=1,6)
      WRITE(KW,287)(J,US(J),J=7,12)
      WRITE(KW,287)(J,US(J),J=13,KU)
  287 FORMAT(1H ,1X,'US(J)',6(2X,I3,'=',F13.4))
      WRITE(KW,92)
      WRITE(KW,288)
  288 FORMAT(1H ,1X,53X,'END OF INITIAL DATA')
      WRITE(KW,253)
      WRITE(KW,253)
      WRITE(KW,92)
C
C ***********************************************************************
C READ THE NUMBER OF BLOCKS AND TEST FOR A BLOCK CARD
```

```
      READ(KR,1)NB
      READ(KR,15)(IBLT(J),J=1,NBLT)
   15 FORMAT(5A1)
      DO 16 J=1,NBLT
      IF(IBL(J).NE.IBLT(J))GO TO 17
   16 CONTINUE
      WRITE(KW,289)NB
  289 FORMAT(1H ,1X,'THERE ARE ',I3,'  BLOCKS TO BE PROCES',
     1'SED IN THIS RUN.')
      WRITE(KW,92)
      WRITE(KW,290)
  290 FORMAT(1H ,1X,'BEGIN FIRST BLOCK')
      WRITE(KW,92)
      WRITE(KW,253)
      ICD=NCD-1
      ICN=NCN-1
      GO TO 18
   17 WRITE(KW,19)(IBLT(J),J=1,NBLT)
   19 FORMAT(1X,5A1,5X,'INITIAL PROGRAM DATA ERROR ')
      STOP
C
C ********************************************************************
C BEGIN BLOCK CALCULATIONS
   18 DO 10 IB=1,NB
C
C    READ BLOCK DATA AND INITIALIZE SELECTED VARIABLES
     0READ(KR,1)NBLCK,NPB,IM,IME,ILF,ILP,IX,IP,IQ,IK,IU,IST,
     1IXH,IPH,IE,INO,INR,IO
      IF(IM.NE.0)GO TO 21
      DO 22 J=1,KS
      M(J)=0
   22 CONTINUE
   21 IF(IME.NE.0)GO TO 23
      DO 24 J=1,KMV
      ME(J)=0
   24 CONTINUE
   23 IF(ILF.NE.0)GO TO 25
      DO 26 J=1,KAV
      LF(J)=0
   26 CONTINUE
   25 IF(ILP.NE.0)GO TO 27
      DO 28 J=1,KAV
      LP(J)=0
   28 CONTINUE
   27 IF(IX.NE.0)GO TO 29
      DO 30 J=1,KX
      X(J)=XS(J)
      XN(J)=XS(J)
   30 CONTINUE
   29 IF(IP.NE.0)GO TO 31
      DO 32 J=1,KNP
      P(J)=PS(J)
   32 CONTINUE
```

```
   31 IF(IE.NE.0)GO TO 33
      DO 34 J=1,KEH
      EH(J)=0.
   34 CONTINUE
   33 IF(IQ.NE.0)GO TO 35
      DO 36 J=1,KS
      Q(J)=0.
   36 CONTINUE
   35 IF(IR.NE.0)GO TO 37
      DO 38 J=1,KS
      R(J)=0.
   38 CONTINUE
   37 IF(IU.NE.0)GO TO 81
      DO 82 J=1,KU
      U(J)=US(J)
   82 CONTINUE
   81 IF(IST.NE.0)GO TO 62
      DO 63 J=1,NST
      TST(J)=-1.0
      UST(J)=0.
   63 CONTINUE
   62 IF(INQ.NE.0)GO TO 64
      DO 65 J=1,KS
      QN(J)=0.
   65 CONTINUE
   64 IF(INR.NE.0)GO TO 66
      DO 67 J=1,KS
      RN(J)=0.
   67 CONTINUE
   66 IF(IO .NE.0)GO TO 113
      DO 77 J=1,NOUT
      IOUT(J)=0
   77 CONTINUE
  113 IF(IXH.NE.0)GO TO 114
      DO 115 J=1,KX
      XH(J)=XSTR(J)
  115 CONTINUE
  114 IF(IPH.NE.0)GO TO 76
      DO 116 J=1,KNP
      PH(J)=PSTR(J)
  116 CONTINUE
C
C     READ IN CHANGES TO SELECTED VARIABLES
   39 FORMAT(5(I5,F11.4))
   40 FORMAT(10(I4,I4))
   76 READ(KR,40)(IBLT(J),JBLT(J),J=1,NIND)
      DO 41 J=1,NIND
      KA=IBLT(J)
      IF(KA.LE.0)GO TO 42
      M(KA)=JBLT(J)
   41 CONTINUE
      GO TO 76
C
```

```
C     SET THE STRUCTURE SELECTOR
   42 KB=0
      DO 176 J=1,KS
      KB=KB+M(J)
      IF(M(J).EQ.1)L(KB)=J
  176 CONTINUE
  199 READ(KR,40)(IBLT(J),JBLT(J),J=1,NIND)
      DO 44 J=1,NIND
      KA=IBLT(J)
      IF(KA.LE.0)GO TO 43
      ME(KA)=JBLT(J)
   44 CONTINUE
      GO TO 199
C
C     READ TIME VARIABLES
   43 READ(KR,3)TZ,DT
      READ(KR,1)NDT,KF,KP
C
C     READ IN CHANGES TO SELECTED VARIABLES
   45 READ(KR,40)(IBLT(J),JBLT(J),J=1,NIND)
      DO 46 J=1,NIND
      KA=IBLT(J)
      IF(KA.LE.0)GO TO 47
      LF(KA)=JBLT(J)
   46 CONTINUE
      GO TO 45
   47 READ(KR,40)(IBLT(J),JBLT(J),J=1,NIND)
      DO 48 J=1,NIND
      KA=IBLT(J)
      IF(KA.LE.0)GO TO 49
      LP(KA)=JBLT(J)
   48 CONTINUE
      GO TO 47
   49 READ(KR,39)(IBLT(J),ABLT(J),J=1,NIN )
      DO 50 J=1,NIN
      KA=IBLT(J)
      IF(KA.LE.0)GO TO 51
      X(KA)=ABLT(J)
      XN(KA)=ABLT(J)
   50 CONTINUE
      GO TO 49
   51 READ(KR,39)(IBLT(J),ABLT(J),J=1,NIN )
      DO 52 J=1,NIN
      KA=IBLT(J)
      IF(KA.LE.0)GO TO 55
      P(KA)=ABLT(J)
   52 CONTINUE
      GO TO 51
   55 READ(KR,39)(IBLT(J),ABLT(J),J=1,NIN )
      DO 56 J=1,NIN
      KA=IBLT(J)
      IF(KA.LE.0)GO TO 57
      Q(KA)=ABLT(J)
```

```
 56 CONTINUE
    GO TO 55
 57 READ(KR,39)(IBLT(J),ABLT(J),J=1,NIN )
    DO 58 J=1,NIN
    KA=IBLT(J)
    IF(KA.LE.0)GO TO 59
    R(KA)=ABLT(J)
 58 CONTINUE
    GO TO 57
 59 READ(KR,39)(IBLT(J),ABLT(J),J=1,NIN )
    DO 84 J=1,NIN
    KA=IBLT(J)
    IF(KA.LE.0)GO TO 83
    U(KA)=ABLT(J)
 84 CONTINUE
    GO TO 59
 83 READ(KR,39)(IBLT(J),ABLT(J),J=1,NIN )
    DO 68 J=1,NIN
    KA=IBLT(J)
    IF(KA.LE.0)GO TO 69
    TST(KA)=ABLT(J)
 68 CONTINUE
    GO TO 83
 69 READ(KR,39)(IBLT(J),ABLT(J),J=1,NIN )
    DO 70 J=1,NIN
    KA=IBLT(J)
    IF(KA.LE.0)GO TO 117
    UST(KA)=ABLT(J)
 70 CONTINUE
    GO TO 69
117 READ(KR,39)(IBLT(J),ABLT(J),J=1,NIN )
    DO 118 J=1,NIN
    KA=IBLT(J)
    IF(KA.LE.0)GO TO 119
    XH(KA)=ABLT(J)
118 CONTINUE
    GO TO 117
119 READ(KR,39)(IBLT(J),ABLT(J),J=1,NIN )
    DO 120 J=1,NIN
    KA=IBLT(J)
    IF(KA.LE.0)GO TO 53
    PH(KA)=ABLT(J)
120 CONTINUE
    GO TO 119
 53 READ(KR,39)(IBLT(J),ABLT(J),J=1,NIN )
    DO 54 J=1,NIN
    KA=IBLT(J)
    IF(KA.LE.0)GO TO 71
    EH(KA)=ABLT(J)
 54 CONTINUE
    GO TO 53
 71 READ(KR,39)(IBLT(J),ABLT(J),J=1,NIN )
    DO 72 J=1,NIN
```

```
      KA=IBLT(J)
      IF(KA.LE.0)GO TO 73
      QN(KA)=ABLT(J)
   72 CONTINUE
      GO TO 71
   73 READ(KR,39)(IBLT(J),ABLT(J),J=1,NIN )
      DO 74 J=1,NIN
      KA=IBLT(J)
      IF(KA.LE.0)GO TO 75
      RN(KA)=ABLT(J)
   74 CONTINUE
      GO TO 73
   75 READ(KR,40)(IBLT(J),JBLT(J),J=1,NIND)
      DO 78 J=1,NIND
      KA=IBLT(J)
      IF(KA.LE.0)GO TO 79
      IOUT(KA)=JBLT(J)
   78 CONTINUE
      GO TO 75
C
C   TEST FOR BLOCK CARD AND CORRECT BLOCK DATA
   79 ISW=1
      ISWI=1
   88 READ(KR,15)(IBLT(J),J=1,NBLT)
      DO 85 J=1,NBLT
      IF(IBL(J).NE.IBLT(J))GO TO 86
   85 CONTINUE
      GO TO (133,10),ISW
   86 WRITE(KW,87)(IBLT(J),J=1,NBLT),NBLOK
   87 FORMAT(1X,5A1,5X,'BLOCK NUMBER  ',I4,'  HAS INPUT DATA',
     1' ERROR')
      GO TO (198,88),ISWI
  198 IF(MREF.EQ.1)GO TO 189
      ISW=2
      GO TO 88
  189 DO 190 J=1,NMRT
      IF(IMRF(J).NE.IBLT(J))GO TO 191
  190 CONTINUE
      JP=1
      GO TO 192
  191 JP=
  192 IF(JP.GE.NP3)GO TO 10
  193 READ(KR,15)(IMRT(J),J=1,NMRT)
      DO 194 J=1,NMRT
      IF(IMRF(J).NE.IMRT(J))GO TO 193
  194 CONTINUE
      JP=JP+1
      GO TO 192
C
C   WRITE BLOCK DESCRIPTION
   20 FORMAT(1H1,5X,53('*'))
   89 FORMAT(1H ,5X,53('*'),/)
   90 FORMAT(1H ,5X,'*',12X,7H BLOCK ,I4,
```

```
  116H CHARACTERISTICS,12X,'*')
 91 FORMAT(1H1)
 92 FORMAT(1H )
 93 FORMAT(1H ,5X,'*',51X,'*')
 940FORMAT(1H ,5X,'*',20H MODEL U V W P Q R  ,
  130HEFF TA SE SH PR TH NO CO CB XD,1X,'*')
 95 FORMAT(1H ,5X,'*',6X,6I2,5X,9I3,1X,'*')
 960FORMAT(1H ,5X,'*',13H DATA PASSES ,I2,
  114H  DATA POINTS ,I4,13H  NO. COEFFS ,I4,1X,'*')
 970FORMAT(1H ,5X,'*',7H TZERO ,E11.4,5H  DT ,E11.4,
  112H  NO. PARAM ,I4,1X,'*')
 980FORMAT(1H ,5X,'*',15H INITIALIZING- ,
  135H  M ME LF LP X P E O R ST QU PN J C,1X,'*')
 990FORMAT(1H ,5X,'*',3X,'C=INITIALIZE',1X,
  1I2,3I3,5I2,3I3,2I2,1X,'*')
133 WRITE(KW,20)
    WRITE(KW,90)NBLOK
    WRITE(KW,93)
    WRITE(KW,94)
    WRITE(KW,95)(M(J),J=1,KS),(ME(K),K=1,KMV)
    WRITE(KW,93)
    WRITE(KW,96)NPB,NDT,KF
    WRITE(KW,97)TZ,DT,KP
    WRITE(KW,93)
    WRITE(KW,98)
   0WRITE(KW,99)IM,IME,ILF,ILP,IXH,IPH,IE,INO,INP,
   1IST,IO,IR,IU,IO
    WRITE(KW,93)
1000FORMAT(1H ,5X,'*',30H COEFFICIENTS TO BE IDENTIFIED,
  121X,'*')
1010FORMAT(1H ,5X,'*',28H PARAMETERS TO BE IDENTIFIED,
  123X,'*')
102 FORMAT(1H ,5X,'*',2X,8(1X,3A1,1X,I1),1X,'*')
103 FORMAT(1H ,5X,'*',2X,5(1X,3A1,1X,I1),13X,'*')
    WRITE(KW,100)
    JT=1
    DO 104 J=1,KAV
    LFW(J)=0
    IF(JT.GT.KF)GO TO 104
    IF(LF(JT).GT.J)GO TO 104
    LFW(J)=1
    JT=JT+1
104 CONTINUE
    JK=8
    JT=0
    JV=0
    DO 105 K=1,KS
    DO 106 J=1,KS
    DO 107 I=J,KS
109 JT=JT+1
    IF(JT.GT.JK)GO TO 108
    KXW(JT)=IXWR(K)
    IXW(JT)=JXWR(J)
```

```
      JXW(JT)=JXWR(I)
      NT=J+(I*(I-1))/2+21*(K-1)
      LXW(JT)=LFW(NT)
      IF((K.EQ.KS).AND.(J.EQ.KS))GO TO 109
      GO TO 107
  108 JV=JV+1
      IF(JV.GE.ISIXT)GO TO 110
      WRITE(KW,102)(KXW(JW),IXW(JW),JXW(JW),LXW(JW),JW=1,JK)
      JT=0
      GO TO 109
  110 IF(JV.GE.ISEVT)GO TO 111
      WRITE(KW,102)(KXW(JW),IXW(JW),JXW(JW),LXW(JW),JW=1,JK)
      JK=6
      JT=0
      GO TO 109
  111 WRITE(KW,103)(KXW(JW),IXW(JW),JXW(JW),LXW(JW),JW=1,JK)
      GO TO 112
  107 CONTINUE
  106 CONTINUE
  105 CONTINUE
  112 WRITE(KW,93)
      WRITE(KW,101)
      JT=1
      DO 127 J=1,KAP
      LFP(J)=0
      IF(JT.GT.KP)GO TO 127
      IF(LP(JT).GT.J)GO TO 127
      LFP(J)=1
      JT=JT+1
  127 CONTINUE
      JK=8
      JV=0
      JT=0
      DO 123 J=1,NGPI
      JA=NGP(J)
      JM=MGP(J)-1
      DO 126 JG=1,JA
      JM=JM+1
  128 JT=JT+1
      IF(JT.GT.JK)GO TO 129
      KXW(JT)=JM
      LXW(JT)=LFP(JM)
      IF((JK.EQ.6).AND.(J.EQ.NGPI).AND.(JG.EQ.JA))GO TO 129
      GO TO 126
  129 JV=JV+1
      IF(JV.GE.ISIXT)GO TO 130
      WRITE(KW,124)(KXW(JW),LXW(JW),JW=1,JK)
      JT=0
      GO TO 128
  130 IF(JV.GE.ISEVT)GO TO 131
      WRITE(KW,124)(KXW(JW),LXW(JW),JW=1,JK)
      JK=6
      JT=0
```

```
      GO TO 128
  131 WRITE(KW,125)(KXW(JW),LXW(JW),JW=1,JK)
      GO TO 132
  126 CONTINUE
  123 CONTINUE
  132 WRITE(KW,93)
      WRITE(KW,89)
  124 FORMAT(1H ,5X,'*',2X,8(I4,1X,I1),1X,'*')
  125 FORMAT(1H ,5X,'*',2X,6(I4,1X,I1),13X,'*')
C
C   SET NOISE PARAMETERS FOR SEA TRIAL
      JR=ME(15)-1
      JV=JR+KS
      DO 136 JK=1,KS
      JR=JR+1
      JV=JV+1
      RV(JK+KS)=R(JK)
      P(JV)=Q(JK)
      RV(JK)=P(JR)
  136 CONTINUE
      ME(20)=IOUT(4)
C
C **************************************************************
C WRITE BLOCK DATA FOR REFERENCE
      WRITE(KW,91)
      WRITE(KW,253)
      WRITE(KW,291)NBLCK,NPB,IB,NB
  291 FORMAT(1H ,1X,'BEGIN BLOCK ',I4,' WITH ',I4,' PASSES',
     1'. THIS BLOCK IS ',I4,' OF ',I4,' TO BE PROCESSED.')
      WRITE(KW,92)
      WRITE(KW,292)
  292 FORMAT(1H ,1X,'THE FOLLOWING IS THE PROGRAM BLOCK ',
     1'DATA AS USED IN THE SEA TRIAL.')
      WRITE(KW,253)
      WRITE(KW,293)NBLCK
  293 FORMAT(1H ,52X,'BLOCK ',I5,' DATA')
      WRITE(KW,92)
      WRITE(KW,294)
  294 FORMAT(1H ,1X,'STATES OR DEGREES OF FREEDOM IN THIS',
     1' BLOCK')
      DO 295 JK=1,KS
      IF(M(JK).EQ.0)GO TO 295
      GO TO (296,297,298,299,300,301),JK
  296 WRITE(KW,302)X(JK)
      GO TO 295
  297 WRITE(KW,303)X(JK)
      GO TO 295
  298 WRITE(KW,304)X(JK)
      GO TO 295
  299 WRITE(KW,305)X(JK)
      GO TO 295
  300 WRITE(KW,306)X(JK)
      GO TO 295
```

```
301 WRITE(KW,307)X(JK)
295 CONTINUE
302CFORMAT(1H ,10X,'U  SURGE  X(1) FT/SEC  AT T=TZ U=',
   1F12.7)
303CFORMAT(1H ,10X,'V  SWAY   X(2) FT/SEC  AT T=TZ V=',
   1F12.7)
304CFORMAT(1H ,10X,'W  HEAVE  X(3) FT/SEC  AT T=TZ W=',
   1F12.7)
305CFORMAT(1H ,10X,'P  ROLL   X(4) RAD/SEC AT T=TZ P=',
   1F12.7)
306CFORMAT(1H ,10X,'Q  PITCH  X(5) RAD/SEC AT T=TZ Q=',
   1F12.7)
307CFORMAT(1H ,10X,'R  YAW    X(6) RAD/SEC AT T=TZ R=',
   1F12.7)
    WRITE(KW,92)
    WRITE(KW,308)
308 FORMAT(1H ,1X,'EFFECTORS USED IN THIS BLOCK')
    DO 309 JK=1,KMV
    IF(ME(JK).EQ.0)GO TO 309
    GO TO (310,311,312,313,314,315,316,317,318),JK
310 WRITE(KW,319)
    GO TO 309
311 WRITE(KW,320)
    GO TO 309
312 WRITE(KW,321)
    GO TO 309
313 WRITE(KW,322)
    GO TO 309
314 WRITE(KW,323)
    GO TO 309
315 WRITE(KW,324)
    GO TO 309
316 WRITE(KW,325)
    GO TO 309
317 WRITE(KW,326)
    GO TO 309
318 WRITE(KW,327)
309 CONTINUE
319 FORMAT(1H ,10X,'TANKS -  5-MERCURY,2-WATER U(1)-U(7)')
320CFORMAT(1H ,10X,'SECONDARY DRAG - FORCES AND MOMENTS',
   1' DUE TO YVV AND ZWW')
321 FORMAT(1H ,10X,'SHROUD - U(8)-U(9) ANGLES IN RADIANS')
322 FORMAT(1H ,10X,'PROPELLOR - U(10) RPS , U(11) RPS/SEC')
323CFORMAT(1H ,10X,'THRUSTERS - U(12) YFWD, U(13) YAFT,',
   1' U(14) ZFWD, U(15) ZAFT ;+U=+Y,+M,-Z,-N ')
324CFORMAT(1H ,10X,'NOISE - GAUSSIAN WHITE ADDITIVE PROC',
   1'ESS AND MEASUREMENT NOISE')
325 FORMAT(1H ,10X,'CONSTANT TERMS - ADDED TO THE PROCESS')
326 FORMAT(1H ,10X,'CENTER OF BUOYANCY NOT ZERO')
327CFORMAT(1H ,10X,'NOISELESS TRAJECTORIES ARE ALSO ',
   1'CALCULATED')
    IF(ME(19).EQ.0)GO TO 328
    WRITE(KW,329)
```

```
329OFORMAT(1H ,10X,'ANGLES ARE INTEGRATED P,Q,R RESULTIN',
   1'G FROM VEHICLE TRAJECTORY')
328 IF(ME(20).EQ.0)GO TO 330
    WRITE(KW,331)
331 FORMAT(1H ,10X,'VEHICLE ANGLES WILL BE PLOTTED')
330 WRITE(KW,92)
    WRITE(KW,332)
332OFORMAT(1H ,1X,'NOISE GENERATION PARAMETERS, Q(6),R(6',
   1'), Q-PROCESS,R-MEASUREMENT')
    WRITE(KW,333)(J,Q(J),J=1,KS)
333 FORMAT(1H ,1X,'Q(J)-',6(2X,I3,'=',F13.4))
    WRITE(KW,334)(J,R(J),J=1,KS)
334 FORMAT(1H ,1X,'R(J)-',6(2X,I3,'=',F13.4))
    WRITE(KW,335)
335OFORMAT(1H ,1X,'WEIGHTING MATRICES,QN(6),RN(6), KFIL ',
   1'USES QN,RN, AND MOREF USES RN')
    WRITE(KW,336)(J,QN(J),J=1,KS)
336 FORMAT(1H ,1X,'QN(J)',6(2X,I3,'=',F13.4))
    WRITE(KW,337)(J,RN(J),J=1,KS)
337 FORMAT(1H ,1X,'RN(J)',6(2X,I3,'=',F13.4))
    WRITE(KW,92)
    WRITE(KW,253)
    WRITE(KW,338)
338 FORMAT(1H ,1X,'INPUT FUNCTIONS USED IN THIS BLOCK')
    IF(ME(1).EQ.0)GO TO 339
    WRITE(KW,319)
    WRITE(KW,340)
340OFORMAT(1H ,12X,'MERCURY TANKS - U(1)-U(3) LIST CONTR',
   1'OL,1-RESERVOIR,2-STBD,3-PORT')
    WRITE(KW,341)
341OFORMAT(1H ,12X,'MERCURY TANKS - U(4)-U(5) TRIM CONTR',
   1'OL,4-FWD,5-AFT')
    WRITE(KW,342)
342OFORMAT(1H ,12X,'WATER VB TANKS- U(6)-U(7) TRIM AND B',
   1'UOYANCY CONTROL,6-FWD,7-AFT')
    WRITE(KW,343)
343OFORMAT(1H ,12X,'TANKS STEP FUNCTION INPUTS - AT MOST',
   1' TEN,TST=START TIME,UST=AMPLITUDE IN LBF LIQUID')
    WRITE(KW,345)
345 FORMAT(1H ,15X,'NEGATIVE TST MEANS NO STEP GENERATED')
    WRITE(KW,344)(J,J=1,10)
344OFORMAT(1H ,3X,'TANK NO.',3X,'LOC.  ',10('STEP',I3,3X))
    JV=1
    JR=10
    DO 346 JT=1,7
    WRITE(KW,347)JT,JV,(TST(J),J=JV,JR)
    WRITE(KW,348)JT,JV,(UST(J),J=JV,JR)
    JV=JR+1
    JR=JR+10
346 CONTINUE
347 FORMAT(1H ,1X,'TST=',I4,5X,I4,2X,10(1X,F8.2,1X))
348 FORMAT(1H ,1X,'UST=',I4,5X,I4,2X,10(1X,F8.2,1X))
    WRITE(KW,91)
```

```
      WRITE(KW,253)
      WRITE(KW,338)
  339 IF(ME(3).EQ.0)GO TO 349
      WRITE(KW,321)
      WRITE(KW,350)
  350 FORMAT(1H ,12X,'SHROUD ANGLES - 8=DELTA P,9=DELTA Y')
      JV=71
      JR=80
      JT=8
      WRITE(KW,351)
  351 FORMAT(1H ,12X,'SHROUD STEP FUNCTION INPUTS - AT MOS',
     1'T TEN,TST=START TIME,UST=AMPL. IN RADIANS')
      WRITE(KW,345)
      WRITE(KW,352)(J,J=1,10)
  352 FORMAT(1H ,3X,'INPUT NO.',2X,'LOC.   ',
     110('STEP',I3,3X))
      WRITE(KW,347)JT,JV ,(TST(J),J=JV,JR)
      WRITE(KW,348)JT,JV ,(UST(J),J=JV,JR)
      JV=JR+1
      JR=JR+10
      JT=JT+1
      WRITE(KW,347)JT,JV,(TST(J),J=JV,JR)
      WRITE(KW,348)JT,JV,(UST(J),J=JV,JR)
  349 WRITE(KW,92)
      IF(ME(4).EQ.0)GO TO 355
      WRITE(KW,322)
      JV=91
      JR=100
      JT=10
      WRITE(KW,353)
  353 FORMAT(1H ,12X,'PROP STEP FUNCTION INPUTS - AT MOST ',
     1'TEN,TST=START TIME,UST=AMPL. IN RPS OR RPS/SEC')
      WRITE(KW,345)
      WRITE(KW,352)(J,J=1,10)
      WRITE(KW,347)JT,JV ,(TST(J),J=JV,JR)
      WRITE(KW,348)JT,JV,(UST(J),J=JV,JR)
      JT=JT+1
      JV=JR+1
      JR=JR+10
      WRITE(KW,347)JT,JV ,(TST(J),J=JV,JR)
      WRITE(KW,348)JT,JV,(UST(J),J=JV,JR)
      IF(ME(1).NE.0)GO TO 355
      WRITE(KW,92)
      WRITE(KW,253)
      WRITE(KW,91)
      WRITE(KW,253)
      WRITE(KW,338)
  355 IF(ME(5).EQ.0)GO TO 354
      WRITE(KW,92)
      WRITE(KW,323)
      JV=111
      JR=120
      WRITE(KW,356)
```

```
3560FORMAT(1H ,12X,'THRUSTER STEP FUNCTION INPUTS - AT M',
   1'OST TEN,TST=START TIME,UST=AMPL. IN RPS')
    WRITE(KW,345)
    WRITE(KW,352)(J,J=1,10)
    DO 357 JT=12,15
    WRITE(KW,347)JT,JV,(TST(J),J=JV,JR)
    WRITE(KW,348)JT,JV,(UST(J),J=JV,JR)
    JV=JR+1
    JR=JR+10
357 CONTINUE
    WRITE(KW,92)
354 IF(ME(19).EQ.0)GO TO 358
    WRITE(KW,329)
    GO TO 360
358 WRITE(KW,359)
359 FORMAT(1H ,10X,'ANGLES ARE NOT INTEGRATED ,BUT ARE ',
   1'STEP FUNCTION INPUTS')
360 IF(ME(20).EQ.0)GO TO 361
    WRITE(KW,331)
    GO TO 362
361 WRITE(KW,363)
363 FORMAT(1H ,10X,'VEHICLE ANGLES WILL NOT BE PLOTTED')
362 WRITE(KW,92)
    WRITE(KW,367)
3670FORMAT(1H ,12X,'ANGLE STEP FUNCTION INPUTS - AT MOST',
   1' TEN,TST=START TIME,UST=AMPL. IN RADIANS')
    WRITE(KW,368)
3680FORMAT(1H ,15X,'VEHICLE ANGLES PHI,THETA,PSI = U(16)',
   1' - U(18)')
    JV=151
    JR=160
    WRITE(KW,345)
    WRITE(KW,352)(J,J=1,10)
    DO 369 JT=16,18
    WRITE(KW,347)JT,JV,(TST(J),J=JV,JR)
    WRITE(KW,348)JT,JV,(UST(J),J=JV,JR)
    JV=JR+1
    JR=JR+10
369 CONTINUE
    WRITE(KW,92)
    WRITE(KW,364)
3640FORMAT(1H ,12X,'U(J) - STARTING VALUES FOR ALL OF ',
   1'THE EFFECTOR STEP FUNCTIONS')
    JR=6
    DO 365 JV=1,13,6
    WRITE(KW,366)(J,U(J),J=JV,JR)
366 FORMAT(1H ,1X,'U(J)-',6(2X,I3,'=',F13.4))
    JR=JR+6
365 CONTINUE
    WRITE(KW,92)
    WRITE(KW,370)
370 FORMAT(1H ,1X,'END OF INPUT FUNCTIONS THIS BLOCK')
    WRITE(KW,253)
```

```
      WRITE(KW,91)
      WRITE(KW,253)
      WRITE(KW,371)
      IF(IPUT(5).EQ.0)GO TO 404
 3710FORMAT(1H ,1X,'X(J) - TOTAL STATE VECTOR USED FOR SE',
     1'A TRIAL GENERATION')
      WRITE(KW,372)(J,X(J),J=1,KX)
  372 FORMAT(1H ,1X,'X(J)-',6(2X,I3,'=',F13.4))
      WRITE(KW,92)
      WRITE(KW,253)
      WRITE(KW,91)
      WRITE(KW,253)
      WRITE(KW,374)
  374 FORMAT(1H ,1X,'XH(J)- STARTING STATE VECTOR')
      WRITE(KW,373)(J,XH(J),J=1,KX)
  373 FORMAT(1H ,1X,'XH(J)',6(2X,I3,'=',F13.4))
      WRITE(KW,92)
      WRITE(KW,253)
      WRITE(KW,91)
      WRITE(KW,253)
      WRITE(KW,375)
  375 FORMAT(1H ,1X,'P(J) - TOTAL PARAMETER VECTOR')
      WRITE(KW,376)(J,P(J),J=1,KX)
  376 FORMAT(1H ,1X,'P(J)-',6(2X,I3,'=',F13.4))
      WRITE(KW,92)
      WRITE(KW,253)
      WRITE(KW,91)
      WRITE(KW,253)
      WRITE(KW,375)
      JV=KX+1
      WRITE(KW,376)(J,P(J),J=JV,KNP)
      WRITE(KW,92)
      WRITE(KW,253)
      WRITE(KW,91)
      WRITE(KW,253)
      WRITE(KW,377)
  377 FORMAT(1H ,1X,'PH(J)- STARTING PARAMETER VECTOR')
      WRITE(KW,378)(J,PH(J),J=1,KX)
  378 FORMAT(1H ,1X,'PH(J)',6(2X,I3,'=',F13.4))
      WRITE(KW,92)
      WRITE(KW,253)
      WRITE(KW,91)
      WRITE(KW,253)
      WRITE(KW,377)
      WRITE(KW,378)(J,PH(J),J=JV,KNP)
      WRITE(KW,92)
      WRITE(KW,253)
      WRITE(KW,91)
      WRITE(KW,253)
      WRITE(KW,381)
  381 FORMAT(1H ,1X,'EH(J) - DIAGONAL OF ERROR COV. MATRIX')
      JV=1
      DO 382 J=1,KEH
```

```
      WRITE(KW,383)J,EH(J)
  383 FORMAT(1H ,1X,'EH(J)-',5X,I5,5X,F20.12)
      J=J+JV
      JV=JV+1
  382 CONTINUE
      WRITE(KW,92)
      WRITE(KW,253)
      WRITE(KW,91)
  404 WRITE(KW,253)
      WRITE(KW,379)
  379 FORMAT(1H ,1X,'IOUT- OUTPUT DESIGNATION VECTOR')
      WRITE(KW,380)(J,IOUT(J),J=1,10)
  380 FORMAT(1H ,1X,'IOUT(J)',10(2X,I2,'=',I4))
      WRITE(KW,92)
      WRITE(KW,253)
C ****************************************************************************
C END OF BLOCK DATA OUTPUT
C
C ****************************************************************************
C RUN A SIMULATED SEA TRIAL FOR THIS BLOCK
      WRITE(KW,384)(X(J),J=1,KS)
  384 FORMAT(1H ,1X,'BEGIN SEA TRIAL- X(J)= ',
     116(2X,F13.6,2X))
      CALL SEATR(IABO,P,X,XN)
C
C   TEST FOR A VALID SEA TRIAL;IF NOT ,ABORT THIS BLOCK
      IF(IABO.EQ.IONE)GO TO 134
      IF(MREF.NE.1)GO TO 188
      DO 185 IP=1,NPB
  186 READ(KR,15)(IMRT(J),J=1,NMRT)
      DO 187 J=1,NMRT
      IF(IMRF(J).NE.IMRT(J))GO TO 186
  187 CONTINUE
  185 CONTINUE
  188 WRITE(KW,135)NBLOK
  135 FORMAT(5X,'BLOCK NUMBER',I5,5X,'SEA TRIAL ABORT')
      GO TO 10
C
C ****************************************************************************
C PLOT THE DATA GENERATED BY THE SEA TRIAL
  134 NPLOT=NBLOK
      NCON=NBLOK
      CALL STOUT(T,UD,Z,ZN,IOUT(2),NDT,NPLOT)
C
C ****************************************************************************
C TURN OFF THE NOISE GENERATOR IN THE OCEAN VEHICLE
      IF(ME(6).NE.0)GO TO 178
      JW=NDT*KB
      DO 180 JK=1,JW
      ZN(JK)=Z(JK)
  180 CONTINUE
      GO TO 179
  178 ME(6)=0
```

```
C
C ***********************************************************************
C BEGIN IDENTIFICATION PASSES OVER SEA TRIAL DATA
  179 DO 11 IP=1,NPB
      READ(KR,40)KF,KP
      IF(MREF.EQ.1)GO TO 137
C
C ***********************************************************************
C EXTENDED KALMAN FILTER PASS OVER DATA
      GO TO 11
C
C ***********************************************************************
C MODEL REFERENCE PASS OVER DATA WITH TWO COEFFICIENTS OR
C PARAMETERS VARIED
C
C   READ MODEL REFERENCE CHANGES TO SELECTED
C   VARIABLES LF,LP,RN
  137 READ(KR,40)(IBLT(J),JBLT(J),J=1,NIND)
      DO 141 J=1,NIND
      KA=IBLT(J)
      IF(KA.LE.0)GO TO 142
      LF(KA)=JBLT(J)
  141 CONTINUE
      GO TO 137
  142 READ(KR,40)(IBLT(J),JBLT(J),J=1,NIND)
      DO 143 J=1,NIND
      KA=IBLT(J)
      IF(KA.LE.0)GO TO 167
      LP(KA)=JBLT(J)
  143 CONTINUE
      GO TO 142
  167 READ(KR,39)(IBLT(J),ABLT(J),J=1,NIN)
      DO 168 J=1,NIN
      KA=IBLT(J)
      IF(KA.LE.0)GO TO 144
      RN(KA)=ABLT(J)
  168 CONTINUE
      GO TO 167
  144 DO 169 JTS=1,KB
      K=L(JTS)
      IF(RN(K).EQ.0)GO TO 166
  169 CONTINUE
C
C   TEST FOR PASS CARD AND CORRECT PASS DATA
      ISW=1
      READ(KR,15)(IMRT(J),J=1,NMRT)
      DO 145 J=1,NMRT
      IF(IMRF(J).NE.IMRT(J))GO TO 146
  145 CONTINUE
      GO TO (147,11),ISW
  146 WRITE(KW,148)(IMRT(J),J=1,NMRT),IB,IP
  148 FORMAT(1X,4A1,5X,'BLOCK NUMBER  ',I4,'  HAS INPUT',
     1' DATA ERROR ,PASS NUMBER   ',I4)
```

```
      IF(NPB.LE.1)GO TO 195
      DO 196 JPT=1,6
      READ(KR,15)(IMRT(J),J=1,NMRT)
      DO 197 J=1,NMRT
      IF(IMRF(J).NE.IMPT(J))GO TO 196
  197 CONTINUF
      IF(NPB.GT.2)GO TO 11
      GO TO 10
  196 CONTINUE
  195 IB=IB+1
      ISW=2
      ISWI=2
      GO TO 88
C
C ******************************************************************
C SET THE INDICES,INITIAL VALUES, AND INCREMENTS OF THE
C COEFFICIENTS AND PARAMETERS (TOTAL OF TWO) TO BE
C VARIED IN MODEL REFERENCE PASS
  147 IF(KF.GT.1)GO TO 138
      IF(KF.GT.0)GO TO 139
      ICDV=LP(1)
      ICNV=LP(2)
      NI=2
      GO TO 140
  139 ICDV=LF(1)+KS
      ICNV=LP(1)
      NI=1
      GO TO 140
  138 ICDV=LF(1)+KS
      ICNV=LF(2)+KS
      NI=0
  140 IF(NI.EQ. )GO TO 153
      IF(NI.EQ.1)GO TO 154
      CD(1)=PH(ICDV)
      CN(1)=PH(ICNV)
      CDSV=P(ICDV)
      CNSV=P(ICNV)
      P(ICDV)=CD(1)
      P(ICNV)=CN(1)
      DELCD=PINC(ICDV)
      DELCN=PINC(ICNV)
      DO 155 K=1,ICD
      CD(K+1)=CD(K)+DELCD
  155 CONTINUE
      DO 156 K=1,ICN
      CN(K+1)=CN(K)+DELCN
  156 CONTINUE
      GO TO 157
  154 CD(1)=XH(ICDV)
      CN(1)=PH(ICNV)
      CDSV=X(ICDV)
      CNSV=P(ICNV)
      X(ICDV)=CD(1)
```

```
      P(ICNV)=CN(1)
      DELCD=XINC(ICDV)
      DELCN=PINC(ICNV)
      DO 158 K=1,ICD
      CD(K+1)=CD(K)+DELCD
  158 CONTINUE
      DO 159 K=1,ICN
      CN(K+1)=CN(K)+DELCN
  159 CONTINUE
      GO TO 157
  153 CD(1)=XH(ICDV)
      CN(1)=XH(ICNV)
      CDSV=X(ICDV)
      CNSV=X(ICNV)
      X(ICDV)=CD(1)
      X(ICNV)=CN(1)
      DELCD=XINC(ICDV)
      DELCN=XINC(ICNV)
      DO 160 K=1,ICD
      CD(K+1)=CD(K)+DELCD
  160 CONTINUE
      DO 161 K=1,ICN
      CN(K+1)=CN(K)+DELCN
  161 CONTINUE
C ***************************************************************
C DESCRIBE COEFFICIENTS AND PARAMETERS BEING VARIED
  157 WRITE(KW,385)
  385 FORMAT(1H ,1X,'COEFFICIENTS AND PARAMETERS IN HOREF')
      IF(NI.GT.1)GO TO 386
      IF(NI.GT.0)GO TO 387
      WRITE(KW,388)ICDV,ICNV
  388 FORMAT(1H ,1X,'TWO COEFFICIENTS - INDEXES ',I4,'AND',
     1I6)
      WRITE(KW,389)CD(1),CN(1),DELCD,DELCN,CDSV,CNSV
  389 FORMAT(1H ,1X,'START AT  ',F13.4,'AND',F13.4,3X,
     1'RUN IN INCREMENTS OF ',F13.4,'AND',F13.4,3X,'SAVE',
     2F13.4,'AND',F13.4)
      GO TO 390
  387 WRITE(KW,391)ICDV,ICNV
  391 FORMAT(1H ,1X,'ONE COEFFICIENT AND ONE PARAMETER - ',
     1'INDEXES ',I4,'  AND',I6)
      WRITE(KW,389)CD(1),CN(1),DELCD,DELCN,CDSV,CNSV
      GO TO 390
  386 WRITE(KW,392)ICDV,ICNV
  392 FORMAT(1H ,1X,'TWO PARAMETERS - INDEXES ',I4,'  AND',
     1I6)
      WRITE(KW,389)CD(1),CN(1),DELCD,DELCN,CDSV,CNSV
  390 WRITE(KW,401)(J,RN(J),J=1,KS)
  401 FORMAT(1H ,'WEIGHTING MATRIX DIAG.',6(2X,I1,2X,F11.4))
      WRITE(KW,402)(J,LF(J),J=1,10)
      WRITE(KW,403)(J,LP(J),J=1,10)
  403 FORMAT(1H ,'J,LP(J)',10(1X,I2,2X,I3))
  402 FORMAT(1H ,'J,LF(J)',10(1X,I2,2X,I3))
```

```
C
C ****************************************************************************
C GENERATE THE MODEL DATA USING SEA TRIAL INPUTS
      DO 149 J=1,NCN
      IF(NI.EQ.2)GO TO 162
      X(ICDV)=CD(1)
      GO TO 163
  162 P(ICDV)=CD(1)
  163 DO 150 I=1,NCD
      CALL MOREF(IABO,P,X)
      IF(IABO.EQ.IONE)GO TO 164
      WRITE(KW,165)NBLOK,IP
  165 FORMAT(7X,'BLOCK NUMBER',I5,' MODEL REFERENCE ABORT',
     1', PASS ',I5)
      GO TO 166
C
C ****************************************************************************
C CALCULATE THE WEIGHTED INTEGRAL SQUARE ERROR COST FUNCTION
  164 NC=(J-1)*NCD+I
      COST(NC)=0.
      DO 151 JSQ=1,NDT
      DO 152 JSQS=1,KB
      K=L(JSQS)
      KZ=JSQ+NDT*(JSQS-1)
      XDIF=Z(KZ)-ZN(KZ)
      XDIF=XDIF*XDIF/RN(K)
      COST(NC)=COST(NC)+XDIF*DT
      IF(COST(NC).GT.EMAX)GO TO 177
  152 CONTINUE
  151 CONTINUE
C
C ****************************************************************************
C INCREMENT THE COEFFICIENTS AND PARAMETERS BEING VARIED
  177 IF(NI.EQ.2)GO TO 170
      X(ICDV)=X(ICDV)+DELCD
      GO TO 150
  170 P(ICDV)=P(ICDV)+DELCD
  150 CONTINUE
      IF(NI.EQ.3)GO TO 171
      P(ICNV)=P(ICNV)+DELCN
      GO TO 149
  171 X(ICNV)=X(ICNV)+DELCN
  149 CONTINUE
C
C ****************************************************************************
C PLOT OVERALL CONTOURS OF COST FUNCTION
      NCONT=NCON+IP
      ISW=1
  393 CALL CONTUR(NCONT,CD,CN,COST,NCD,NCN,0)
      IF(IOUT(3).EQ.0)GO TO 166
C
C   PLOT 5 SLICES OF COST FUNCTION ALONG 2'ND COEFFICIENT
      DO 172 I=1,NCD
```

```
      Z(I)=CD(I)
      Z(NCD+I)=COST(I)
      Z(2*NCD+I)=COST(3*NCD+I)
      Z(3*NCD+I)=COST(5*NCD+I)
      Z(4*NCD+I)=COST(7*NCD+I)
      Z(5*NCD+I)=COST(10*NCD+I)
  172 CONTINUE
      CALL PLCT(NCONT,Z,NCD,6,0)
C
C    PLOT 5 SLICES OF CUST FUNCTICN ALONG 1'ST COEFFICIENT
      J=NCN-NCD+1
      I=0
      DO 173 ITS=J,NCN
      I=I+1
      ITV=(ITS-1)*NCD
      Z(I)=CN(ITS)
      Z(NCD+I)=COST(ITV+1)
      Z(2*NCD+I)=COST(ITV+4)
      Z(3*NCD+I)=COST(ITV+6)
      Z(4*NCD+I)=COST(ITV+8)
      Z(5*NCD+I)=COST(ITV+11)
  173 CONTINUE
      CALL PLOT(NCONT,Z,I,6,0)
      IF(IOUT(1).EQ.0)GO TO 166
      GO TO (394,166),ISW
  394 ISW=2
      DO 395 J=1,NCN
      DO 396 I=1,NCD
      NC=(J-1)*NCD+I
      IF(COST(NC))397,398,399
  397 COST(NC)=-COST(NC)
      GO TO 399
  398 COST(NC)=1.0E-40
  399 COST(NC)=ALOG(COST(NC))
  396 CONTINUE
  395 CONTINUE
      WRITE(KW,400)
  400 FORMAT(1H ,1X,'LOG BASE E OF COST FUNCTION CONTOURS')
      GO TO 393
  166 IF(NI.EQ.0)GO TO 174
      IF(NI.EQ.1)GO TO 175
      P(ICDV)=CDSV
      P(ICNV)=CNSV
      GO TO 11
  175 P(ICNV)=CNSV
      X(ICDV)=CDSV
      GO TO 11
  174 X(ICDV)=CDSV
      X(ICNV)=CNSV
C
C ****************************************************************
C END OF PASS
   11 CONTINUE
```

```
C
C ****************************************************************
C END OF BLOCK
   10 CCNTINUE
      STOP
      END
```

APPENDIX A20

SAMPLE DSRV INPUT DATA DECK FOR APPENDIX A19

```
Card #
 1  *    6  20 126  27   1    9    9    2 258    3   18 576   10    5 180   10    5 587   4
 2  *    1   0   1   11   11   47    1    3   16
 3  *    15    1620100 1200   1.00E-40     1.00E20
 4  *   14   72  229  277  285  302  310  320  332  352  359  374  381  386  572
 5  *   28   35   5    5    5    4    6   10    4    4    4    4    6    6
 6  *  0 0 0 0 0 0
 7  *    0    0    0    0    0    0    0    0    0   44   72  103  229  266  560  572  578  586    0    0
 8  *  1 0 0 0 0 0 0 0 0 0 0.0 0 0 0 0 0 0 0 0 0-1 0 0 1 0-1 0 0 0 0 0 0 0 0 0 0 0 0
 9  *  0 0 1 0 0-1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-1 1 0 1 0-1 0 0 1 0 0 0 0 0 0 0
10  *  0 0 0 0 1 0 0-1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-1 0 0 1 0-1 0 0 0 0 0 0 0 0
11  *  0 0 0 0 0 0
12  * 35 0 0 0 0 0 0 0 0 0 0 0 1 2 3 0 4 0 5 0 6 035 0 035 035 0 7 8 0 0.0 9 010 0 0 0
13  * 111235 03735 0 0 013 01415 0 0 016 034 01718 0 03534 0363435192034213 4 0 0 022 0
14  * 34 023 035 03735 0 0 024 0 025 026 0 0 027 028 034 035 0 036 035 029 0 0343031 0
15  * 3233 0 0 0 0
16  *  7 2 2 4 3 1 8101216 510 3 8 3 5 7 7 6 91511 9 5 910 9
17  * -144.0   0.         0.         0.         0.         0.         0.        -4320.0 0.         793.0
18  * 0.        8300.0   0.         0.        -3600.0 0.        -770.      0.        0.         917.
19  * 0.       -4550.     0.         0.         0.         0.        -770.      0.        -410000.0.
20  * 0.        830.      0.         0.         0.        -410000.37800.      0.        452000.00.
21  * 0.        450000.00.173
22  * 2404.     93.       93.        210.       210.       200.       200.      -3.37     -3.37     -3.37
23  * 18.9     -18.4      12.75     -12.3       0.         2.21      -2.21      0.        0.        0.
24  * 0.        3.03      -2.21     -2.21      -2.34      -2.50      -2.46      -2.28
25  * 1.0      -0.15      183.69750.0142 -0.0051923.47524-1.760631.0           0.15      137.7731
26  * 0.0142   0.0031952 3.475231.76063132 34.32317.50631-0.003553.521262376.295 0.075
27  * 41.33100 3234.32317.50631-0.00355-0.000050.003555-0.075  -41.3311-46.95    -23.4
28  * -23.55
29  * 0.        15.       40.        135.       170.       130.       0.08      -0.008    -0.0179   -0.0029
30  * 0.08      0.        1.32       1.715     -0.314     -14.4      0.         10.       50.       70.
31  * 65.       90.       95.        110.       130.       170.       180.       0.005     0.035     0.015
32  * -0.0732  -0.02      0.02       0.0732    -0.015     -0.0325   -0.01       0.05     -0.25      0.75
33  * 0.93      2.4      -1.2       -6.27       3.45       5.725      1.9        24.59     14.64     1.994
```

34	*	.0000001	0.	0.	10.	20.	35.	45.	130.	140.	165.
35	*	175.	180.	0.	-.0088	0.	-.00099	0.	.0015	0.	.00829
36	*	0.	0.08	.168	-.008	.02665	-.0179	-.2129	-.0029	-1.37075	.08
37	*	0.	5.	15.	45.	55.	65.	75.	80.	100.	105.
38	*	115.	125.	135.	165.	175.	180.	0.	.003	0.	-.002
39	*	0.	-.00982	0.	.00732	0.	-.00382	0.	-.00175	0.	.00225
40	*	0.	.005	-.01	.035	.125	.015	.5883	-.0732	-.6588	.0732
41	*	1.0993	-.015	.20375	-.0325	-.40375	-.01				
42	*	755.	-58.	-3.8	365.	-172.	-45.	755.	60.	22.	365.
43	*	-13.	22.	-30.	-12.	-30.	-12.	530.	-6.5	-4.05	468.
44	*	-155.	-29.	530.	80.	15.2	463.	-8.	15.2	-765.	-306.
45	*	765.	306.	26.	22.	131.	-0.21	-25.5			
46	*	-1.	-0.6	-0.4	-0.1	0.0	0.1	0.5	0.7	1.0	0.125
47	*	1.75	6.667	2.5	-4.2	-8.65	-2.55	-0.5	2.225	3.2	5.1667
48	*	4.75	4.75	5.17	2.045	0.75	-1.	-0.6	-0.5	-0.1	0.0
49	*	0.1	0.4	0.6	1.0	0.5	3.5	3.75	4.0	-2.5	-6.667
50	*	-1.75	-0.125	0.8	2.5	5.225	4.75	4.75	5.1667	3.2	2.225
51	*	-1.0	-0.2	0.2	1.0	-3.375	-5.75	-3.375	0.475	0.	-0.475
52	*	-1.0	-0.6	-0.1	0.1	0.5	1.0	-3.5	-0.5	-1.5	-0.5
53	*	-3.5	-1.8	0.12	0.	-0.12	1.8	-1.0	-0.8	-0.45	-0.1
54	*	0.	0.1	0.5	1.0	270.0	48.57143	240.	90.	-90.	-72.5
55	*	-6.	199.	21.85714	109.	93.	93.	93.25	60.	-1.	-0.5
56	*	-0.1	0.	0.1	0.45	0.7	1.0	-8.0	-72.5	-40.	80.
57	*	217.143	64.	163.3333	-57.0	-86.75	-84.	-94.	-97.7143	-28.8	-98.3333
58	*	1.06	1.13	-.696	.99	1.04	-.696	.0000001	-1.	1.	-1.
59	*	-.65	-.55	-.45	-.35	-.15	.15	.45	.55	.65	.75
60	*	1.	0.	16.25	0.	49.17	0.	-51.0567	0.	53.	0.
61	*	16.	0.	.125	10.5775	1.75	23.8765	5.667	-.59167	-6.65	-37.
62	*	-2.35	-12.75	-.75	-1.	-.65	-.45	-.15	.15	.35	.45
63	*	.55	.65	1.0	0.	41.29	0.	-51.39	0.	49.17	0.
64	*	16.25	0.	0.5	27.3125	9.75	1.0417	-6.667	-23.8765	-1.75	-10.6875
65	*	-.125	-1.0	-.25	-.15	.15	.25	1.0	0.	-23.75	0.
66	*	23.75	0.	-3.375	-9.3125	-5.75	-9.3125	-3.375	-1.	-.65	-.55

67	*	-.15	-.05	.05	.15	.55	.65	1.0	0.	32.	0.
68	*	-12.	0.	12.	0.	-32.	0.	-3.5	16.3	-.3	-2.1
69	*	-1.5	-2.1	-.3	16.5	-5.5	-1.	-.85	-.75	-.5	-.4
70	*	-.15	.05	.15	.45	.55	1.	0.	-2214.290.		1914.29
71	*	0.	-1650.	175.	0.	605.	0.	270.	-1612.1448.571431005.714		
72	*	240.	-7.5	-93.75	-72.5	-371.75	-6.	-1.	-.55	-.45	-.05
73	*	.15	.4	.5	.65	.75	1.	0.	-645.	0.	1448.215
74	*	0.	-1531.430.		993.33330.		-8.	-362.75	-72.5	-.08675	217.143
75	*	829.715	64.	-581.667163.3333							
76	*	0.	0.	0.	0.	0.	0.	2.	2.	2.	50.
77	*	50.	50.								
78	*	0.	0.	0.	0.	0.	0.				
79	*	140369.	0.	0.	0.1335	140369.	0.	0.	0.		
80	*	4365.0	32.17259								
81	*	0.2	0.05	0.05	.005	.005	.005	-16.7	0.00	0.00	0.00
82	*	0.	0.	0.	0.	0.	0.	0.	0.	-3600.	0.
83	*	-770.	0.	3720.	0.	-793.0	0.	-830.	0.	-124.	-346.
84	*	0.	-112.	0.	264.	0.	3600.	0.	0.	0.	0.
85	*	770.	0.	1040.	0.	0.	0.	0.	0.	2.43	0.
86	*	-195.	-55.9	0.	-207.	0.	-3720.	0.	793.	-444.	0.
87	*	0.	0.	0.	0.	0.	0.	330.	0.	0.	0.
88	*	264.	142.	0.	638.	0.	-1870.	0.	-793.	-33000.	0.
89	*	592.	0.	0.	0.	-1390.	0.	-592.	0.	0.	0.
90	*	-66.	0.	-3360.	2760.	0.	0.	0.	-830.	0.	0.
91	*	-26600.	0.	0.	0.	0.	0.	793.	0.	406000.	0.
92	*	0.	0.	-3600.	0.	0.	598.	0.	-935.	0.	-770.
93	*	0.	0.	-793.	0.	-406000.0.		-32000.	0.	0.	0.
94	*	0.	0.	-19.6	0.	0.	0.	0.	0.	0.	0.
95	*	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
96	*	0.	0.	0.	0.	-241.	0.	0.	0.	0.	642.
97	*	0.	0.	0.	0.	0.	0.	0.	0.	9240.	0.
98	*	0.	0.	0.	0.	-2.43	0.	0.	-158.	0.	0.
99	*	0.	0.	0.	0.	-10400.	0.	0.	0.	0.	0.

100	*	0.	0.	0.	0.	0.	0.	642.	0.	0.	0.
101	*	0.	-3670.	0.	0.	0.	0.	0.	0.	0.	0.
102	*	-4290.	0.	0.	0.	0.	0.	-66.	0.	0.	-3000.
103	*	0.	0.	0.	0.	0.	0.	-32000.	0.	0.	0.
104	*	0.	0.	0.	0.	0.	0.	0.	0.	2880.	0.
105	*	0.	0.	0.	-4290.	0.	0.	0.	0.	0.	0.
106	*	0.	0.	-49700.	0.	0.	0.	0.	0.		
107	*	140369.	0.	0.	0.1335	140369.	0.	0.	0.		
108	*	2404.	93.	93.	210.	210.	200.	200.	0.	0.	0.
109	*	0.	0.	0.	0.	0.	0.	0.	0.		
110	*	0.02	0.00	0.00	0.00	0.00	0.00	1.67	0.0	0.0	0.0
111	*	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	360.00	0.0
112	*	77.00	0.0	372.00	0.0	79.30	0.0	83.00	0.0	12.40	34.60
113	*	0.0	11.20	0.0	26.40	0.0	360.00	0.0	0.0	0.0	0.0
114	*	77.00	0.0	104.00	0.0	0.0	0.0	0.0	0.0	0.24	0.0
115	*	19.50	5.59	0.0	20.70	0.0	372.00	0.0	79.30	44.40	0.0
116	*	0.0	0.0	0.0	0.0	0.0	0.0	83.00	0.0	0.0	0.0
117	*	26.40	14.20	0.0	63.50	0.0	187.00	0.0	79.30	3300.00	0.0
118	*	59.20	0.0	0.0	0.0	159.00	0.0	59.20	0.0	0.0	0.0
119	*	6.60	0.0	336.00	276.00	0.0	0.0	0.0	83.00	0.0	0.0
120	*	2650.00	0.0	0.0	0.0	0.0	0.0	79.30	0.0	40599.98	0.0
121	*	0.0	0.0	360.00	0.0	0.0	59.80	0.0	93.50	0.0	77.00
122	*	0.0	0.0	79.30	0.0	40599.98	0.0	3200.00	0.0	0.0	0.0
123	*	0.0	0.0	1.96	0.0	0.0	0.0	0.0	0.0	0.0	0.0
124	*	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
125	*	0.0	0.0	0.0	0.0	24.10	0.0	0.0	0.0	0.0	64.20
126	*	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	924.00	0.0
127	*	0.0	0.0	0.0	0.0	0.24	0.0	0.0	15.80	0.0	0.0
128	*	0.0	0.0	0.0	0.0	1040.00	0.0	0.0	0.0	0.0	0.0
129	*	0.0	0.0	0.0	0.0	0.0	0.0	64.20	0.0	0.0	0.0
130	*	0.0	357.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
131	*	429.00	0.0	0.0	0.0	0.0	0.0	6.60	0.0	0.0	300.00
132	*	0.0	0.0	0.0	0.0	0.0	0.0	3200.00	0.0	0.0	0.0

133	*	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	288.00	0.0
134	*	0.0	0.0	0.0	429.00	0.0	0.0	0.0	0.0	0.0	0.0
135	*	0.0	0.0	4970.00	0.0	0.0	0.0	0.0	0.0		
136	*	14.40	0.0	0.0	0.0	0.0	0.0	0.0	432.00	0.0	79.30
137	*	0.0	830.00	0.0	0.0	360.00	0.0	77.00	0.0	0.0	91.70
138	*	0.0	453.00	0.0	0.0	0.0	0.0	77.00	0.0	40999.98	0.0
139	*	0.0	83.00	0.0	0.0	0.0	40999.98	3780.00	0.0	45199.96	0.0
140	*	0.0	44999.98	0.02	240.40	9.50	9.50	21.00	21.00	20.00	20.00
141	*	0.34	0.34	0.34	1.89	1.84	1.27	1.23	0.0	0.22	0.22
142	*	0.0	0.0	0.0	0.0	0.31	0.22	0.22	0.28	0.25	0.25
143	*	0.25	0.10	0.01	13.57	0.00	0.00	2.35	0.18	0.10	0.01
144	*	13.78	0.00	0.00	2.35	0.18	323.43	1.76	0.00	0.35	37.64
145	*	0.01	4.13	323.43	1.76	0.00	0.00	0.00	0.01	4.15	4.69
146	*	2.34	2.35	0.0	1.50	4.00	13.50	17.00	18.00	0.01	0.00
147	*	0.00	0.00	0.01	0.0	0.10	0.17	0.03	1.44	0.0	1.00
148	*	5.00	7.00	8.50	9.00	9.50	11.00	15.00	17.00	18.00	0.00
149	*	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00
150	*	0.02	0.07	0.69	0.74	0.12	0.63	0.34	0.57	0.19	2.46
151	*	1.46	0.20	0.00	0.0	0.0	1.00	2.00	3.50	4.50	13.00
152	*	14.00	16.50	17.50	13.00	0.0	0.00	0.0	0.00	0.0	0.00
153	*	0.0	0.00	0.0	0.01	0.02	0.00	0.00	0.00	0.02	0.00
154	*	0.14	0.01	0.0	0.50	1.50	4.50	5.50	6.50	7.50	8.00
155	*	10.00	10.50	11.50	12.50	13.50	16.50	17.50	18.00	0.0	0.00
156	*	0.0	0.00	0.0	0.00	0.0	0.00	0.0	0.00	0.0	0.00
157	*	0.0	0.00	0.0	0.00	0.00	0.00	0.01	0.00	0.05	0.01
158	*	0.07	0.01	0.11	0.00	0.02	0.00	0.04	0.00	75.50	5.80
159	*	0.38	36.50	17.20	4.50	75.50	6.00	2.20	36.50	1.30	2.20
160	*	3.00	1.20	3.00	1.20	53.00	0.65	0.40	46.60	15.50	3.80
161	*	53.00	8.00	1.52	46.60	0.80	1.52	76.50	50.60	76.50	50.60
162	*	2.60	2.20	13.10	0.07	2.55	0.10	0.06	0.04	0.01	0.0
163	*	0.01	0.05	0.07	0.10	0.01	0.17	0.67	0.25	0.42	0.36
164	*	0.25	0.05	0.22	0.52	0.52	0.47	0.47	0.52	0.20	0.07
165	*	0.10	0.06	0.05	0.01	0.0	0.01	0.04	0.05	0.10	0.05

166	*	0.35	0.87	0.40	0.25	0.67	0.17	0.01	0.08	0.26	0.52
167	*	0.47	0.47	0.52	0.32	0.22	0.10	0.02	0.02	0.10	0.34
168	*	0.57	0.54	0.05	0.0	0.05	0.10	0.06	0.01	0.01	0.06
169	*	0.10	0.35	0.03	0.15	0.03	0.35	0.18	0.01	0.0	0.01
170	*	0.18	0.10	0.03	0.04	0.01	0.0	0.01	0.05	0.10	27.00
171	*	4.86	24.00	9.00	9.00	7.25	0.60	19.90	2.19	10.80	9.30
172	*	9.30	9.32	6.00	0.10	0.05	0.01	0.0	0.01	0.04	0.07
173	*	0.10	0.80	7.25	4.00	3.00	21.71	6.40	16.33	5.70	8.67
174	*	8.40	8.40	9.77	2.88	9.83	0.11	0.11	0.07	0.10	0.10
175	*	0.07	0.00	0.10	0.10	0.10	0.06	0.05	0.04	0.03	0.01
176	*	0.01	0.04	0.05	0.06	0.07	0.10	0.0	1.62	0.0	4.92
177	*	0.0	7.11	0.0	6.30	0.0	1.60	0.0	0.01	1.06	0.17
178	*	2.39	0.67	0.10	0.86	3.70	0.25	1.27	0.07	0.10	0.06
179	*	0.04	0.01	0.01	0.03	0.04	0.05	0.06	0.10	0.0	4.12
180	*	0.0	5.14	0.0	4.92	0.0	1.62	0.0	0.05	2.73	0.87
181	*	0.10	0.67	2.39	0.17	1.07	0.01	0.10	0.02	0.01	0.01
182	*	0.02	0.10	0.0	2.37	0.0	2.37	0.0	0.34	0.93	0.57
183	*	0.93	0.34	0.10	0.06	0.05	0.01	0.00	0.00	0.01	0.05
184	*	0.06	0.10	0.0	3.20	0.0	1.20	0.0	1.20	0.0	3.20
185	*	0.0	0.35	1.63	0.07	0.21	0.15	0.21	0.03	1.63	0.35
186	*	0.10	0.08	0.07	0.05	0.04	0.01	0.00	0.01	0.04	0.05
187	*	0.10	0.0	221.43	0.0	191.43	0.0	165.00	17.50	0.0	66.50
188	*	0.0	27.00	161.21	4.36	100.57	24.00	0.75	9.87	7.25	57.17
189	*	0.60	0.10	0.05	0.04	0.00	0.01	0.04	0.05	0.05	0.07
190	*	0.10	0.0	64.50	0.0	144.82	0.0	155.14	0.0	99.33	0.0
191	*	0.80	36.27	7.25	0.01	21.71	62.97	6.40	58.17	16.33	0.0
192	*	0.0	0.0	0.0	0.0	0.0	0.20	0.20	0.20	5.00	5.00
193	*	5.00	0.0	0.0	0.0	0.0	0.0	0.0	14036.89	0.0	0.0
194	*	0.01	14036.89	0.0	0.0	0.0	436.50	3.22			
195	*	1	0.1000	2	0.0250	3	0.0250	4	0.0025	5	0.0025
196	*	6	0.0025	7	-25.0500	8	0.0	9	0.0	10	0.0
197	*	11	0.0	12	0.0	13	0.0	14	0.0	15	0.0
198	*	16	0.0	17	0.0	18	0.0	19	-5400.0000	20	0.0

199	*	21	-1155.0000	22	0.0	23	1360.0000	24	0.0	25	-1189.5000
200	*	26	0.0	27	-1245.0000	28	0.0	29	-186.0000	30	-519.0000
201	*	31	0.0	32	-168.0000	33	0.0	34	132.0000	35	0.0
202	*	36	1800.0000	37	0.0	38	0.0	39	0.0	40	0.0
203	*	41	385.0000	42	0.0	43	520.0000	44	0.0	45	0.0
204	*	46	0.0	47	0.0	48	0.0	49	1.2150	50	0.0
205	*	51	-292.5000	52	-83.8500	53	0.0	54	-310.5000	55	0.0
206	*	56	-5580.0000	57	0.0	58	396.5000	59	-666.0000	60	0.0
207	*	61	0.0	62	0.0	63	0.0	64	0.0	65	0.0
208	*	66	0.0	67	415.0000	68	0.0	69	0.0	70	0.0
209	*	71	132.0000	72	71.0000	73	0.0	74	319.0000	75	0.0
210	*	76	-2805.0000	77	0.0	78	-1189.5000	79	-49500.0000	80	0.0
211	*	81	296.0000	82	0.0	83	0.0	84	0.0	85	-2085.0000
212	*	86	0.0	87	-888.0000	88	0.0	89	0.0	90	0.0
213	*	91	-96.0000	92	0.0	93	-5040.0000	94	1380.0000	95	0.0
214	*	96	0.0	97	0.0	98	-1245.0000	99	0.0	100	0.0
215	*	101	-39900.0000	102	0.0	103	0.0	104	0.0	105	0.0
216	*	106	0.0	107	396.5000	108	0.0	109	200000.0000	110	0.0
217	*	111	0.0	112	0.0	113	-5400.0000	114	0.0	115	0.0
218	*	116	299.0000	117	0.0	118	-1402.5000	119	0.0	120	-1155.0000
219	*	121	0.0	122	0.0	123	-1189.5000	124	0.0	125	-609000.
220	*	126	0.0	127	-48000.0000	128	0.0	129	0.0	130	0.0
221	*	131	0.0	132	0.0	133	-29.4000	134	0.0	135	0.0
222	*	136	0.0	137	0.0	138	0.0	139	0.0	140	0.0
223	*	141	0.0	142	0.0	143	0.0	144	0.0	145	0.0
224	*	146	0.0	147	0.0	148	0.0	149	0.0	150	0.0
225	*	151	0.0	152	0.0	153	0.0	154	0.0	155	-361.5000
226	*	156	0.0	157	0.0	158	0.0	159	0.0	160	321.0000
227	*	161	0.0	162	0.0	163	0.0	164	0.0	165	0.0
228	*	166	0.0	167	0.0	168	0.0	169	4620.0000	170	0.0
229	*	171	0.0	172	0.0	173	0.0	174	0.0	175	-3.6450
230	*	176	0.0	177	0.0	178	-237.0000	179	0.0	180	0.0
231	*	181	0.0	182	0.0	183	0.0	184	0.0	185	-15600.0000

232	*	186	0.0	187	0.0	188	0.0	189	0.0	190	0.0
233	*	191	0.0	192	0.0	193	0.0	194	0.0	195	0.0
234	*	196	0.0	197	321.0000	198	0.0	199	0.0	200	0.0
235	*	201	0.0	202	-5505.0000	203	0.0	204	0.0	205	0.0
236	*	206	0.0	207	0.0	208	0.0	209	0.0	210	0.0
237	*	211	-6435.0000	212	0.0	213	0.0	214	0.0	215	0.0
238	*	216	0.0	217	-99.0000	218	0.0	219	0.0	220	-4500.0000
239	*	221	0.0	222	0.0	223	0.0	224	0.0	225	0.0
240	*	226	0.0	227	-48000.0000	228	0.0	229	0.0	230	0.0
241	*	231	0.0	232	0.0	233	0.0	234	0.0	235	0.0
242	*	236	0.0	237	0.0	238	0.0	239	1440.0000	240	0.0
243	*	241	0.0	242	0.0	243	0.0	244	-6435.0000	245	0.0
244	*	246	0.0	247	0.0	248	0.0	249	0.0	250	0.0
245	*	251	0.0	252	0.0	253	-74550.0000	254	0.0	255	0.0
246	*	256	0.0	257	0.0	258	0.0				
247	*	1	-216.0000	2	0.0	3	0.0	4	0.0	5	0.0
248	*	6	0.0	7	0.0	8	-6480.0000	9	0.0	10	396.5000
249	*	11	0.0	12	4150.0000	13	0.0	14	0.0	15	-5400.0000
250	*	16	0.0	17	-1155.0000	18	0.0	19	0.0	20	453.5000
251	*	21	0.0	22	-6795.0000	23	0.0	24	0.0	25	0.0
252	*	26	0.0	27	-1155.0000	28	0.0	29	-615000.	30	0.0
253	*	31	0.0	32	415.0000	33	0.0	34	0.0	35	0.0
254	*	36	-615000.	37	19900.0000	38	0.0	39	226000.0000	40	0.0
255	*	41	0.0	42	225000.0000	43	0.0865	44	1202.0000	45	46.5000
256	*	46	46.5000	47	105.0000	48	105.0000	49	100.0000	50	100.0000
257	*	51	-5.0550	52	-5.0550	53	-5.0550	54	9.4500	55	-27.6000
258	*	56	6.3750	57	-1[illegible].4500	58	0.0	59	1.1050	60	-3.3150
259	*	61	0.0	62	0.0	63	0.0	64	0.0	65	1.5400
260	*	66	-3.5150	67	-3.3150	68	-4.2600	69	-3.7500	70	-3.6900
261	*	71	-3.4200	72	0.5000	73	-0.2250	74	91.8487	75	0.0071
262	*	76	-0.0048	77	11.7576	78	-2.6409	79	0.5000	80	0.0750
263	*	81	69.8865	82	0.0071	83	0.0016	84	11.7376	85	0.9803
264	*	86	1617.1614	87	8.8032	88	-0.0053	89	1.7606	90	188.1974

265	*	91	0.0375	92	20.6655	93	1617.1614	94	8.8032	95	-0.0053
266	*	96	-0.0000	97	0.0018	98	-0.1125	99	-61.9966	100	-70.4250
267	*	101	-35.1000	102	-35.3250	103	0.0	104	7.5000	105	20.0000
268	*	106	67.5000	107	85.0000	108	90.0000	109	0.0400	110	-0.0120
269	*	111	-0.0268	112	-0.0043	113	0.0400	114	0.0	115	0.6600
270	*	116	0.9575	117	-0.4710	118	-21.6000	119	0.0	120	5.0000
271	*	121	25.0000	122	35.0000	123	42.5000	124	45.0000	125	47.5000
272	*	126	55.0000	127	65.0000	128	85.0000	129	90.0000	130	0.0025
273	*	131	0.0175	132	0.0075	133	-0.1098	134	-0.0300	135	0.0100
274	*	136	0.0366	137	-0.0225	138	-0.0487	139	-0.0150	140	0.0250
275	*	141	-0.3750	142	0.3750	143	3.4650	144	1.2000	145	-1.8000
276	*	146	-9.4050	147	1.7250	148	2.8625	149	0.9500	150	12.2950
277	*	151	7.3200	152	0.9970	153	0.0000	154	0.0	155	0.0
278	*	156	5.0000	157	10.0000	158	17.5000	159	22.5000	160	65.0000
279	*	161	70.0000	162	82.5000	163	87.5000	164	90.0000	165	0.0
280	*	166	-0.0132	167	0.0	168	-0.0015	169	0.0	170	0.0007
281	*	171	0.0	172	0.0041	173	0.0	174	0.0400	175	0.0840
282	*	176	-0.0120	177	0.0153	178	-0.0268	179	-0.3193	180	-0.0043
283	*	181	-2.0561	182	0.0400	183	0.0	184	2.5000	185	7.5000
284	*	186	22.5000	187	27.5000	188	32.5000	189	37.5000	190	40.0000
285	*	191	50.0000	192	52.5000	193	57.5000	194	62.5000	195	67.5000
286	*	196	82.5000	197	87.5000	198	90.0000	199	0.0	200	0.0015
287	*	201	0.0	202	-0.0030	203	0.0	204	-0.0132	205	0.0
288	*	206	0.0037	207	0.0	208	-0.0132	209	0.0	210	-0.0025
289	*	211	0.0	212	0.0011	213	0.0	214	0.0025	215	-0.0150
290	*	216	0.0175	217	0.0625	218	0.0075	219	0.2941	220	-0.1098
291	*	221	-0.9882	222	0.0066	223	0.5496	224	-0.0225	225	0.1019
292	*	226	-0.0467	227	-0.6035	228	-0.0150	229	377.5000	230	-37.0000
293	*	231	-5.7000	232	132.5000	233	-258.0000	234	-67.5000	235	377.5000
294	*	236	30.0000	237	11.0000	238	182.5000	239	-19.5000	240	11.0000
295	*	241	-45.0000	242	-18.0000	243	-45.0000	244	-18.0000	245	265.0000
296	*	246	-9.7500	247	-6.0750	248	234.0000	249	-232.5000	250	-57.0000
297	*	251	265.0000	252	40.0000	253	7.5000	254	234.0000	255	-12.0000

298	*	256	7.6000	257	-1147.5000	258	-459.0000	259	382.5000	260	153.0000
299	*	261	13.0000	262	11.0000	263	65.5000	264	-0.3150	265	-38.2500
300	*	266	-1.5000	267	-0.9000	268	-0.6000	269	-0.1500	270	0.0
301	*	271	0.0500	272	0.2500	273	0.3500	274	0.5000	275	0.0625
302	*	276	0.8750	277	3.3335	278	1.2500	279	-6.3000	280	-12.9750
303	*	281	-3.5250	282	-0.7500	283	1.1125	284	1.6000	285	2.5833
304	*	286	2.3750	287	2.3750	288	2.5850	289	1.0225	290	0.3750
305	*	291	-1.5000	292	-0.9000	293	-0.7500	294	-0.1500	295	0.0
306	*	296	0.0500	297	0.2000	298	0.3000	299	0.5000	300	0.2500
307	*	301	1.7500	302	4.3750	303	2.0000	304	-3.7500	305	-10.0005
308	*	306	-2.6250	307	-0.1375	308	0.4000	309	1.3000	310	2.6125
309	*	311	2.3750	312	2.3750	313	2.5833	314	1.6000	315	1.1125
310	*	316	-1.5000	317	-0.3000	318	0.1000	319	0.5000	320	-5.0625
311	*	321	-2.6250	322	-5.0625	323	0.2375	324	0.0	325	-0.7125
312	*	326	-1.5000	327	-0.9000	328	-0.1500	329	0.0500	330	0.3000
313	*	331	0.5000	332	-5.2500	333	-0.4500	334	-2.2500	335	-0.4500
314	*	336	-5.2500	337	-2.7000	338	0.0600	339	0.0	340	-0.1600
315	*	341	0.9000	342	-1.5000	343	-1.2000	344	-0.6750	345	-0.1500
316	*	346	0.0	347	0.0500	348	0.2500	349	0.5000	350	135.0000
317	*	351	24.2857	352	120.0000	353	45.0000	354	-135.0000	355	-108.7500
318	*	356	-9.0000	357	99.5000	358	10.9286	359	54.0000	360	46.5000
319	*	361	46.5000	362	45.6250	363	30.0000	364	-1.5000	365	-0.7500
320	*	366	-0.1500	367	0.0	368	0.0500	369	0.2250	370	0.3500
321	*	371	0.5000	372	-12.0000	373	-108.7500	374	-60.0000	375	40.0000
322	*	376	108.5715	377	52.0000	378	81.6666	379	-85.5000	380	-150.1250
323	*	381	-120.0000	382	-120.0000	383	-146.5714	384	-43.2000	385	-147.4999
324	*	386	0.5300	387	0.5550	388	-1.0440	389	0.4200	390	0.5200
325	*	391	-1.0440	392	0.0000	393	-1.5000	394	0.5000	395	-1.5000
326	*	396	-0.9750	397	-0.3250	398	-0.6750	399	-0.5250	400	-0.2250
327	*	401	0.0750	402	0.2250	403	0.2750	404	0.3250	405	0.3750
328	*	406	0.5000	407	0.0	408	8.1250	409	0.0	410	24.5850
329	*	411	0.0	412	-75.5850	413	0.0	414	31.5000	415	0.0
330	*	416	0.0000	417	0.0	418	0.0625	419	5.2887	420	0.8750

331	*	421	11.9384	422	5.3335	423	-1.4875	424	-12.9750	425	-55.5000
332	*	426	-5.5250	427	-19.1250	428	-1.1250	429	-1.5000	430	-0.9750
333	*	431	-0.6750	432	-0.2250	433	0.0750	434	0.1750	435	0.2250
334	*	436	0.2750	437	0.3250	438	0.5000	439	0.0	440	20.6250
335	*	441	0.0	442	-77.0850	443	0.0	444	24.5850	445	0.0
336	*	446	3.1250	447	0.0	448	0.2500	449	13.0563	450	4.3750
337	*	451	0.5207	452	-10.0005	453	-35.8147	454	-2.6250	455	-16.0313
338	*	456	-0.1875	457	-1.5000	458	-0.3750	459	-0.2250	460	0.0750
339	*	461	0.1250	462	0.5000	463	0.0	464	-35.6250	465	0.0
340	*	466	11.8750	467	0.0	468	-5.0625	469	-13.9688	470	-3.6250
341	*	471	-13.9688	472	-5.0625	473	-1.5000	474	-0.9750	475	-0.8250
342	*	476	-0.2250	477	-0.0750	478	0.0250	479	0.0750	480	0.2750
343	*	481	0.3250	482	0.5000	483	0.0	484	16.0000	485	0.0
344	*	486	-12.0000	487	0.0	488	6.0000	489	0.0	490	-48.0000
345	*	491	0.0	492	-5.2500	493	8.1500	494	-0.4500	495	-3.1500
346	*	496	-2.2500	497	-3.1500	498	-0.4500	499	8.1500	500	-5.2500
347	*	501	-1.5000	502	-1.2750	503	-1.1250	504	-0.7500	505	-0.6000
348	*	506	-0.2250	507	-0.0250	508	0.0750	509	0.2250	510	0.2750
349	*	511	0.5000	512	0.0	513	-3321.4346	514	0.0	515	957.1448
350	*	516	0.0	517	-2475.0000	518	87.5000	519	0.0	520	332.5000
351	*	521	0.0	522	135.0000	523	-2418.2097	524	24.2857	525	502.8569
352	*	526	120.0000	527	-11.2500	528	-148.1250	529	-108.7500	530	-557.6250
353	*	531	-9.0000	532	-1.5000	533	-0.3250	534	-0.6750	535	-0.0750
354	*	536	0.0750	537	0.2000	538	0.2500	539	0.3250	540	0.3750
355	*	541	0.5000	542	0.0	543	-967.5000	544	0.0	545	724.1074
356	*	546	0.0	547	-2297.1448	548	0.0	549	496.6665	550	0.0
357	*	551	-12.0000	552	-544.1250	553	-106.7500	554	-0.1301	555	108.5715
358	*	556	414.3574	557	32.0000	558	-872.5005	559	81.6666	560	0.0
359	*	561	0.0	562	0.0	563	0.0	564	0.0	565	0.0
360	*	566	1.0000	567	1.0000	568	1.0000	569	25.0000	570	25.0000
361	*	571	25.0000	572	0.0	573	0.0	574	0.0	575	0.0
362	*	576	0.0	577	0.0	578	70184.5000	579	0.0	580	0.0
363	*	581	0.0667	582	70184.5000	583	0.0	584	0.0	585	0.0

```
*  586, 2181.5000  587     16.0863
*    7
* BLOCK
* 1150   4   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
*    1   1
*    3   1   4   1   9   1
* 0.       1.
*   47   1   1
*    1   1
*    1   1
*
*
*
*
*
*     91 0.0          92 2.5           93 5.0           94 7.5            95 10.0
*     96 12.5         97 15.0          98 17.5          99 20.0          101 0.0
*    102 25.0
*     91 0.1          92 0.05          93 0.05          94 0.05           95 0.05
*     96 0.05         97 0.05          98 0.05          99 0.05          101 0.02
*    102 -0.02
*
*
*
*
*      1 1.
*    1   1   2   1   3   1   4   0   5   1
* BLOCK
*    1   1
*    1   1
*    1   1
*      1 1.
* PASS
```

```
397  *     1   1
398  *     1   1
399  *     1 229
400  *      1 1.
401  *  PASS
402  *     1   1
403  *     1   1
404  *     1 230
405  *      1 1.
406  *  PASS
407  *     1   1
408  *     1   1
409  *     1 231
410  *      1 1.
411  *  PASS
412  *  2150  11   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
413  *     1   1   4   1
414  *     4   1   9   1  19   1  .3   1
415  *  0.       1.
416  *    47   1   1
417  *     1   1
418  *     1   1
419  *
420  *
421  *
422  *
423  *
424  *     91 0.0          92 2.5          93 5.0          94 7.5          95 10.0
425  *     96 12.5         97 15.0         98 17.5         99 20.0        101 0.0
426  *    102 25.0
427  *     91 0.1          92 0.05         93 0.05         94 0.05         95 0.05
428  *     96 0.05         97 0.05         98 0.05         99 0.05        101 0.02
429  *    102 -0.02
```

```
*
*
*
*
*
*    1   1   2   1   3   1   4   1   5   0
*  BLOCK
*    1   1
*    1   1
*    1   1
*     1 1.          3 0.1           4 0.01          5 0.01
*  PASS
*    1   1
*    1   1
*    1 229
*     1 1.          3 0.1           4 0.01          5 0.01
*  PASS
*    1   1
*    1   1
*    1 230
*     1 1.          3 0.1           4 0.01          5 0.01
*  PASS
*    1   1
*    1   1
*    1 231
*     1 1.          3 0.1           4 0.01          5 0.01
*  PASS
*    1   1
*    1   1
*    1 578
*     1 1.          3 0.1           4 0.01          5 0.01
*  PASS
*    1   1
```

```
463  *     1   1
464  *     1 581
465  *      1 1.                3 0.1              4 0.01              5 0.01
466  *  PASS
467  *     1   1
468  *     1   1
469  *     1 582
470  *      1 1.                3 0.1              4 0.01              5 0.01
471  *  PASS
472  *     0   2
473  *     1   1
474  *     1  37   2 229
475  *      1 1.                3 0.1              4 0.01              5 0.01
476  *  PASS
477  *     0   2
478  *     1   1
479  *     1   1   2  22
480  *      1 1.                3 0.1              4 0.01              5 0.01
481  *  PASS
482  *     2   0
483  *     1   1   2  70
484  *     1   1
485  *      1 1.                3 0.1              4 0.01              5 0.01
486  *  PASS
487  *     2   0
488  *     1   1   2  73
489  *     1   1
490  *      1 1.                3 0.1              4 0.01              5 0.01
491  *  PASS
492  *  3150  15   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
493  *     1   1   3   1   5   1
494  *     4   1   9   1  19   1   2   1   3   1
495  *  0.       1.
```

```
496  *    47    1    1
497  *     1    1
498  *     1    1
499  *
500  *
501  *
502  *
503  *
504  *     91 0.0         92 2.5         93  5.0        94 7.5         95 10.0
505  *     96 12.5        97 15.0        98 17.5        99 20.0       101 0.0
506  *    102 25.0
507  *     91 0.1         92 0.05        93  0.05       94 0.05        95 0.05
508  *     96 0.05        97 0.05        98  0.05       99 0.05       101 0.02
509  *    102 -0.02
510  *
511  *
512  *
513  *
514  *
515  *     1    1    2    1    3    1    4    1    5    0
516  *  BLOCK
517  *     1    1
518  *     1    1
519  *     1    1
520  *      1 1.          3 0.1          4 0.01          5 0.01
521  *  PASS
522  *     1    1
523  *     1    1
524  *     1  229
525  *      1 1.          3 0.1          4 0.01          5 0.01
526  *  PASS
527  *     1    1
528  *     1    1
```

```
529  *      1 230
530  *       1 1.            3 0.1            4 0.01            5 0.01
531  *   PASS
532  *      1   1
533  *      1   1
534  *      1 231
535  *       1 1.            3 0.1            4 0.01            5 0.01
536  *   PASS
537  *      1   1
538  *      1   1
539  *      1  15
540  *       1 1.            3 0.1            4 0.01            5 0.01
541  *   PASS
542  *      1   1
543  *      1   1
544  *      1  42
545  *       1 1.            3 0.1            4 0.01            5 0.01
546  *   PASS
547  *      1   1
548  *      1   1
549  *      1  29
550  *       1 1.            3 0.1            4 0.01            5 0.01
551  *   PASS
552  *      1   1
553  *      1  13
554  *      1  17
555  *       1 1.            3 0.1            4 0.01            5 0.01
556  *   PASS
557  *      1   1
558  *      1  15
559  *      1  27
560  *       1 1.            3 0.1            4 0.01            5 0.01
561  *   PASS
```

```
562  *      1    1
563  *      1   43
564  *      1  578
565  *        1 1.                3 0.1                4 0.01               5 0.01
566  *   PASS
567  *      2    0
568  *      1   46     2    53
569  *      1    1
570  *        1 1.                3 0.1                4 0.01               5 0.01
571  *   PASS
572  *      2    0
573  *      1   48     2    65
574  *      1    1
575  *        1 1.                3 0.1                4 0.01               5 0.01
576  *   PASS
577  *      2    0
578  *      1   88     2    95
579  *      1    1
580  *        1 1.                3 0.1                4 0.01               5 0.01
581  *   PASS
582  *      1    1
583  *      1   95
584  *      1  581
585  *        1 1.                3 0.1                4 0.01               5 0.01
586  *   PASS
587  *      1    1
588  *      1    1
589  *      1   15
590  *        1 1.                3 0.1                4 0.01               5 0.01
591  *   PASS
592  *   4150    4    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
593  *      1    1    0    1    5    1    4    1
594  *      4    1    0    1   19    1    2    1    3    1
```

```
*  0.        1.
*     47     1     1
*      1     1
*      1     1
*
*
*
*
*
*     91 0.0            92 2.5            93 5.0            94 7.5            95 10.0
*     96 12.5           97 15.0           98 17.5           99 20.0          101 0.0
*    102 25.0
*     91 0.1            92 0.05           93 0.05           94 0.05           95 0.05
*     96 0.05           97 0.05           98 0.05           99 0.05          101 0.02
*    102 -0.02
*
*
*
*
*
*      1     1     2     1     3     1     4     1     5     0
*  BLOCK
*      1     1
*      1     1
*      1     1
*       1 1.             3 0.1             4 0.01            5 0.01
*  PASS
*      1    '1
*      1     1
*      1  229
*       1 1.             3 0.1             4 0.01            5 0.01
*  PASS
*      1     1
```

```
*      1    1
*      1  230
*       1 1.            3 0.1              4 0.01              5 0.01
*   PASS
*      1    1
*      1    1
*      1  231
*       1 1.            3 0.1              4 0.01              5 0.01
*   PASS
*   5150    2    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
*      1    1    3    1    5    1    4    1    6    1
*      4    1    9    1   19    1    2    1    3    1
*   0.        1.
*     47    1    1
*      1    1
*      1    1
*
*
*
*
*
*      91 0.0          92 2.5             93 5.0              94 7.5              95 10.0
*      96 12.5         97 15.0            98 17.5             99 20.0            101 0.0
*     102 25.0
*      91 0.1          92 0.05            93 0.05             94 0.05             95 0.05
*      96 0.05         97 0.05            98 0.05             99 0.05            101 0.02
*     102 -0.02
*
*
*
*
*       3 0.1           5 0.01             6 0.01
*      1    1    2    1    3    1    4    1    5    0
```

```
BLOCK
    1     1
    1     1
    1     1
      1 1.                3 0.1               4 0.01              5 0.01
PASS
    1     1
    1     1
    1   229
      1 1.                3 0.1               4 0.01              5 0.01
PASS
6150      2     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0
    1     1     2     1     3     1     4     1     5     1     6     1
    4     1     9     1    19     1     2     1     3     1
0.         1.
   47     1     1
    1     1
    1     1
     91 0.0               92 2.5              93 5.0              94 7.5              95 10.0
     96 12.5              97 15.0             98 17.5             99 20.0            101 0.0
    102 25.0
     91 0.1               92 0.05             93 0.05             94 0.05             95 0.05
     96 0.05              97 0.05             98 0.05             99 0.05            101 0.02
    102 -0.02
```

```
*
*     1    1    2    1    3    1    4    1    5    0
*   BLOCK
*     1    1
*     1    1
*     1    1
*      1 1.                2 0.1                3 0.1                4 0.01               5 0.01
*      6 0.01
*   PASS
*     1    1
*     1    1
*     1  229
*      1 1.                2 0.1                3 0.1                4 0.01               5 0.01
*      6 0.01
*   PASS
*   3150   15    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
*     1    1    3    1    5    1
*     4    1    9    1   19    1    2    1    3    1    6    1
*   0.        1.
*    47   . 1    1
*     1    1
*     1    1
*
*
*      1 2.5
*      1 0.25
*
*     91 0.               92 5.                93 10.               94 15.               95 20.
*    101 0.              102 25.
*     91 0.1              92 0.1               93 0.1               94 0.1               95 0.1
*    101 0.02            102 -0.02
*
*
```

```
*
*
*
*     1    1    2    1    3    1    4    1    5    0
*   BLOCK
*     1    1
*     1    1
*     1    1
*       1 1.                3 0.1                4 0.01               5 0.01
*   PASS
*     1    1
*     1    1
*     1  229
*       1 1.                3 0.1                4 0.01               5 0.01
*   PASS
*     1    1
*     1    1
*     1  230
*       1 1.                3 0.1                4 0.01               5 0.01
*   PASS
*     1    1
*     1    1
*     1  231
*       1 1.                3 0.1                4 0.01               5 0.01
*   PASS
*     1    1
*     1    1
*     1  15
*       1 1.                3 0.1                4 0.01               5 0.01
*   PASS
*     1    1
*     1    1
*     1   42
```

```
760  *       1 1.            3 0.1            4 0.01            5 0.01
761  *  PASS
762  *      1   1
763  *      1   1
764  *      1  29
765  *       1 1.            3 0.1            4 0.01            5 0.01
766  *  PASS
767  *      1   1
768  *      1  13
769  *      1  17
770  *       1 1.            3 0.1            4 0.01            5 0.01
771  *  PASS
772  *      1   1
773  *      1  15
774  *      1  27
775  *       1 1.            3 0.1            4 0.01            5 0.01
776  *  PASS
777  *      1   1
778  *      1  43
779. *      1 578
780  *       1 1.            3 0.1            4 0.01            5 0.01
781  *  PASS
782  *      2   0
783  *      1  46   2  53
784  *      1   1
785  *       1 1.            3 0.1            4 0.01            5 0.01
786  *  PASS
787  *      2   0
788  *      1  48   2  25
789  *      1   1
790  *       1 1.            3 0.1            4 0.01            5 0.01
791 .*  PASS
792  *      2   0
```

```
*      1   88    2   95
*      1    1
*        1 1.                3 0.1              4 0.01              5 0.01
*   PASS
*      1    1
*      1    1
*      1   15
*        1 1.                3 0.1              4 0.01              5 0.01
*   PASS
*      1    1
*      1   95
*      1  561
*        1 1.                3 0.1              4 0.01              5 0.01
*   PASS
*
*
*
*
*
*
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SECTION 9

BIBLIOGRAPHY AND BIOGRAPHY (B)

"THE AIM OF PRINCES AND PHILOSOPHERS IS TO IMPROVE."

GOTTFRIED WILHELM LEIBNITZ (1702)

THIS SECTION PRESENTS A COMBINED BIBLIOGRAPHY OF 276 REFERENCES AND A BIOGRAPHICAL NOTE CONCERNING THE AUTHOR. THE BIBLIOGRAPHY COVERS ALL OF THE AREAS DISCUSSED AND UTILIZED IN THIS THESIS. AN OUTLINE OF MOST OF THE PERTINENT BIBLIOGRAPHY AREAS IS PRESENTED BELOW.

- MODERN CONTROL THEORY
  - ADAPTIVE CONTROL
  - SYSTEM IDENTIFICATION
  - ESTIMATION AND FILTERING THEORY
- OCEAN ENGINEERING
  - DYNAMICS OF OCEAN VEHICLE MOTIONS
  - VEHICLE MATHEMATICAL MODELING
  - VEHICLE EQUATIONS OF MOTION
  - MOTION CONTROL OF OCEAN VEHICLES

CHAPTER B1

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CHAPTER B2

BIOGRAPHICAL NOTE

Lt. Michael N. Hayes was born in Abilene, Texas, on September 2, 1941. Four years later the family moved to Ohio, where he attended primary schools in Wakeman and in Massillon. He was graduated from Washington High School, Massillon, Ohio, in June 1959.

Mr. Hayes enlisted as a Seaman Recruit in the United States Navy in September, 1959. He was graduated from the U.S. Naval Guided Missiles School, Dam Neck, Virginia, in July, 1960, and was assigned to the U.S.S. Kitty Hawk (CVA-63) until June, 1961. He was graduated from the U.S. Naval Preparatory School, Bainbridge, Maryland, in August, 1961.

In September, 1961, Mr. Hayes began 4 years of undergraduate education at North Carolina State University, Raleigh, N.C., under the auspices of the Naval Enlisted Scientific Education Program (NESEP). He was graduated in June, 1965, with the degrees of Bachelor of Science in both Electrical Engineering and Applied Mathematics and with the degree of Master of Electrical Engineering.

After attending the U.S. Naval Officer Candidate School at Newport, R.I., Mr. Hayes was commissioned an Ensign in the U.S. Navy in October, 1965, and assigned for duty at the Boston Naval Shipyard, as a Ship Superintendent, supervising naval ship construction and repair, and attended evening classes at Boston University in business administration and at Northeastern University in pure mathematics.

In June, 1967, Mr. Hayes entered M.I.T. as a U.S. Naval Post-graduate student in Curriculum 13A with specialized studies in electrical engineering. After completing coursework and a thesis entitled "Dynamic Models of Binary Plate Distillation Columns," he was awarded the degrees of Master of Science in Electrical Engineering and Electrical Engineer in September, 1969. He is a member of Sigma Xi, Tau Beta Pi, Eta Kappa Nu, Phi Eta Sigma, and is an Eagle Scout.

Lt. Hayes is married to the former Ann Marie Graney of Stoughton, Massachusetts. They have a three-year-old son, Stephen, and a one-year-old son, Kevin. Lt. Hayes is the son of Mr. and Mrs. Ross N. Hayes of Massillon, Ohio. He has a brother, James, and sisters, Ellen and Roxy.

Thesis
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Parametric identification of nonlinear stochastic systems applied to ocean vehicle dynamics.